A Vibert Douglas "Immensities of Time and Space" Publication #102

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IMMENSITIES OF TIME AND SPACE

BY

A. VIBERT DOUGLAS, M. B. E., M. Sc.

FROM THE SMITHSONIAN REPORT FOR 1925, PAGES 147-155

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IMMENSITIES OF TIME AND SPACE[1]

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Within the last twelve months three workers in the realms of mathematical astronomy and cosmogony have brought their researches to conclusions of tremendous importance and widespread interest. So closely are their problems interwoven that it may prove stimulating to consider in some detail the aim of each investigation, the line of argument, and the outstanding results thus far achieved.

STAR'S MASS AND LUMINOSITY

Last year there was given to the Royal Astronomical Society by Prof. A. S. Eddington, F. R. S., a paper which has aroused great interest among astronomers the world over. For many years Professor Eddington has been investigating the radiative properties of a giant star—that is, a star of gigantic size and low density, so low that it could be considered as obeying what in physics are known as the "perfect gas" laws. These investigations led him to the conclusion that the total luminosity of such a star depended chiefly upon its mass and temperature, being almost uninfluenced by other factors. To test the validity of this formula relating luminosity to mass, he first evaluated the constants involved in it from the known values of mass and luminosity of the bright star Capella. He then plotted his relation as shown in the accompanying graph by the curved line. Next he collected all the available data from every possible source, giving both the masses and luminosities of stars and these he plotted individually on his graph. Their close

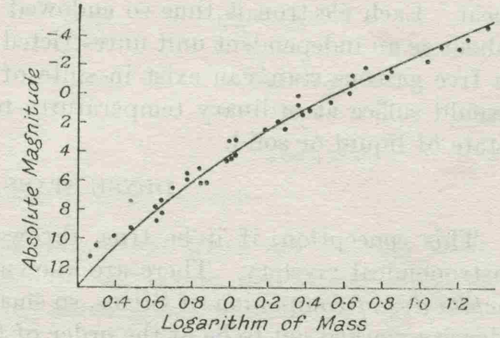

Fig. 1.—Diagram of luminosity—mass relation.

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proximity to the theoretical curve is very striking, and forms a strong confirmation of the validity of the theoretical work.

Curiosity, we suppose, led him to wonder where the points would lie representing similar data for the dwarf stars. Now the dwarf stars were so named by Prof. Henry Norris Russell because they are so dense, due to gradual contraction, that it was thought impossible that the gas laws could hold true in their case. Hence a relation between mass and luminosity explicitly based upon strict adherence to the gas laws would not be expected to hold good for a star of the dwarf type. But, *mirabile visu*, the plotted points for data taken from dwarf stars conform as well to the theoretical curve as in the case of the giants.

This was an astounding fact suggesting as it did that the dwarf star, no matter what its density, was in the state of atomic agitation of a perfect gas. The explanation given by Eddington seems plausible, though it has not been accepted unanimously—namely, that where matter is subject to such high temperatures as exist in the stars, temperatures to be measured in millions of degrees, each atom is reduced in effective volume a hundred thousand times since its revolving electrons, whose outermost orbits determine its normal effective volume, are all stripped off or ionized by the intensity of heat. Each electron is thus so endowed with energy that it moves about as an independent unit unrestricted to any atomic orbit. Thus a free gaseous state can exist in spite of much closer packing than would suffice at ordinary temperatures to reduce the matter to the state of liquid or solid.

DENSE STARS

This conception, if it be true, throws light on a long-standing astronomical mystery. There are known to be a few stars, like the *white dwarf* companion of Sirius, so small yet so massive that their density worked out to be of the order of fifty thousand times that of water—an absurdity it was thought, an impossible result, something radically wrong somewhere in the observations or calculations. But instead of finding something wrong with the calculations, Eddington's work suggests that the trouble lay in our thinking the result absurd, in our failure to realize the tremendous difference between the state of matter at terrestrial temperatures and at stellar temperatures.

GIANTS AND DWARFS

If, then, all the stars are in a true gaseous condition, it is necessary to modify all ideas and calculations based on the old point of view that the dwarf stars were not obeying the same laws of pressure,

volume, and temperature as the giant stars. Prof. H. N. Russell's famous Giant and Dwarf theory of Stellar Evolution suggested that a star begins its career as a very large mass of gas, highly inflated and much less dense than air. This would slowly contract by gravitational influence, growing hotter and hotter and brighter and brighter. This period would embrace its life as a giant star.

When contraction had reached such a point that, upon the old view, the gas laws could no longer be considered as even approximately representing the state of the star, then at this critical point of balance between mass, density, and temperature, further gravitational contraction would of necessity be accompanied by decrease of temperature and of luminosity. This period constituted the dwarf stage of a star's career. Now this theory is thrown into a new light by Eddington's results. If the mass of a star be constant throughout

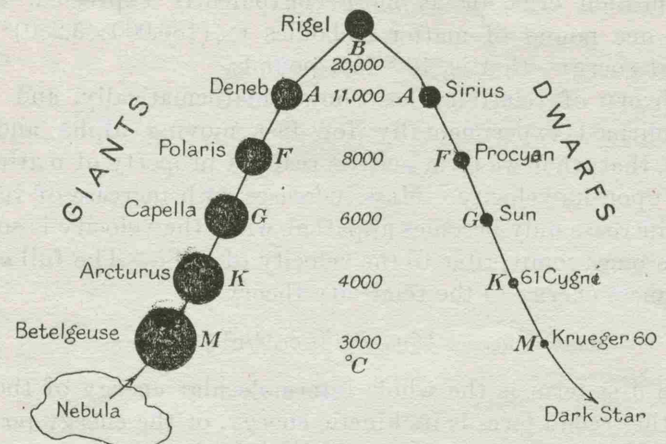

Fig. 2.—Diagram of stellar evolution

its life, Eddington's formula shows that there can be very little alteration in luminosity in spite of contraction, and hence the evolution of a single star as outlined above is an impossibility. If as an alternative interpretation the mass of a star is gradually diminishing, being actually consumed away to maintain the supply of energy which a star is continuously radiating, then Russell's theory of evolution may be retained as indicating the probable sequence of stellar change, but with this difference that, though the effective or surface temperature may decrease during the latter stages, the internal temperature will continue to rise.

LOSS OF MASS

A great deal thus turns on whether the mass of a radiating star can actually be considered to be diminishing. There are, furthermore, two ways of looking at this question.

One suggestion is that by collision of a very infrequent type between an electron and a proton (the ultimate particles of matter known to the physicist, carrying the unit electrical charges, negative and positive) their impact results in complete annihilation of their mass—that is, of the matter which they form—a definite amount of radiant energy being the equivalent result. This is highly speculative, there being no observed phenomenon in nature to prove definitely that such a transformation can take place; but it is theoretically possible, for matter and energy are essentially the same thing. Matter is one of many forms of energy and in the usual units of measurement of energy, the energy equivalent of a mass m grams of matter is $m\,c^2$ ergs where c is the velocity of light, 3×10^{10} cm. per second. In other words, one gram of matter represents a potential concentration of energy equivalent to nine hundred million million million ergs, or as more conveniently expressed, 9×10^{20} ergs; or one pound of matter embodies $1\times(186{,}000\times5{,}280)^2$ foot-pounds of energy—that is, 10^{18} foot-pounds.

The theory of relativity has shown mathematically, and it has been confirmed experimentally for fast moving alpha and beta particles, that what we term mass is really a property of matter that depends upon its velocity. Mass increases with increase of velocity, but this increase only becomes apparent when the velocity is so great that it becomes comparable to the velocity of light. The full expression for mass energy in the relativity theory is

$$mc^2 = m_0c^2 + \tfrac{1}{2}m_0v^2 + \tfrac{3}{8}m_0v^4/c^2 + \text{———}$$

Here the first term is the whole intermolecular energy of the mass at rest; the second term is its kinetic energy, or the energy possessed by it in virtue of its motion; the third and subsequent terms will be quite negligible for small values of the velocity v of the mass, but become appreciable one by one as the velocity v approaches the velocity of light. From this it follows that in order for the mass of a star to diminish as a result (or perhaps we should say as the cause) of its radiation, it is not necessary to postulate the annihilation of matter, but simply the loss in mass resulting from loss in velocity. From the above equation it can be seen that this might well be a tremendous reservoir of available energy.

The idea that continued stellar radiation might imply gradual loss in stellar mass was not new. It had been suggested some years ago by Dr. J. H. Jeans, but the facts above explained brought this idea forward with a new significance. Doctor Jeans saw that if this idea be really true, many astronomical problems are seen in a new light, and many investigations require reconsideration. This he has done and only recently has he published his results.

AGE OF SUN

One of the questions which has provoked controversy between geologist, physicist, and astronomer for many years is the probable age of the earth, an estimate of the age of the sun being, of course, an upper limit to the age of the solar system. Jeans attacks the problem from the new point of view, and his argument is as follows: The sun is radiating away its mass at the rate of about 4,200,000 tons per second and, if it were once as massive a star as Sirius now is, then it has been radiating for 7×10^{12} years. This means an age of at least a million million years, and is several thousand times greater than any previous estimate—a figure so great that it baffles comprehension, and staggers even the imagination.

GALAXY EXPANDING

There is an interesting consequence of loss of mass by radiation which has an important bearing upon our system and upon the galaxy of stars about us. If our sun be gradually diminishing in mass, the law of conservation of momentum requires that the planets move gradually off in ever expanding orbits. Similarly, if the mass of our galaxy as a whole be gradually diminishing, the stars must be opening out, spreading farther apart from the common center of gravity and therefore from one another. Jeans estimates that 10^{12} years ago this galaxy was packed sixty-four times more closely than it is at the present time.

BINARY STARS

This modifies various problems of cosmogony in a remarkable manner. The orbits of binary stars have long been a mystery, because no mutual force between two such stars was known which could account for their being in such eccentric and large orbits about their common center of gravity. Jeans points out that it is no longer necessary to look for such a force, that with the enlarged time scale for the galaxy (greater than 10^{10} years) outside influences become not only possible but very highly probable—that is to say, the normal orbits of a binary system may be perturbed by the gravitational pull of a passing star approaching more closely than is usual. The chance of such an influence being brought to bear upon a binary is greatly increased by the closer packing of the system in bygone ages. Basing his calculations upon the observed percentage of decidedly modified binary orbits and the probability of outside influence, Jeans obtains an estimate of the age of the galaxy which confirms his previous result of 10^{12} years.

MASS AND VELOCITY

A statistical study of the masses and the velocities of stars revealed a striking correlation, which though incomplete pointed toward equipartition of energy. No satisfactory explanation of this was offered until Jeans dispelled the clouds of mystery by showing that it is essentially the same problem as that just discussed. The mutual influences of the stars, originally sixty-four times as closely packed as now, have resulted in the course of 10^{12} years in bringing some measure of order out of primeval disorder. It is analogous to the behavior of a mixture of gases—if undisturbed by external influences the tendency would be for those molecules most massive to move most slowly while the lighter molecules moved with the greater speed, the kinetic energy or product of mass by velocity squared, tending to be the same for all the molecules. This also formed a basis, though of a truth a somewhat shaky basis, for again confirming the age of the galaxy as 10^{12} years.

SOLAR SYSTEMS

The nebular hypothesis of Laplace and the planetesimal hypothesis of Chamberlin have gone into the history of science as great and lasting monuments to their originators, both of whom were powerful and constructive thinkers. As a direct development from Chamberlin's hypothesis, Jeans some years ago worked out upon a rigorous mathematical basis his tidal theory of the origin of the solar system. In view of the contrast between his conclusions in 1919 and his present conclusions in the light of the researches under review, it is worth while to consider the tidal theory in some detail.

An analysis of the equilibrium forms of rotating fluids under various conditions amenable to mathematical treatment, revealed the fact that within a rotating gaseous mass there are no forces which could combine to produce a series of planets such as encircle our sun. Hence, following the lead of Chamberlin, Jeans called in the aid of a passing star to supply the force necessary to disrupt the parent sun. It is obvious that a close approach of one star to another would draw out tides, one on each side of the star considered, the tidal arm on the side nearest to the tide-producing star being slightly the greater, and therefore more readily drawn out to a distance producing its instability. The matter in this arm would then break away from the parent sun and follow orbits about the sun in directions governed by the direction of travel of the passing star. Condensation would gradually take place about any points in the tidal arm where there happened to be a local concentration of gas. Each of these nuclei would become one of the planets, and its orbit and other individual characteristics would be determined, in part at least, by the influence of the "resisting medium" through which it moved, this medium being composed of the vast millions of gaseous particles scattered hither and thither into space about the sun by the cataclysmic disruption of the tidal arm.

The formation of satellites by the planets is again an evidence of tidal action, but in this case the sun itself was the tide-producing agency which caused the disruption of the planets when each passed its perihelion for the first time. That some satellites so formed eventually became detached from their parent planets to be captured by other planets is one of the interesting results of the action of the resisting medium.

Dr. Harold Jeffreys has recently proposed several modifications of the above tidal theory. The outstanding point of difference is that Jeffreys limits the size of the ancestral sun to 40 million kilometers in diameter, whereas Jeans had presupposed a much less dense sun of diameter 8,000 million kilometers; the former figure is approximately the diameter of Mercury's orbit, the latter figure is greater than the diameter of the orbit of Neptune. The evidence in favor of the smaller figure seems to be fairly strong.

UNKNOWN PLANETS

The point of interest in both these forms of the tidal theory is that they led to the belief that our solar system was possibly unique in the galaxy of stars, because the chance of two stars approaching closely enough to produce tidal disruption—namely 10^{10} kilometers—was only once in 10^{10} years, which was the whole age of the galaxy then considered possible. With the much greater time scale now proposed by Jeans, and considering also the much closer packing in those early millions of years, the probability of the close approach of two stars becomes decidedly great. Hence the conclusion now reached by Jeans is that of the myriad stars we see about us—not the majority—but a considerable number are probably suns to a family of planets. Like our own solar system in many respects, these numerous other systems may be, yet differing probably from it and from one another in all the details. Whether upon some favored planets in some of these many systems there have been developed physical conditions as on this earth, rendering them fit cradles for the advent of life we know not, and it is beyond the scope of the mathematical physicist and astronomer to speculate further.

STABILITY OF GALAXY

An investigation of very great interest has been carried out during the last year by Dr. Ludvik Silberstein on the question of the permanence of star clusters, in particular, the great star cluster or galaxy near the center of which our solar system finds itself. Silberstein bases his calculations upon the four-dimensional space-time relations of de Sitter, and from this starting point he last year deduced a relation which could be evaluated in terms of observed astronomical data in such a way as to give a numerical value for an invariant characteristic of spacetime called by the mathematician the radius of curvature. This quantity, symbolized by R, has the finite value of 10^{12} astronomical units, that is 10^{12} times the distance from earth to sun. This theory, with the consequent value of R, has not been universally accepted, but this does not detract from the interest of the subsequent reasoning by which Silberstein deduces a criterion of stability in terms of the total mass of a system of material bodies (molecules or stars) and the radius of the system. Associated with any given mass there is a critical distance. If a star be at a greater distance than this critical value from the center of gravity, its orbit will of necessity be a hyperbola. This means that sooner or later it will desert the system forever. On the other hand, if its distance from the mass center be less than the critical value it will describe an elliptic orbit, thus remaining indefinitely within the system.

This criterion has been applied to those globular clusters far out in space beyond our own galaxy, for which the astronomer has been able to form estimates of their size and mass. They are found to be considerably less massive than our galaxy and very much more closely packed, so closely packed that the calculated critical radius is much greater than the dimensions of the clusters, which may, therefore, from the point of view of this theory, be considered as stable aggregates of stars.

The reverse is the case of our own galaxy. Much too widely scattered for its mass, its radius exceeds the critical value for stability, and therefore this theory predicts that it will suffer from what Doctor Silberstein terms " hyperbolic desertion " until its ranks be reduced and its volume diminished to such an extent that the criterion might perchance be satisfied. In its present form it is, like the Roman Empire, far too inflated to be enduring.

Densities can be treated in a similar manner. Silberstein evaluates the critical density of matter in space in terms of his finite curvature invariant R, the gravitational constant and the velocity of light—three fundamental quantities in this complex universe. Any aggregation of matter of less than this critical density will be unstable and tend to dissipate, whereas any aggregation of density exceeding this value will be in a state of stability. The galaxy of stars in which our system finds itself is estimated to have a density fifty-two times too small to satisfy the conditions for permanence.

Having surveyed the future, let us, in the light of this same theory, glance backward in an endeavor to trace the origin of a stellar cluster. Silberstein considers the possibility of a gaseous mass or nebula giving rise to millions of individual concentrations of matter, and thus forming the individual stars of a cluster. This was essentially the primary postulate of Laplace, though he was considering the relatively minute case of a nebula giving rise to a solar system—an impossible hypothesis in the light of modern knowledge. But as an explanation of the evolution of a small galaxy of stars, like many of the star clusters revealed by the telescope, it is by no means to be discarded—it may well be the true solution of the problem, as was pointed out by Jeans some years ago. When, however, an attempt is made to explain the origin of our galaxy in this manner, it is found to be incapable of satisfying all the conditions. Our galaxy, to quote yet another analogy taken by Doctor Silberstein from the history of mankind, must have developed, like the far-flung British Empire, by the aggregation into one conglomerate whole of many remnants of previous systems, systems long since scattered to the four winds.

Guided by some of the great thinkers of to-day, our thoughts have traversed æons of time, contemplating some of the changes taking place with majestic deliberation throughout the vastnesses of space. " Time rolls his ceaseless course." A million million years suffice for the birth of a star and its early development; a few hundred thousand years will tell the tale of the life of mankind upon this planet; and as for man, an individual man, the years of his life are three score years and ten, and yet such is the power of a great mind that, despite the brevity of its allotted span, it can wrestle with the problems of nature and learn something at least of the immensities of space and time.

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