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SPECTROSCOPIC ABSOLUTE MAGNITUDES AND PARALLAXES OF 200 A-TYPE STARS

BY

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ABSTRACT

The spectroscopic absolute magnitudes and parallaxes of 200 A stars have been determined.

The material used was the large collection of 1-prism slit spectrograms of the Yerkes Observatory. Eighty stars having reliable trigonometric or cluster parallaxes were made the basis of calibration and seven criteria have been found giving correlations with absolute magnitude, namely—width of K, Hδ, λ4481, and intensity ratios λ4215/λ4227, λ4233/λ4227, λ4535/λ4481, λ4549/λ4481.

Relative to the standard stars, the systematic error of the magnitudes is −0.04 and the probable error ±0.5. Relative to 108 Mt. Wilson spectroscopic magnitudes the systematic error is +0.09 and the probable error ±0.3.

The Shajn Double Star test applied to 12 pairs gives satisfactory evidence in favour of the accuracy of the magnitudes. The magnitudes show correlation with proper motion but not with reduced proper motion.

A comparison is given between Mt. Wilson, Arcetri and the writer's average magnitudes for each spectral subdivision and evidence is given in support of the claim that the magnitudes herein determined have greater individual accuracy than can be obtained by adopting the mean magnitude method.

INTRODUCTION

During the summer of 1925 the writer held an appointment as Volunteer Research Assistant for four months at the Yerkes Observatory, Williams Bay, Wis. The Director, Dr. E. B. Frost, offered the writer, as a research problem, the determination of the spectroscopic absolute magnitudes and parallaxes of A-type stars. For this purpose there were available several thousand spectrograms of about 500 stars of spectral classification A0 to A9, or taking the next sub-class above and below, B9 to F0. A selection from these was to be made including as many stars as possible for which reliable group or trigonometric parallaxes were available, these to form the basis of calibration.

All these spectrograms were taken at the Yerkes Observatory with the 40-inch refractor and Bruce Spectrograph attachment. They are 1-prism spectrograms having a dispersion of 30A to the millimetre at λ4500, while from Hβ to Hε approximately 691A, is 33 mm.

This problem presented many features of interest and of difficulty, for though considerable work has been done on spectroscopic magnitudes of the later type stars (F, G, K, M), comparatively little has been done upon spectra of Class A. In later type spectra (viz., F to M) there are many metallic lines; in spectra earlier in type than A (viz., B stars) there are helium lines, hydrogen lines and spark lines of silicon and of a few other elements; but in the spectra of early A stars, especially A0, there are comparatively few lines well defined with the exception of strong hydrogen lines and the H and K lines of ionized calcium.

Table I summarizes the criteria upon which other investigators have depended in their determinations of spectroscopic absolute magnitudes, and for the sake of completeness the criteria used in the present investigation are included:

TABLE I

Spec. Class	Investigator	Ref.	Criteria
B	Adams, Joy	3	Correlation with spectral class based chiefly on helium and hydrogen lines as in Harvard system, but with "nebulous" and "sharp" subdivisions
B	Edwards	4 5 6	λλ4471, 4388 (helium) with Hγ, λ4144 (helium) with Hδ
B8-A3	Lindblad	8	Comparison of regions λλ3884-3907 and λλ3907-3935
B9-A9	Adams, Joy	2	Correlation with spectral class based chiefly on λλ4026, 4471 (helium) and λλ4326, 4384 (Fe)
A	Struve	9	Width of λ4481 (Mg^+)
A	Douglas	—	Width of Hδ, K, λ4481; intensity ratios from λλ4215, 4227, 4233, 4481, 4535, 4549
A-F5	Abetti	10	Correlation with spectral class involving trigonometrical, cluster, and all available spectroscopic data from Mt. Wilson, Victoria, Sidmouth and Arcetri
F-M	Adams, Joy, etc.	1	Selected ratios from λλ4072, 4077, 4215, 4250, 4271, 4290, 4455, 4462.
F-M	Lindblad	7	Arc line λ3900; Cyanogen bands λλ4144-4184, 4227-4272, 3993
F-M	Rimmer	11	Selected ratios from λλ4072, 4077, 4215, 4227, 4250, 4271, 4290, 4444, 4455, 4462
F-M	Young, Harper	12	Selected ratios from λλ4072, 4077, 4162, 4168, 4215, 4247, 4250, 4258, 4271, 4290, 4455, 4482, 4489, 4494, 4496, etc.
F-M	Macklin	13	Selected ratios from λλ4174, 4227, 4290, 4326, 4387, 4444

Examination of the Yerkes spectrograms made it evident that new criteria would have to be sought.

The comparison of the density of the regions used with such success by Lindblad is ruled out in the present case because the range of the Yerkes 1-prism plates is from just to the red of Hβ to just to the violet of K, approximately λ4900 to λ3920. Thus the bands at λλ3884, 3907 are beyond the range of these spectrograms.

The widths of the hydrogen lines should, theoretically, show a correlation, but that this extends throughout the range of A spectra is doubted by Harvard investigators.[33], [34]. For the early A stars, however, the writer felt that this line of attack should not be ignored. Likewise the width of λ4481 correlated with luminosity by Struve[9] for 36 stars of types A0 to F0 appeared to warrant careful consideration.

In order to gain familiarity with spectra of this type and to find if possible new criteria upon which to work, the writer made a random selection of plates covering a wide range of absolute magnitudes as determined by Adams. These were studied in pairs on the Hartmann Spectrocomparator which allows of the simultaneous view under magnification and equal illumination of the two spectra in exact contiguity throughout their entire range. The result of the preliminary study and measurements was that the following criteria were adopted:

(1) Widths of λ4481, Hδ, K.

(2) Intensity ratios: $\frac{4215}{4227}, \frac{4233}{4227}, \frac{4535}{4481}, \frac{4549}{4481}$.

PART I

CORRELATION CURVES

Methods.—In order to avoid any conscious or unconscious bias in the measurements and intensity estimates, the writer decided to make no reductions whatever until all the measurements had been made. The required data for a sufficiently large number of stars were obtained before leaving Yerkes Observatory in September, 1925, and the compilation and reduction of this data was commenced subsequently. Thus the correlations which have been obtained are felt to be free from prejudice and subject only to the personal factor which is inevitable in all such work.

The number of spectrograms of any one star varied from one in comparatively few cases to several score in certain cases of spectroscopic binaries where orbit determination had necessitated many plates being taken. Usually there were several plates and of these the two best were selected, special effort being made to get good definition at the extreme left—the H and K region. After preliminary examination under low power magnification to establish the essential similarity of the two, one was selected for final measurement. Where there were distinct differences between the two, both were measured and other plates of the same star examined. Cases of uncertainty were generally attributable to spectroscopic binary effects where considerable care was necessary lest apparent breadth of line was a result of Doppler displacement rather than an effect of the kind sought. The final measurement, in general, of one plate for each star has obvious advantages and, apart from the writer's own tests, it appears justified by the consistency of records of the widths of Hγ and Hδ made from different plates of the same star by means of a Moll microphotometer.[34]

The measurement of line widths involves a large uncertainty where the lines are wide and nebulous; in particular, in the A stars, the "wings" of the hydrogen lines are a very prominent feature of these spectra and it is a matter of individual opinion as to where the "line" begins and ends. All that can be hoped is that the measures made by one individual with magnification and illumination kept approximately uniform, will be consistent within themselves.

The measurement or estimate of line intensity presents difficulties which different investigators have sought to overcome in various ways. The writer depended solely on eye estimates endeavouring always to integrate mentally the total intensity from wings to line centre. It is this integrated intensity that is really sought and not merely the central maximum intensity obtained by wedge extinction methods. Eye estimates are relied upon in much of the work of Harvard investigators, even where the comparisons are made between lines on different plates. In the present case, however, comparisons were made only between closely adjacent lines on the same plate, and for this purpose an eye-piece giving magnification 3 was used.

The widths of lines were measured on a Gaertner machine under magnification 15. In order to eliminate temperature or other effects causing lack of complete uniformity in dispersion a factor was recorded for every plate measured, namely the distance between the titanium-iron spark comparison lines λλ3963-3969, or when these were indistinct, λλ3981-3998. All measured widths were subsequently reduced to the standard given by the average plate factor 0.370 mm. in the former and 0.950 mm. in the latter case.

Each plate was weighted at the time of measurement on the basis: Good 3, Fair 2, Poor 1. Where the plate was not equally good throughout, separate weights were recorded for the different regions as required.

Known Parallaxes.—The stars for which data had been obtained included 31 belonging to the Taurus and Ursa Major Clusters for which reliable parallaxes were available from the work of Rasmuson;[14] also 49 for which trigonometrical parallaxes were given in Schlesinger's General Catalogue of Parallaxes.[15] The Cluster parallaxes being absolute were used as they stand,[16] the trigonometric parallaxes however are differential, but 35 were made use of by W. S. Adams[2] and his value for the corresponding absolute parallax was taken in each case, the remaining 11 were reduced to absolute by the addition of an average factor $+0''.004$.[17] Of these 49, only one had a negative parallax and as it had the value $-0''.003$ the reduction to absolute brought it up to $+0''.001$ thus obviating any necessity of having to introduce the effect of negative parallaxes into the reduction of the data.

These 80 stars formed the basis upon which the absolute magnitude relations were determined.

Absolute Magnitude.—The absolute magnitude of each of these stars was calculated from the relation

$$M=m+5+5\log p$$

where M and m are the absolute and apparent magnitudes respectively and p the absolute parallax. The values of m were taken in every case from the Henry Draper Catalogue—photometric magnitude.

Spectral Classification.—In assigning the spectral class to each star studied, the writer was definitely influenced and guided by the Mt. Wilson classification rather than the Harvard H. D. classification. Following the lead of the Mt. Wilson investigators, the writer classified all the spectrograms studied as *s* or *n* according as the absorption lines were sharp, narrow and clean cut or nebulous, wide and hazy. It seemed evident from the outset that there were many spectra which could not be said to be either definitely sharp or decidedly nebulous; these were labelled *ns* or *sn* and put with the group which they most resembled.

The writer decided to carry out separate reductions for the *s* and *n* stars and having done these quite independently the resulting curves should provide unbiased evidence as to whether this grouping has a physical reality or whether the stars form a homogeneous class.

Reduction Procedure.—With absolute magnitude as abscissae and either line width or intensity ratio as ordinates, plots were made for each of the seven criteria in the case of the *n*-stars and likewise for the *s*-stars, fourteen plots in all. Cluster stars were differentiated from trigonometric parallax stars so that any systematic difference would become apparent. As was hoped, no such effect was evident.

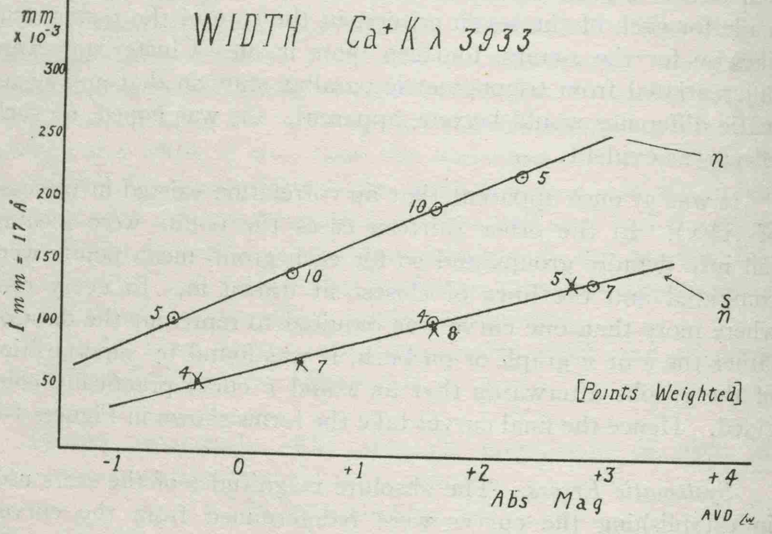
It was at once apparent that no correlation existed in the case of Hδ(*n*). In the other thirteen cases the points were seen to fall into definite groups and so for each group mean points were computed and the lines of closest fit drawn in. In every case where more than one curve was required to represent the data on either the *s* or *n* graph or on both, it was found by superposition of the graphs afterwards that an *s* and *n* curve practically coincided. Hence the final curves take the forms shown in Figures 1-7.

Systematic Errors.—The absolute magnitudes of the stars used in establishing the curves were redetermined from the curves. Weights were assigned to the estimate from each of the seven criteria based upon its apparent reliability and then the mean magnitude calculated. Comparison was made for each criterion separately with the standard value (cluster or trigonometric), and wherever possible with Adams' value.

In the tables the symbols used have the following significance:

S = the standard value of absolute magnitude whether dependent upon cluster or trigonometric parallax.
A = the spectroscopic absolute magnitude, as determined at Mt. Wilson.
D = ditto, as determined by the writer, from all the criteria applicable.
1 = ditto, as determined by the writer from width of K.
2 = ditto, from width of Hδ.
3 = ditto, from width of λ4481.
4 = ditto, from Ratio λ4215/λ4227.
5 = ditto, from Ratio λ4233/λ4227.
6 = ditto, from Ratio λ4535/λ4481.
7 = ditto, from Ratio λ4549/λ4481.
N = Weight (number of stars).
e = systematic error.
r = probable error.

1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1 mm = 17 Å
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250
200
150
100
50
n
s
n
5
10
10
5
5
7
4
8
4
7
[Points Weighted]
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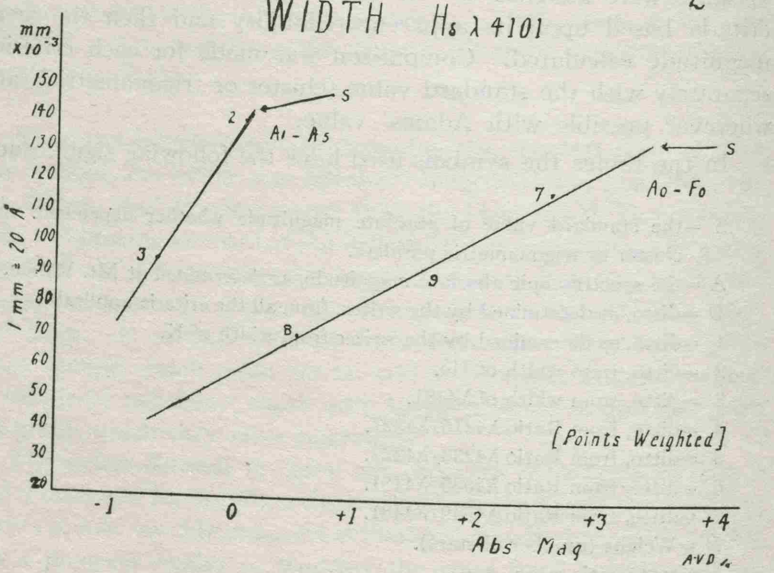
0
+1
+2
+3
+4
Abs Mag
AVD

2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mm x 10^-3
1 mm = 20 Å
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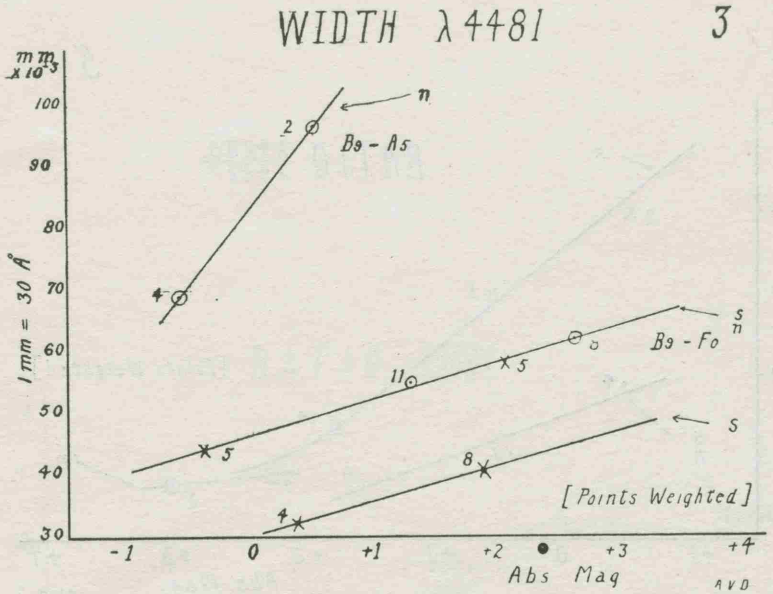
140
130
120
110
100
90
80
70
60
50
40
30
20
2
A1 - A5
3
s
A0 - F0
7
9
8
[Points Weighted]
-1
0
+1
+2
+3
+4
Abs Mag
AVD

3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1 mm = 30 Å
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80
70
60
50
40
30
n
2
B9 - A5
4
s
n
B9 - F0
11
5
5
s
8
4
[Points Weighted]
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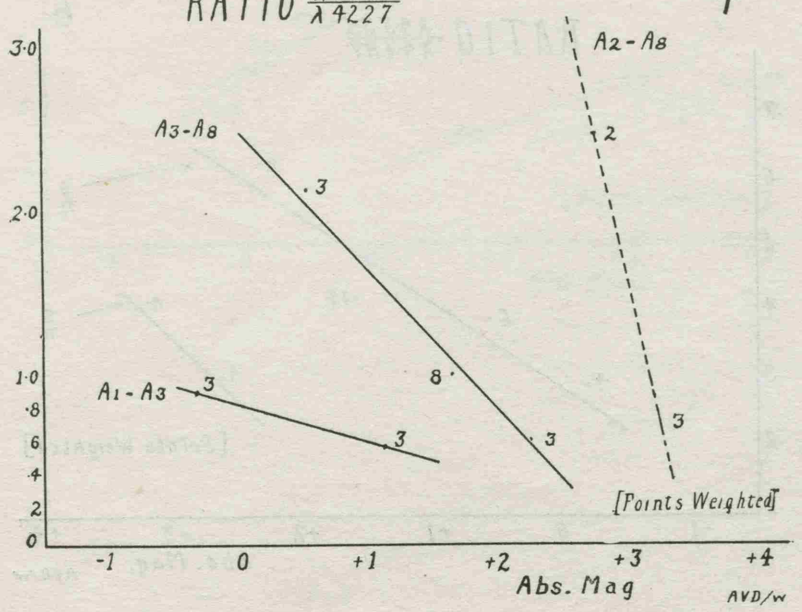
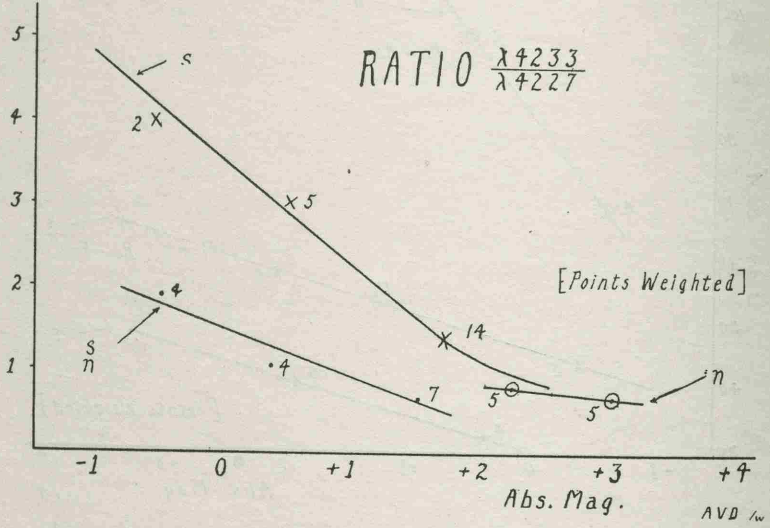
0
+1
+2
+3
+4
Abs Mag
AVD

4


RATIO λ4215 / λ4227
A2 - A8
3·0
2
A3 - A8
3
2·0
1·0
·8
·6
·4
·2
0
A1 - A3
3
3
8
3
3
[Points Weighted]
-1
0
+1
+2
+3
+4
Abs. Mag
AVD

5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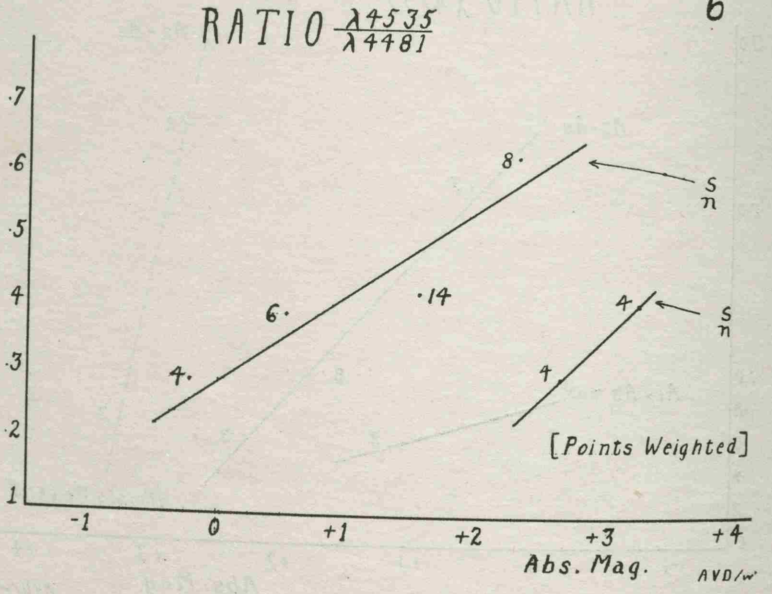
[Points Weighted]
s
2 ×
× 5
× 14
s
n
•4
•4
•7
5
5
n
5
4
3
2
1
-1
0
+1
+2
+3
+4
Abs. Mag.
AVD /w

6


RATIO λ4535/λ4481
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6•
•14
4•
s
n
4
4
s
n
[Points Weighted]
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.6
.5
4
.3
.2
1
-1
0
+1
+2
+3
+4
Abs. Mag.
AVD/w

7

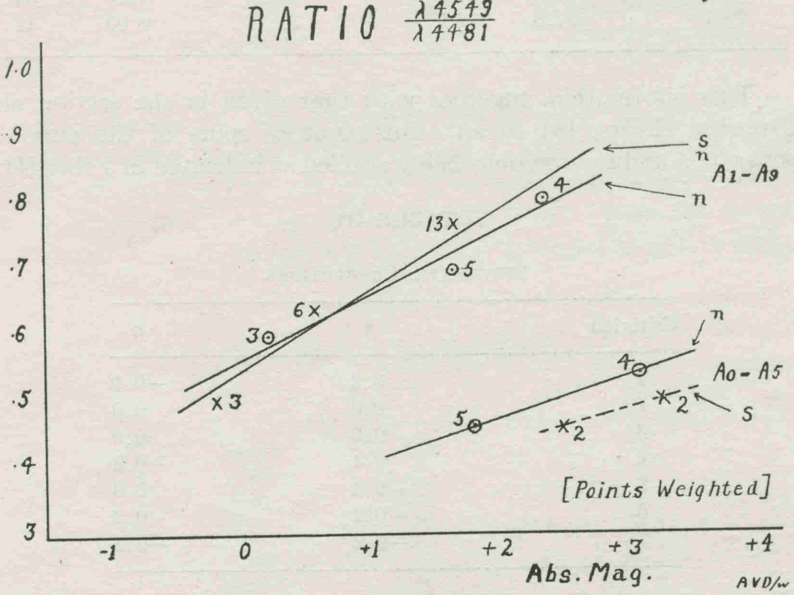
RATIO λ4549/λ4481
s
n
A1 - A9
n
⊙4
13×
⊙5
6×
3⊙
×3
n
4⊙
A0 - A5
5⊙
×2
×2
s
[Points Weighted]
1.0
.9
.8
.7
.6
.5
.4
3
-1
0
+1
+2
+3
+4
Abs. Mag.
AVD/w

TABLE II

SYSTEMATIC ERRORS

Comparison	e	N	Comparison	e	N
S−1	−0.01	65	A−1	0.33	56
S−2	0.05	30	A−2	0.45	26
S−3	−0.10	50	A−3	0.25	40
S−4	0.30	26	A−4	0.54	19
S−5	−0.01	50	A−5	0.26	42
S−6	−0.34	43	A−6	0.10	34
S−7	−0.28	50	A−7	0.00	42

This information, together with that given in the section on Probable Errors, led to an adjustment of some of the curves, separate *n* and *s* corrections being applied as indicated in Table III.

TABLE III

SYSTEMATIC CORRECTIONS

Criterion	s	n
1	0.2	−0.1
2	0.0	0.0
3	0.0	0.0
4	0.4	+0.2
5	−0.2	0.0
6	−0.2	−0.2
7	−0.2	−0.2

After applying these corrections the weighted mean magnitude was calculated for each star and the systematic error determined. It was noted that one star was largely responsible for the error and as this star (Boss 606) is the one for which the differential parallax is negative, it seemed fair to omit it from the total and consider that the resulting low value of the systematic error indicated that the criteria were satisfactory from this point of view. In Table IV the value of *e* is given both with and without Boss 606. It may be remarked that for this star the absolute magnitude derived from the absolute parallax +0″.001 and apparent magnitude 5.4 is −4.6, whereas the writer's determination is +0.5. It is not included in the Mt. Wilson list.

TABLE IV

COMPARISON OF SYSTEMATIC ERRORS

	e	N	Remarks
S−D	−0.11	74	Boss 606 included
S−D	−0.04	73	Boss 606 excluded
S−A	−0.16	69	Boss 606 excluded
A−D	+0.15	66	Boss 606 excluded

Probable Errors.—The average error, or mean deviation, was found for each star with respect to each estimate individually and to the weighted mean. In Table V the probable errors[18] corresponding to these average errors are set forth. Boss 606 is not included; if it were the errors would all be increased by 5% to 10%.

TABLE V

SUMMARY OF PROBABLE ERRORS

	r	N	Remarks
S−1	±0.75	64	
S−2	0.59	30	
S−3	0.67	49	
S−4	0.74	26	
S−5	0.57	49	
S−6	0.67	42	
S−7	0.76	49	
S−D	0.52	73	
S−A	0.60	69	
A−D	0.45	65	
S−D	0.48	71	Boss 3960, 6031 omitted
S−A	0.57	67	Boss 3960, 6031 omitted

It is gratifying to find that the probable error of the luminosities determined in the present investigation is somewhat less than that of the Mt. Wilson results. In their determination of luminosities of later type stars the probable errors vary for various types from ± 0.64 to ± 0.00, the average probable error being given as $\pm 0^m.40$.[19] That the corresponding figure in the case of the more difficult A type stars should be as low as $\pm 0^m.5$ seems to be all that can be hoped for at the present stage of our knowledge of the complex conditions which evidently exist in atmospheres of stars of this interesting class.

Theoretical Considerations.—While it must be explicitly understood that the relationships that have been obtained are empirical, it is also to be remembered that there is a theoretical basis for relationships of the kind found. Considering first the question of the *width* of an absorption line, there are at least four points to be taken into account:

(*a*) Minimum width
(*b*) Doppler widening
(*c*) Rayleigh scattering
(*d*) Stark effect.

(*a*) For a given instrument and a given slit width, there is a minimum width to be expected for an absorption line of any given wave length. For the Bruce Spectrograph with slit width 0.05 mm. and the ratio $f/D = 19$ the minimum width of λ4481 on a 1-prism plate has been calculated by O. Struve[9] to be 0.68A. This is based upon the theories of Schuster[20] and Newall[21] who find that two lines cannot be resolved into separate lines if the difference in the wave length, $d\lambda$ is less than λ/P or at most, $\lambda/2P$, where P the purity of the spectrum is some fraction of the resolving power, R. This $P = pR$ where $0 < p < 1$. If the slit were of infinitely narrow width then $p = 1$ and $P = R$. For the case under consideration, the value of p is believed to lie between 1/6 and 1/7. A glance at the graphs (Figs. 1-3) shows that the lowest measured values are just about at the calculated minimum, the great majority lying well above.

(*b*) A line may be considerably widened as a result of Doppler shifts due to the translatory motion of the radiating and absorbing atoms. That this is not the main cause of widening is evident from the observed fact that line width tends to increase as temperature decreases and pressure increases in the stellar atmosphere. If the widening were chiefly dependent upon thermal agitation the reverse would be true. Stewart,[22] following the treatment of this question by Lorentz, believes the Doppler effect to be negligible except in the case of hydrogen where it may become appreciable though not dominant.

(*c*) An important factor in producing width is undoubtedly *Rayleigh scattering*, "the intrinsic lack of sharpness in the 'tuning' of the active (scattering) molecules." This does not involve "absorption" in the restricted use of the word advocated by Stewart[23] who limits absorption to cases involving transformation of radiant to thermal energy. Scattering does not involve such transformation. The radiant energy taken up momentarily by an atom is re-radiated without important change in frequency and the resultant opacity is due to diffusion in direction of the incident beam of radiation.

Stewart deduces the relation for line width Δ

$$\Delta = 5.8\ 10^{-13}\lambda/\sqrt{n}$$

where n is the number of atoms in a column of 1 cm^2 cross section in the line of sight above the reversing layer. Thus it is evident that the greater the density of the stellar atmosphere the greater the width of the absorption line, a relation borne out by the graphs in Figs. 1-3.

(*d*) Merton[24] was perhaps the first to draw attention to the broadening effect which will be produced in star spectra by the natural influences of their own *electric and magnetic* fields upon the radiating and absorbing particles. Conditions in the stellar atmosphere, producing closer packing and frequent collisions will thus be accompanied by widening of the absorption lines due to Stark effect and to abnormal electron orbit distortions.

Hulbert[26] has discussed the breadth of the Balmer lines of hydrogen in the stellar spectra by combining the Stark theory with the Saha theory of high temperature ionization. He finds that the observed width in *A*-type stars far exceeds the theoretical width unless either the pressure is equivalent to several atmospheres or there are a very large number of free electrons present. The former assumption is ruled out on many astrophysical grounds, the latter is quite admissible.

The effect of *stellar rotation* may well be a disturbing factor. Lack of observational data renders the question incapable of rigorous treatment, but it may be pointed out that the rate of rotation would tend to increase as a star lost mass through radiation. It is however linear velocity at the extremity of a diameter and not merely angular velocity that affects line width and furthermore the orientation of the star relative to the line of sight is of the utmost importance.

Other causes of broadening are mentioned by Rayleigh,[25, 27] as, for example, the possible rotation of the radiating particles, but it is probable that in general none of these effects become of primary importance.

When we come to investigate the theoretical basis for correlations such as those in Figs. 4-7, the *ratio* is found, in every case, to be that of an *arc* to a *spark* line or vice versa. Indeed these lines were selected with this in mind, because it is an obvious fact that conditions which weaken an arc line may enhance a line arising from an ionized atom. In the early type stars where effective temperature is high and pressure low there is a high degree of ionization, and the proportion of neutral atoms is low. As pressure increases and temperature falls, which corresponds roughly to lower luminosity, the amount of ionization falls off and the intensity of the arc lines is increased. This increase will, of course, not continue indefinitely, there being a definite set of conditions corresponding to maximum intensity for any line. Fowler and Milne,[28,29,30] have evaluated this in terms of the ionization potential, the energy of the given excited state, the partial electron pressure and various constants.

The lines involved in this investigation are the following: λ4215.5 is due to once ionized strontium and has the series relation[31] $1s^2-1p^2$.

λ4226.7 from the normal calcium atom has the series relation $1S-1P$. It grows gradually stronger through the A, F, G, M stars. It is unblended and a very satisfactory basis for comparison.

λ4233 is less simple. It is a blend of ionized iron (Fe$^+$ λ4233.16) $2p^4-1d'^4$, and normal iron (Fe λ4233.6) $1d'^7-md^7$. In general its behaviour is that of a spark line in the A stars, but an occasional anomalous intensity may be due to complications arising from the blend. Near it are two manganese lines λ4235.2, λ4235.1 of type $1d^4-1p'^4$; but blending with these could only occur in extreme cases of stars having line characteristic n.

λ4481 arising from ionized magnesium has series relation $2d^2-3f^2$. It is a close doublet λλ4481.33, 4481.13. It is an admirable line to study being visible on every spectrogram and well situated in the region of best definition and intensity. It is said to reach maximum[32] at A2, its subsequent rise being attributed by C. H. Payne to blending with iron which predominates in the cooler stars. As the present writer is not interested in relative intensities of any one line at different spectral classes but in ratios of two lines, it is sufficient for the present purpose that the intensity of λ4481 increases less rapidly than the intensity of λ4535 and λ4549.

λ4535 is due to neutral titanium. ($1f^5-2f'^5$) Four Ti lines seem to fall close together here: λλ4536.00, 4535.92, 4535.58, 4534.78, the last two being the strongest according to laboratory intensity estimates.

λ4549 appears to be a blend of lines of different kinds; neutral titanium gives rise to λ4548.77 ($1f^5-2f'^5$) of laboratory intensity (35), while ionized titanium has a line at λ4549.64 ($1h^2-1g'^4$) intensity (25) and ionized iron gives λ4549.48 ($2f^2-1d'^4$) intensity (4). Adams describes this line as Fe$^+$ Ti$^+$ but the writer's data go to prove that the blend as a whole behaves as an unionized line, the ratio λ4549/λ4481 growing greater as luminosity diminishes.

Thus it is to be expected that the ratios λ4215/λ4227 and λ4233/λ4227 will have a negative slope, whereas the ratios λ4535/λ4481 and λ4549/λ4481 will have a positive slope as in Figs. 4-7. There is certainly no theoretical basis for straight line relations, but the data at the writer's disposal warranted no other more complicated representation.

Correlation Curves.—The writer believes this to be the first time that definite relations have been obtained for the width of K and the width of Hδ with absolute magnitude. That these are real relations seems undoubted.

In the case of Fig. 1 the Bravais-Pearson correlation coefficient has been evaluated for each curve. Let x be the deviation of the individual star magnitudes from the weighted mean and y the corresponding deviation of line width, then the *correlation coefficient* is given by

$$\frac{\Sigma xy}{\sqrt{\Sigma x^2 . \Sigma y^2}.}$$

In the case of Fig. 2, it has been evaluated for the main curve ($A0-F0$). The results are as follows:

Width K (n)	0.67	Prob. error	±0.067
" K (sn)	0.67	" "	±0.063
" Hδ($A0-F0$)	0.68	" "	±0.074

In the application of all these correlation curves, it is evident that uncertainty may easily arise as a result of their multiple character. In general, it has been found that a glance at the spectrogram, or the brief description of it on the writer's records, was sufficient to indicate the general brightness and hence to determine which curve was most applicable in the case of each criterion. Where uncertainty remained, and either one of two curves seemed equally applicable, the usual procedure adopted was to take the mean, and in general to weight this lower than unique values from other criteria.

These curves are definitely *not* built up upon spectral classification, and therefore in so far as the writer has been successful in eliminating the influence of spectral class from the mind in interpreting the curves, the resulting absolute magnitudes should be something more than just averages for spectral class as are the Mt. Wilson[2] and Arcetri[10] determinations of magnitudes of A stars.

PART II

RESULTS AND DISCUSSION

The correlation curves obtained and tested as described in Part I, have been employed to give values of the absolute magnitudes of some 170 stars for which trigonometric or cluster parallaxes are not available.

The weighting of the values from each criterion was done as already described and the weighted mean value alone is recorded in Table VI. In some cases where the data were very meagre it was thought best to exclude these stars pending further study; they belong in most instances to the group Aon which are by far the most difficult to handle in the light of the present investigation.

Table VI contains the spectroscopic absolute magnitudes and parallaxes of two hundred A stars as determined by the writer. In addition to these, six stars are included which belong to the set of standard stars upon which the correlation curves were based, but for these six reliable determinations from the present criteria were impossible.

Successive columns of Table VI give (1) the Boss Number (Preliminary General Catalogue); (2) the visual apparent magnitude; (Henry Draper Catalogue); (3) the reduced proper motion, H; (4) (5) the absolute magnitude, M, and parallax, p, as determined trigonometrically (T) or by group motion (G); (6) (7) (8) the spectral class, spectroscopic absolute magnitude and parallax as determined at Mt. Wilson; (9) (10) (11) the class and spectroscopic M and p as determined by the writer from the Yerkes Observatory spectrograms. The designation "Yerkes" cannot be given to these three columns as might at first glance seem more appropriate, because to do so would be to set the official stamp of approval of that institution upon these results. For defects in this work the Yerkes Observatory is in no way responsible and criticisms of these results must be borne solely by the writer.

TABLE VI

SPECTROSCOPIC MAGNITUDES AND PARALLAXES OF 200 A STARS

Boss	H.D. m	H	Trig. or Gr.		Mt. Wilson			Douglas		
			M	p	Class	M	p	Class	M	p
				″			″			″
10	2.2	3.8	..	..	A1n	1.1	0.060	A1n	1.0	0.059
43	4.4	3.0	..	..	A1n	1.5	.026	A3n	1.7	.029
50	4.5	..	0.6	0.017T	A0n	1.1	.021	A0n	0.2	.014
145	5.0	2.8	..	..	..	..	..	A1n	1.3	.018
154	5.5	2.7	..	..	A2s	1.1	.014	A1s	0.9	.012
203	3.9	4.9	1.5	.033T	..	..	..	A6n	2.0	.042
246	5.2	6.4	..	..	..	..	..	A4n	2.2	.025
269	4.9	2.1	..	..	..	..	..	A4n	1.6	.022
295	5.3	4.0	..	..	A1n	1.5	.018	A2n	1.4	.017
300	4.7	..	0.9	.017T	..	..	..	A2n	2.3	.033
314	2.8	5.2	..	..	A3n	2.0	.068	A5n	2.5	.087
368	5.5	6.2	..	..	..	..	..	A5n	2.5	.025
370	5.5	4.2	..	..	..	..	..	A2s	0.9	.012
422	4.8	5.5	..	..	..	..	..	A0n	0.7	.015
423	4.8	5.5	..	..	..	..	..	A2s	0.6	.014
428	2.7	..	1.9	.068T	..	..	..	A5s	1.9	.069
441	4.8	..	2.1	.029T	..	..	..	A7n	2.1	.029
446	4.7	..	1.2	.020T	A4s	2.0	.029	A3n	2.0	.029
449	4.1	2.3	..	..	A1n	1.1	.025	A1s	1.2	.026
452	5.4	6.2	..	..	..	..	..	A1n	1.4	.016
463Ft	5.2	..	..	..	A3n	1.8	.022	A3s	1.5	.018
463Br	4.3	2.1	..	..	..	..	..	A3s	0.4	.017
466	5.4	2.2	..	..	A2n	1.5	.016	A1n	2.3	.024
476	5.1	3.2	..	..	..	..	..	A5s	1.9	.023
480	4.8	5.8	..	..	..	..	..	A1n	1.8	.025
482	3.1	..	−0.7	.017T	..	..	..	A3s	1.3	.044
522	5.1	3.9	..	..	A1n	0.9	.014	A0n	0.3	.011
550	4.6	0.5	..	..	A3s	1.4	.021	A3s	2.0	.030
560	4.3	2.3	..	..	..	..	..	A0n	0.8	.020
597	5.8	2.3	..	..	..	..	..	A2n	1.6	.014
606	5.4	..	−4.6	.001T	..	..	..	A1n	0.5	.010
628	5.2	6.0	..	..	..	..	..	A6s	2.4	.028
629	4.4	..	2.2	.036T	..	..	..	A7n	1.9	.032
666	5.3	..	..	..	A0n	1.1	.014	A0n	1.2	.015
674Br	5.3	1.4	1.6	.018T	A3s	1.0	.014	A2s	1.2	.015
677	5.2	4.4	..	..	..	..	..	A2n	1.1	.015
730	5.0	4.4	..	..	..	..	..	A1n	0.9	.015

TABLE VI—*continued*

Boss	H.D. m	H	Trig. or Gr.		Mt. Wilson			Douglas		
			M	p	Class	M	p	Class	M	p
				″			″			″
791	5.0	2.6	..	..	A0n	0.9	0.015	A0n	1.1	0.016
850	5.4	4.6	..	..	..	..	..	A4s	1.1	.014
883	5.4	5.7	..	..	..	..	..	A3n	2.5	.026
923	5.1	−0.5	..	..	..	..	..	A1s	0.5	.012
932	3.9	−1.8	..	..	..	..	..	A4s	1.0	.026
971	5.4	−1.1	..	..	..	..	..	A8s	2.5	.026
974	5.1	..	..	..	A1n	1.1	.015	A0n	1.2	.017
986	4.6	4.0	..	..	..	..	..	A0s	0.6	.016
998	5.3	4.2	..	..	..	..	..	A1s	0.3	.010
1007	5.3	..	2.4	0.027G	A9n	2.8	.031	A6n	2.0	.022
1022	4.8	..	2.1	.029G	A6n	2.4	.034	A5s	2.0	.028
1023	5.1	2.4	..	..	A3n	2.0	.017	A2n	2.1	.025
1026	4.4	4.6	1.3	.025G	A3s	1.8	.030	A2n	1.5	.026
1027	5.4	6.0	2.8	.030G	A3n	2.1	.022	A1n	1.7	.018
1029	4.2	..	1.3	.026G	A3s	1.4	.028	A3s	1.7	.032
1033	4.4	..	1.6	.028G	A2n	1.8	.030	A2n	..	..
1034	4.6	..	1.8	.028G	A0n	1.3	.022	A2n	..	..
1046	3.6	..	0.7	.026G	A3s	1.6	.042	A5s	1.7	.042
1047	5.1	..	2.3	.028G	A2n	2.0	.024	A4n	2.1	.025
1051	5.7	..	2.8	.026G	A3n	2.1	.019	A3n	2.6	.024
1054	4.8	..	2.1	.029G	A5s	2.0	.028	A5s	2.1	.029
1067	4.8	..	1.9	.027G	A2n	2.0	.028	A2n	1.2	.019
1087	4.3	..	1.6	.028G	A3n	2.1	.036	A3n	1.8	.032
1088	5.3	4.4	..	..	A4s	2.0	.022	A6s	2.3	.025
1090	4.9	..	1.6	.023G	A3n	2.2	.028	A3n	2.2	.029
1092	5.6	..	2.3	.023G	A5s	2.2	.020	A5s	2.5	.024
1095	5.0	3.3	..	..	A4s	1.7	.022	A3s	1.8	.023
1114	5.4	..	2.6	.029G	A8s	2.6	.027	A5n	2.1	.022
1117	5.4	..	1.4	.016T	..	..	..	A7s	1.9	.020
1122	5.4	..	1.9	.020G	A7s	2.2	.024	A6s	1.8	.019
1143	5.1	..	2.2	.026G	A2n	1.7	.021	A1n	1.7	.021
1153	4.5	1.7	..	..	..	..	..	A0n	1.9	.030
1194	4.7	..	1.6	.024G	A3n	2.1	.030	A4n	1.4	.022
1220	2.9	..	0.5	.033G	A1n	1.5	.052	A0n	1.6	.055
1244	5.1	1.1	..	..	..	..	..	A7s	2.6	.032
1268	5.2	1.2	..	..	..	..	..	A6s	2.3	.026
1352	5.3	3.0	..	..	..	..	..	A0n	1.3	.016
1392	4.9	3.4	..	..	..	..	..	A0n	0.9	.016
1452	5.3	2.3	..	..	..	..	..	A1s	0.7	.012

TABLE VI—*continued*

Boss	H.D. m	H	Trig. or Gr.		Mt. Wilson			Douglas		
			M	p	Class	M	p	Class	M	p
				″			″			″
1453	4.9	0.6	..	..	A2s	1.4	0.020	A1s	0.6	0.014
1482	2.7	..	−0.9	0.019T	A1s	0.6	.037	A1s	0.1	.030
1488	5.3	0.6	..	..	..	..	..	A0n	0.1	.009
1492	5.2	4.0	..	..	..	..	..	A7s	1.8	.022
1516	5.0	−2.6	..	..	..	..	..	A6s	3.2	.044
1575	4.4	..	1.5	.026T	A2s	1.2	.024	A1n	0.7	.018
1690	1.9	..	0.5	.053T	A2s	0.9	.063	A2s	0.3	.048
1714	5.1	5.1	..	..	..	..	..	A1n	1.4	.018
1716	4.9	1.8	..	..	A1n	1.1	.014	A1n	1.1	.017
1759	5.2	3.3	..	..	..	..	..	A0s	0.4	.011
1763	3.6	..	0.4	.023T	..	..	..	A0n	0.5	.024
1782	5.3	0.7	..	..	..	..	..	A4n	1.7	.019
1853	4.1	−0.7	..	..	..	..	..	A0n	0.5	.019
1886	3.6	..	1.5	.038T	A2n	1.5	.038	A2n	1.8	.044
1928	4.5	2.9	..	..	..	..	..	A1s	0.7	.017
1968	4.8	0.0	..	..	..	..	..	A8s	2.0	.028
1974	5.3	0.9	..	..	..	..	..	A5n	2.3	.025
2051	5.1	1.3	..	..	A7n	2.6	.031	A5n	2.0	.024
2078	5.0	3.4	..	..	A0n	1.1	.016	A1n	1.1	.017
2088	5.3	4.5	..	..	..	..	..	A0s	0.7	.012
2091	5.4	3.8	..	..	..	..	..	A1s	1.0	.013
2120	4.6	2.9	..	..	..	..	..	A0n	0.7	.017
2138	5.1	4.8	..	..	..	..	..	A0n	0.6	.013
2185	5.5	−1.0	..	..	A3n	2.0	.019	A3n	1.9	.019
2237	4.0	..	−0.6	.012T	A0n	0.9	.024	A0n	0.1	.017
2264	5.4	5.0	..	..	..	..	..	A6s	2.2	.023
2327	4.7	..	−0.3	.010T	..	..	..	A1n	0.4	.014
2339	5.6	3.0	..	..	..	..	..	A5s	1.2	.013
2398	5.5	5.5	..	..	..	..	..	A2n	2.0	.020
2404	3.1	..	2.9	.090T	A4n	2.2	.066	A4n	1.9	.058
2407	4.3	2.9	..	..	A4s	1.7	.030	A4s	1.8	.032
2479	3.8	..	−0.1	.017T	A1n	1.3	.032	A0n	..	..
2495	4.0	..	1.8	.037T	B9n	0.6	.021	A0n	1.9	.038
2559	4.5	1.6	..	..	..	..	..	A3n	1.9	.030
2584	5.2	1.7	..	..	A5n	2.4	.026	A5n	2.1	.024
2637	4.5	−0.7	..	..	A1s	0.9	.015	A2s	1.2	.022
2642	5.3	..	3.2	.039T	A5n	2.2	.024	A4n	2.3	.025
2655	5.3	1.6	..	..	..	..	..	A1n	0.0	.009
2692	4.5	3.1	..	..	A2n	1.8	.028	A2n	1.3	.023

TABLE VI—*continued*

Boss	H.D. m	H	Trig. or Gr.		Mt. Wilson			Douglas		
			M	p	Class	M	p	Class	M	p
				″			″			″
2697	4.5	1.9	..	..	..	..	..	A0s	0.0	0.013
2729	3.5	4.6	..	..	..	..	..	A4s	1.2	.035
2735	5.4	6.4	..	..	..	..	..	A7s	2.6	.028
2754	4.9	1.9	..	..	..	..	..	A3s	1.1	.017
2900	4.8	3.6	..	..	..	..	..	A0s	0.0	.011
2930	2.4	..	0.8	0.046G	A3s	1.0	0.054	A3s	0.3	.038
2932	4.4	..	0.1	.014T	..	..	..	A4s	0.2	.014
2972	2.6	..	2.2	.085G	A2n	1.7	.066	A1n	1.0	.048
2974	3.4	..	0.0	.021T	A2s	0.9	.032	A3s	0.2	.023
2987	4.8	4.8	..	..	A0n	0.7	.016	A1s	0.2	.012
2990	4.1	4.0	..	..	B9s	−0.2	.014	B9s	0.0	.015
3023	5.3	3.9	..	..	B9n	0.8	.012	B9n	0.4	.010
3063	5.5	6.4	..	..	A4n	2.0	.020	A6n	1.9	.019
3088	5.1	4.3	..	..	A1n	1.5	.019	A1n	1.2	.017
3097	5.2	3.7	..	..	A1n	0.8	.015	A1s	0.6	.012
3101	2.2	..	2.5	.114T	A2n	1.7	.079	A5s	2.4	.110
3117	2.5	..	0.6	.041G	A0n	0.9	.048	A0n	0.7	.044
3126	5.2	3.8	..	..	..	..	..	A0s	0.7	.013
3139	4.6	2.2	..	..	..	..	..	A3n	1.4	.023
3182	5.1	1.7	..	..	..	..	..	F0n	2.9	.036
3190	3.4	..	1.7	.045G	A0n	0.9	.032	A0n	..	..
3210	4.0	3.1	..	..	..	..	..	A2s	1.2	.028
3240	5.2	2.5	..	..	..	..	..	A6n	3.0	.036
3244	5.0	0.6	..	..	..	..	..	A4s	2.6	.033
3266	5.4	1.9	..	..	A3s	1.0	.014	A3s	1.0	.013
3277	5.4	5.0	..	..	B9n	0.6	.010	B9n	0.2	.009
3283	4.8	4.1	..	..	..	..	..	A0s	0.3	.013
3309	5.0	5.6	..	..	B9n	0.6	.014	B9n	0.4	.012
3310	5.5	5.0	..	..	B9n	0.6	.010	A0n	0.7	.011
3323	5.2	5.5	..	..	A6n	2.4	.027	A7s	2.4	.028
3354	5.8	3.0	..	..	..	..	..	A0s	0.4	.008
3356	5.3	2.8	..	..	..	..	..	A0n	0.2	.009
3370	5.4	7.3	..	..	..	..	..	A8s	3.3	.028
3371	2.9	..	1.1	.044T	A1s	0.6	.034	A1s	1.1	.044
3409	4.4	..	−0.4	.011T	A2s	0.9	.020	A2s	−0.3	.011
3450	5.1	2.3	..	..	..	..	..	B9n	0.3	.011
3474	2.4	..	0.6	.044G	A2s	1.1	.054	A2s	0.6	.044
3475	4.0	..	2.3	.046G	A8s	2.0	.052	A5s	1.8	.036
3480	4.0	..	2.1	.042G	A1n	1.1	.026	A1n	2.2	.044

TABLE VI—*continued*

Boss	H.D. m	H	Trig. or Gr.		Mt. Wilson			Douglas		
			M	p	Class	M	p	Class	M	p
				″			″			″
3506	4.9	..	0.6	0.014G	A3s	1.5	0.021	A5s	0.7	0.014
3508	3.4	5.7	..	..	..	..	..	A0n	0.9	.032
3509	5.5	1.7	..	..	..	..	..	A0n	0.8	.011
3512	4.6	5.6	..	..	..	..	..	A0n	0.7	.017
3518	4.9	..	3.5	.052T	A1n	1.5	.021	A1n	1.8	.024
3526	5.5	..	1.4	.015T	A7n	2.5	.018	A7n	2.5	.025
3530	5.3	6.2	..	..	..	..	..	A1n	1.0	.014
3561	5.5	6.4	..	..	..	..	..	A1s	0.6	.010
3612	4.3	1.9	..	..	A1n	1.1	.023	A0n	1.4	.026
3654	4.6	..	1.6	.025T	A4n	2.2	.034	A5n	1.8	.028
3666	4.3	..	2.4	.041T	A1n	1.1	.023	A0n	1.3	.025
3692	5.1	..	1.2	.017T	B9n	0.8	.014	B9n	0.9	.014
3722	3.0	4.3	..	..	A3n	2.0	.062	A5n	2.3	.072
3749	4.9	0.6	..	..	..	..	..	A2s	−0.3	.009
3752	4.4	..	0.4	.016G	A0n	0.9	.020	A0n	0.5	.017
3787	2.9	3.5	..	..	F1n	2.9	.100	A6n	2.2	.072
3911	5.5	1.7	..	..	A2n	1.5	.016	A2n	1.8	.018
3928	3.1	−0.7	..	..	..	..	..	A2s	1.5	.048
3939	5.1	5.0	..	..	..	..	..	A6s	2.5	.030
3960Br	4.2	..	−0.2	.013T	A4n	2.4	.044	A7n	2.7	.050
3961	2.3	..	0.3	.041G	A0n	0.9	.052	A1n	0.8	.050
3998	3.9	..	0.5	.021T	A0n	1.1	.026	A0n	0.8	.024
4004	5.5	4.5	..	..	..	..	..	A1s	0.5	.010
4009	3.7	..	0.6	.023G	A1n	1.1	.030	A0n	0.0	.018
4016	3.6	3.4	..	..	A0s	0.4	.022	A0s	1.0	.030
4022	5.8	0.8	..	..	A6s	2.3	.020	A8s	1.9	.017
4026	3.8	..	1.5	.035T	A6s	1.8	.042	A4s	1.7	.038
40[illegible]8	5.2	5.2	..	..	A0n	0.9	.014	A0n	1.1	.015
4072	5.0	6.4	..	..	A5n	2.5	.032	A4n	1.8	.023
4081	4.8	1.0	..	..	..	..	..	A3n	1.5	.022
4229	5.6	2.5	..	..	B9s	0.4	.009	B9s	0.4	.009
4232	5.6	2.3	..	..	..	..	..	B9n	0.2	.008
4376	3.2	..	0.4	.028T	A0n	0.9	.035	A0n	1.0	.036
4581	3.7	..	1.5	.037T	A5s	1.8	.042	A5s	2.1	.048
4747	5.1	3.7	..	..	..	..	..	A1n	1.1	.016
4749	5.1	4.1	..	..	..	..	..	A1n	1.3	.017
4752	4.3	1.5	1.4	.026T	A5s	2.2	.037	A4s	1.5	.028
4754	5.9	2.6	3.3	.030T	A1n	1.5	.013	A1n	1.9	.016
4761	4.4	..	3.1	.056T	A2n	2.0	.034	A4n	2.6	.044

TABLE VI—*continued*

Boss	H.D. m	H	Trig. or Gr.		Mt. Wilson			Douglas		
			M	p	Class	M	p	Class	M	p
4802	4.5	2.7	..	..	..	..	..	A1n	1.3	0.023
4803	5.4	3.8	..	..	..	..	..	A1n	1.8	.019
4824	3.3	..	−1.3	0.012T	B9s	−0.2	0.020	B9n	−0.2	.020
4858	3.0	..	0.8	.037T	B9n	0.8	.036	B9n	...	..
4988	3.9	..	0.2	.018T	A1n	1.1	.028	A1n	1.0	.026
5048	3.0	..	1.7	.055T	A1n	1.1	.042	B9n	..	..
5062	0.9	..	2.5	.203T	A2n	1.7	.145	A1n	2.0	.166
5186	5.0	1.1	..	..	..	..	..	A1n	1.6	.021
5187	4.0	−4.6	..	..	..	..	..	B9s	0.6	.021
5337	3.8	..	0.6	.023T	A1n	1.6	.037	A1s	1.0	.028
5480	2.6	..	2.2	.084T	A2n	1.3	.056	A2n	2.5	.096
5600	3.0	..	3.3	.115T	A3n	2.2	.068	A6s	2.2	.069
6031	4.9	..	3.1	.043T	A3s	1.2	.018	A3s	0.7	.014

Comparison with Mt. Wilson Magnitudes.—Of the 200 stars in Table VI, 108 are also contained in the list of Adams and Joy.[2] Comparing the two classifications a close agreement is found as is to be expected; only occasionally is a star classed *n* or *s* by Mt. Wilson given the reverse class by the writer and the decimal subclass is seen to be rarely very different.

Taking the writer's *n* and *s* groups and comparing them with the Mt. Wilson values of absolute magnitude the systematic and probable errors were computed as follows:

TABLE VII

	Systematic error e	Probable error r	No. of Stars N
n	+0.07	±0.31	68
s	+0.14	±0.30	40
All	+0.09	±0.31	108

Thus it is seen that there is a fair agreement, the present magnitudes being systematically smaller by about $0^{m}.1$ or in other words, the stars are here given brighter than at Mt. Wilson by $0^{m}.1$.

Comparison with Arcetri Magnitudes.—Of the 81 stars in Table VI for which neither trigonometrical, group nor Mt. Wilson spectroscopic values exist, 22 were found amongst the 275 stars of early type determined by Abetti at Arcetri.[10] His values are given in Table VIII for comparison.

TABLE VIII

SPECTROSCOPIC ABSOLUTE MAGNITUDES

Boss	Abetti	Douglas	Remarks
597	1.7	1.6	
730	0.4	0.9	
850	0.9	1.1	
932	1.2	1.0	
971	2.4	2.5	Systematic error −0.027
1352	1.4	1.3	
1452	1.4	0.7	
1488	1.1	0.1	
1759	1.1	0.4	
1974	2.1	2.3	
2088	0.6	0.7	
2754	0.6	1.1	
3139	1.3	1.4	Probable error ±0.30
3182	1.8	2.9	
3283	0.4	0.3	
3354	0.4	0.4	
3356	0.4	0.2	
3749	0.8	−0.3	
3928	1.2	1.5	
3939	2.1	2.5	
4802	1.2	1.3	
4803	1.8	1.8	

Shajn Double Star Test.—As a means of testing the accuracy of spectroscopic parallaxes, Shajn[35] has made use of the relation which should exist between the apparent and absolute magnitudes of components of multiple star systems. Since the parallax should be the same for each component it follows that

$$\Delta M - \Delta m = 0$$

where ΔM is the difference in spectroscopic absolute magnitude of the two components and Δm is their difference in apparent magnitude. Applying this test to the Mt. Wilson determinations Shajn finds that for 55 double stars of late type

$$\Delta M - \Delta m = \pm 0.28 \text{ to } \pm 0.48$$

while for 139 early type (B, A) doubles,

$$\Delta M - \Delta m = \pm 1.08 \text{ to } \pm 1.24$$

though it is only just to state that these figures hardly do Mt. Wilson justice since Shajn applied Mt. Wilson curves to obtain the magnitude of stars whose spectral classification he took as given by Harvard.

Attention having been called to Shajn's work by Dr. O. Struve, the writer compiled a list of A-type double stars of sufficient apparent magnitude and angular separation to allow of separate spectrograms being obtainable. By the courtesy of the Director, the writer was given permission to use the Yerkes 40-inch telescope throughout the week September 6th-13th, 1925, and every effort was made to obtain these spectrograms. Unfortunately, the nights were unpropitious and only seven plates were secured, but the Yerkes observers have very kindly taken spectrograms of several of these stars and sent them to the writer for examination. Thus it has been possible to apply Shajn's test to the star systems given in Table IX.

In two cases the deviation from zero is somewhat large. For the K Tauri pair, Mt. Wilson shows a departure from zero of −0.7, and Rasmuson's group parallaxes give magnitudes having a deviation +0.5. In the case of the ζ Lyrae pair the trigonometrical values are in good accord, deviation +0.3, but the Mt. Wilson magnitudes give a deviation −2.3.

For the eleven systems, however, the average deviation

$$\Delta M - \Delta m = \pm 0.34$$

is as low as can be expected considering the probable error involved in each determination.

TABLE IX

Boss	Name	Class	Spec. p	Spec. M	H.D. m	ΔM −Δm
			″			
422	5γ ArietisN	A0n	0.015	0.7	4.8	
423	“ S	A2s	.014	0.6	4.8	+0.1
463	α′ Piscium Ft	A3s	.018	1.5	5.2	
463	α² “ Br	A3s	.017	0.4	4.3	+0.2
1026	65 K Tauri Br	A2n	.026	1.5	4.4	
1027	67 K² “ Ft	A1n	.018	1.7	5.4	−0.8
3354	32′ H Camelopardi Ft	A0s	.008	0.4	5.8	
3356	32² “ “ Br	A0n	.009	0.2	5.3	−0.3
3370	12 α′ Can. Ven. Ft	A8s	.028	3.3	5.4	
3371	12 α² “ “ Br	A1s	.044	1.1	2.9	−0.3
3474	79 ζ¹ Urs. Maj. (Mizar)	A2s	.044	0.6	2.4	
3475	79 ζ² “ “	A5s	.036	1.8	4.0	−0.4
3480	80 g “ “ (Alcor)	A1n	.044	2.2	4.0	0.0
4229	16 Draconis Ft	B9s	.009	0.4	5.6	
4232	17 “ Br	B9n	.008	0.2	5.6	+0.2
4747	ϵ′ Lyrae Br	A1n	.016	1.1	5.1	
4749	5 ϵ² Lyrae	A1n	.017	1.3	5.1	+0.2
4752	6 ζ′ Lyrae Br	A4s	.028	1.5	4.3	
4754	6 ζ² Lyrae Ft	A1n	.016	1.9	5.9	−1.2
4802	63 θ′ Serpentis Br	A1n	.023	1.3	4.5	
4803	63 θ² “ Ft	A1n	.019	1.8	5.4	−0.4
5186	30 o′ Cygni Ft	A1n	.021	1.6	5.0	
5187	31 o² “ Br	B9s	.021	0.6	4.0	0.0

Reduced Proper Motion and Magnitude.—Reduced proper motion is defined as the same function of apparent magnitude and proper motion as is absolute magnitude of apparent magnitude and parallax.

$$M = m + 5 + 5 \log p$$
$$H = m + 5 + 5 \log \mu$$

where μ is the total proper motion and H is the reduced proper motion. Since μ varies with distance as well as with space velocity, it seems logical to suppose that considered statistically M and H should exhibit a strong correlation. With this in mind the correlation coefficient was worked out rigorously from the data in Table VI, where values of H are recorded for 129 stars including all those for which no trigonometrical or cluster parallaxes are given.

Instead of finding a reasonable correlation the coefficient came out to be 0.030. This low value is not due to a few exceptional stars, but is thoroughly representative of the data, over half the stars having x and y deviations of opposite sign. Adams records a very close correlation between his spectroscopic parallaxes and proper motion, and Struve has demonstrated a like relationship in the case of the writer's parallaxes, concluding that there is all reason to say that these spectroscopic parallaxes agree perfectly with the expected distribution of μ, the remaining dispersion being due largely to the peculiar motions of the stars and in part to the probable error of the spectroscopic parallaxes. It thus seems that in the process of formation of the H function from μ the essence of the correlation with M is lost.

Some Problems of A Stars.—(1) It is disappointing that not one of the criteria is single-valued. In the stars of Class A we are face to face with a serious problem. Some as yet unknown or unrecognized factor is playing an important part in determining the character of the spectrum. Stars of this type are evidently at a critical stage of development, the transition from the giant stage to the dwarf stage. On the older theory of Russell[36,37] this might be thought of as the transition from the state of a perfect gas to a denser state where the gas laws begin to break down, but since Eddington[38] has shown that the dwarf stars of the main sequence are probably to be regarded as also conforming to the perfect gas laws, on account of the high degree of ionization produced by their great central temperatures, it now becomes necessary to look elsewhere for the cause of complexity. Why do stars turn down the main sequence? Jeans[39] explains it in terms of the automatic reduction in the rate of production of radiant energy at the centre of a star when its central temperature exceeds about 30 million degrees. Fowler and Guggenheim[40] have given quantitative evidence in favour of the assumptions of Eddington and Jeans that at these temperatures there would be 99% ionization. As a star approaches complete ionization, its radiation will be unable to increase further. Though its central temperature will be maintained, its density will continue to increase, accompanied by an increase in the absorption coefficient, and thus luminosity will gradually fall off.

There is in this theory, however, no direct clue to the interpretation of the spectra of stars at the transition stage. The writer's material provides independent evidence that the Mt. Wilson subdivisions, *n* and *s* (according as the absorption lines are nebulous or sharp), are of the utmost importance in forming magnitude correlations but do not represent two distinct classes of stars, there being all gradations in line character from the extremely hazy and ill-defined to the extremely sharp, clean-cut, narrow line.

That there is some as yet unrecognized factor in the atmospheres of A stars seems certain. Harvard investigators have stressed this, pointing out that a one dimensional classification of A stars is inadequate.[41] The Henry Draper classification is based primarily upon the intensity of H and K and is consistent on this basis. Mt. Wilson investigators have adopted a different basis. If the helium lines λλ4026, 4471, 4636 are showing in a Draper A0 star, they call it B9 and they classify the A stars chiefly by the number and intensity of the metallic lines without reference to the intensity of H and K. The writer followed Mt. Wilson fairly closely until gradually a personal classification was felt to be shaping itself. The presence of the helium " raies ultimes" was considered sufficient to warrant the designation B9, unless there were just a trace of these lines accompanied by well developed K. An A0 star usually showed only λλ4481, 4227, 4233, 4215, 4549, 4535 with barest traces of anything else (except of course H, K, and the Balmer lines of hydrogen), and even these lines too weak in general for relative intensity estimates to be made. Growing intensity of these lines and the appearance of other metallic lines marked the A1 to A5 stars, but no star was classed by the writer later than A5 no matter how many lines were up unless the hydrogen lines were beginning to stand out less conspicuously. In an A5 star the hydrogen lines are sinking into comparability with the stronger metallic lines and in an F0 star there is equality between the outstanding metallic lines and the diminishing hydrogen lines.

(2) Shapley and Fairfield[34] found no correlation to exist between the width of hydrogen lines and absolute magnitude, but they found slight correlation between width and reduced proper motion for late B and early A stars, indicating that there is a tendency for narrow line stars to have low space velocities. No explanation has been hazarded. The present material throws some light on the question. The *n* and *s* stars must be dealt with separately at least in the case of Hδ—there being no apparent correlation between widths of *n*-lines with absolute magnitude, but a strong correlation in the case of the *s*-line stars (Fig. 2). This together with the failure to find a general correlation between M and H indicates the possibility that the suggested relationship with space velocity is illusory.

(3) A perplexing problem is presented by the stars of c-characteristic whose spectra exhibit lines so narrow and sharp as to resolve the usual HεH blend into two distinct lines of independently measurable widths. Of the 250 stars studied, 10 stars fall into this class. Three of these stars have well-established parallaxes and in each case the writer's *M* was too low. That the present criteria failed badly in the case of Sirius shook faith in the applicability of these criteria to stars of this extreme class, and pending further study they have been omitted from Table VI.

(4) Certain stars have individual peculiarities which offer problems upon which as yet it is premature to attempt explanations. A few of these may be noted.

Boss 370. 43 ωCassiopaiae A2s. It is very unusual to find λ4535 equal to and λ4549 greater than λ4481, while simultaneously λλ4215, 4233 are much more intense than λ4227.

Boss 1516. 17 Leporis A6s. λ4481 is very faint and a line at about λ4546 is very strong. This star has the H.D. class A0 which the writer finds difficult to understand for though the K line is certainly faint there are many metallic lines very sharp and intense and Hβ, Hγ, Hδ are waning.

Boss 3749. 29 πBoötis (Br) A2s. The line λ3984 is strong. It is probably the line of unknown origin recorded by Belopolsky[42] in αCanum Venaticorum and by Lockyer and Baxandall[43] in αAndromedae. Another unusual line to be outstanding is λ4137, a very weak Fe^+ line undoubtedly blended with some line of unknown origin.[44]

Boss 476 12 K Arietis A5s
" 3409 51 θ Virginis A2s
" 3506 78 o " A5s
" 3561 84 Urs. Maj. A1s

In these stars a pair of *chromium* lines are present with unusual intensity; they are Cr^+λλ4558.89, 4588.43.

Twenty-four *strontium stars* are amongst those in Table VI, remarkable for the intensity of the Sr^+line λ4215. The Boss number, the name, the writer's classification and magnitude, also the average magnitude for that class are given in Table X.

TABLE X

STRONTIUM STARS

Boss	Name	Class (D)	M (D)	$\overline{M}$ (D)	Remarks
370	ω Cass	A2s	0.9	0.7	
423	γ Arie	A2s	0.6	0.7	
463	α^2 Pisc	A3s	0.4	1.1	Si+λλ4128,31 also strong
476	κ Arie	A5s	1.9	1.9	
550	ι Cass	A3s	2.0	1.1	
674	ε Arie	A2s	1.2	0.7	
677	−3° 470 Erid	A2s	1.1	0.7	Sr+ λ4078 also strong
850	+70° 257 Camel	A4s	1.1	1.4	
923	τ^9 Erid	A1s	0.5	0.7	Si+λλ4128,31 also strong
1117	4 Camel	A7s	1.9	2.3	Sr+λ4078 also strong
1122	+11° 646 Ori	A6s	1.8	2.3	Sr+λ4078 also strong
1268	19 Aur	A6s	2.3	2.3	
1453	ξ Aur	A1s	0.6	0.7	
1492	2 Monoc	A7s	1.8	2.3	Sr+λ4078 also strong
1968	97 G Pup	A8s	2.0	2.3	
2339	49 b Canc	A5s	1.2	1.9	Si+λλ4128,31 also strong
2932	60 b Leo	A4s	0.2	1.4	Si+λλ4128,31 also strong
3266	21 Com. Ber.	A3s	1.0	1.1	Sr+λ4078 also strong
3475	ζ U. Maj.	A5s	1.8	1.9	
3506	o Virg	A5s	0.7	1.9	
3561	84 U. Maj.	A1s	0.6	0.7	
3749	π Boötis	A2s	0.3	0.7	very abnormal
4022	β Drac	A8s	1.9	2.3	Sr+λ4078 also strong
4026	ε Serp	A4s	1.7	1.4	
6031	κ Pisc	A3s	0.7	1.1	

The comparison between M and $\overline{M}$ indicates that on the whole the strontium stars are $0^m.24$ brighter than the average stars of the same spectral type, but the scantiness of the data makes generalizations dangerous. C. H. Payne,[45] arguing from proper motion relations and a few individual cases of dwarf strontium stars, concludes that there is no sufficient justification for the statement[46] that these stars are "distinctly brighter than the average." The writer has made as the criterion for the inclusion of a star in this class not any arbitrary scale of absolute line intensity but the one condition that λ4215 > λ4227. As indicated in the Table, the other member of the Sr^+doublet λ4077.7 is sometimes also of outstanding intensity.

The *silicon stars* are represented by fifteen given in Table XI, one or both of the pair of Si^+lines λλ4128.1, 4131.1 being unusually prominent.

A comparison between the magnitudes of these stars and the mean magnitude for their respective classes indicates that they are brighter on the average by $0^m.5$.

TABLE XI

SILICON STARS

Boss	Name	Class (D)	M (D)	$\overline{M}$ (D)	Remarks
463	α^2 Pisc Br	A3s	0.4	1.1	4128,31 both strong
923	36 τ^9 Erid.	A1s	0.5	0.7	4128,31 both strong
998	56 Tauri	A1s	0.3	0.7	4128,31 both strong
1046	78θ^2 Tauri	A5s	1.7	1.9	4128 strong
1117	4 Camel	A7s	1.9	2.3	4128 strong
1482	37 θ Aurigae	A1s	0.1	0.7	4128,31 strong
2088	+79° 265 Camel	A0s	0.7	0.5	" "
2339	49b Cancri	A5s	1.2	1.9	" "
2754	30 H Urs. Maj.	A3s	1.1	1.1	" "
2900	45 ω Urs. Maj.	A0s	0.0	0.5	4128,31 prominent
2932	60b Leonis	A4s	0.2	1.4	4128,31 strong
3409	51 θ Virginis	A2s	−0.3	0.7	4128,31 fairly strong
3474	79 ζ Urs. Maj.	A2s	0.6	0.7	4128,31 fairly strong
3506	78 o Virginis	A5s	0.7	1.9	4128,31 strong
3749	29π Boötis Br	A2s	−0.3	0.7	4128,31 strong

It is worth noticing that frequently, though by no means always, do the lines of Si^+, Cr^+ and Sr^+ occur with unusual intensity in the same star. Evidently conditions favouring one, favour also the others, and the absence of any one or two given the third is a matter of the abundance of the element in the stellar atmosphere.

Mean Magnitude and Spectral Class.—One of the main problems of the A stars is the question as to whether it is possible to interpret the spectra with individual accuracy or whether the Mt. Wilson and Arcetri method of merely adopting the mean magnitude for spectral class is all that can be done at present.

This investigation has been a definite attempt to maintain the former position. How far it has been successful it is difficult to say. The following table together with Tables IV, V, present the evidence for and against the writer's claim that the magnitudes herein determined have a greater individual accuracy than can be obtained by following the Mt. Wilson and Arcetri methods. What is the true interpretation of this evidence, the writer is not in a position to say, the decision must rest with the critical astronomer.

TABLE XII

AVERAGE DEVIATION

Class	A–D				$\bar{D}$–D			
	n ±	N	s ±	N	n ±	N	s ±	N
B9	0.22	5	0.10	2	0.21	7	0.23	3
A0	.41	17	.60	1	.38	31	.27	9
A1	.34	15	.42	8	.43	29	.26	15
A2	.45	8	.56	5	.37	12	.51	10
A3	.18	6	.40	8	.31	10	.52	11
A4	.42	6	.30	3	.31	9	.51	8
A5	.43	6	.29	8	.20	8	.35	11
A6– –F0	.38	5	.22	5	.36	10	.32	17
All	0.37	68	0.36	40	0.37	116	0.37	84

In Table XII a comparison is given separately for the *n* and *s* stars between the average deviations of the present magnitudes (D) from the Mt. Wilson magnitudes (A) and from the mean magnitude ($\bar{D}$) per spectral class. As previously, N indicates the number of stars.

In Table XIII the mean absolute magnitudes for each spectral class are given. The Mt. Wilson figures are averages for the individual means of Adams and Joy.[47] The Arcetri figures are taken from Abetti's diagram,[48] upper full curves, the *s* and *sn* curves being averaged.

The general agreement is good. The Arcetri values are based on all the material available from every source, trigonometrical, group and spectroscopic data being all included and they are to be given greatest weight. The Mt. Wilson values are smoothed by graphical means from the original data of Adams and Joy[49] on 101 *n*-stars and 48 *s*-stars. The writer's values are here given unsmoothed. The group of A6-F0 includes only 5 stars that are later than A7.

TABLE XIII

AVERAGE ABSOLUTE MAGNITUDE

Class	Mt. Wilson		Arcetri		Douglas			
	n	s	n	s	n	N	s	N
B9	0.6	−0.2	0.6	0.3	0.3	7	0.3	3
A0	0.9	0.2	0.8	0.5	0.8	31	0.5	9
A1	1.3	0.6	1.1	0.7	1.3	29	0.7	15
A2	1.7	1.0	1.3	1.0	1.7	12	0.7	10
A3	2.0	1.3	1.5	1.1	2.0	10	1.1	11
A4	2.2	1.6	1.7	1.3	2.0	9	1.4	8
A5	2.3	1.8	1.8	1.5	2.2	8	1.9	11
A6 –F0	2.7	2.4	2.4	1.9	2.3	10	2.3	17
						116		84

Further Investigations.—This work is the preliminary to a more extensive investigation which the writer hopes to carry out in the near future, involving the whole of the A stars of which the Yerkes Observatory has spectrograms. With the permission of the Director of the Yerkes Observatory, many of the spectrograms studied in the course of this work will be re-studied and many others not yet measured will be examined. As Dr. J. S. Plaskett has remarked, the determination of spectroscopic magnitudes is a matter of closer and closer approximations towards the truth. In the light of greatly enlarged material, the criteria used in the present work will probably require modification and readjustment. It is hoped that new criteria may be found, especially in regard to the A0 stars, so many of which had to be omitted from the present list.

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NOTES AND REFERENCES

1. W. S. Adams—Mt. W. Contrib. No. 199; Ap. J. 53, 1921.
2. W. S. Adams—Mt. W. Contrib. No. 244; Ap. J. 56, 1922.
3. W. S. Adams—Mt. W. Contrib. No. 262; Ap. J. 57, 1923.
4. D. L. Edwards—M.N.R.A.S. 83, 47, 1922.
5. D. L. Edwards—M.N.R.A.S. 84, 366, 1924.
6. D. L. Edwards—M.N.R.A.S. 85, 439, 1925.
7. B. Lindblad—Ap. J. 55, 85, 1922.
8. B. Lindblad—Nova Acta R. Soc. Sci. Upsaliensis, IV, 6, 5, 1925.
9. O. Struve—Dissertation, University of Chicago—unpublished.
10. G. Abetti—Parallassi Spettroscopiche di 275 Stelle, Pubb. R. Univ. d. Studi di Firenze (R. Osservatorio Astrofisico di Arcetri) Nr. 42, 1925.
11. W. B. Rimmer—Memoirs R.A.S. 62, 4, 1923.
12. R. K. Young and W. E. Harper—Pub. Dom. Ap. Obsy. III, 1, 1924.
13. H. Macklin—M.N.R.A.S. 85, 444, 1925.
14. N. H. Rasmuson—Meddelanden fran Lunds Astronomiska Observatorium, II, 26, 1921.
15. F. Schlesinger—General Catalogue of Parallaxes, Advance Copy, 1924.
16. Rasmuson's magnitudes are relative to the siriometer scale instead of to the usual 10 parsec scale. Since 10 parsecs = 2.06265 siriometers, it is necessary to add $1^{m}.57$ to all Rasmuson's absolute magnitudes to bring them into line with absolute magnitude on the 10 parsec basis.
17. Van Rhijn—Pub. Kapteyn Astron. Lab., Groningen, 37.
18. Probable error $= 0.8453 \times$ average error.
19. See Ref. 1 (p. 15).
20. Schuster—Ap. J. 21, 192, 1905.
21. Newall—M.N.R.A.S. 65, 636, 1905.
22. J. Q. Stewart—M.N.R.A.S. 85, 732, 1925.
23. J. Q. Stewart—Ap. J. 59, 1924.
24. Merton—Proc. Roy. Soc. 92, 322, 1915.
25. E. O. Hulburt—Ap. J. 55, 399, 1922.
26. E. O. Hulburt—Ap. J. 59, 177, 1924.
27. Rayleigh—Phil. Mag. 29, 274, 1915.
28. R. H. Fowler and Milne—M.N.R.A.S. 83, 403, 1923.
29. R. H. Fowler and Milne—M.N.R.A.S. 84, 499, 1924.
30. R. H. Fowler—M.N.R.A.S. 85, 970, 1925.
31. H. N. Russell—Mt. W. Contr. No. 286; Ap. J. 61, 223, 1925.
32. C. H. Payne—Stellar Atmospheres, 1925 (p. 127).
33. C. H. Payne—Harvard Circ. 263.
34. P. Fairfield—Harvard Circ. 264.

35. G. Shajn (Pulkovo)—Ap. J. 62, 104, 1925.
36. H. N. Russell—Ap. J. 26, 147, 1910.
37. H. N. Russell—The Obsy. 36, p. 324 and 37, p. 165.
38. A. S. Eddington—M.N.R.A.S. 84, 308, 1924.
39. J. H. Jeans—Nature, Jan. 2, 1926.
40. R. H. Fowler and E. A. Guggenheim—M.N.R.A.S. 85, 939, 1925.
41. H. Shapley—Rep. Spectr. Class. Comm., I.A.U., 1925.
42. Belopolsky—Astr. Nach. 196, 1, 1913.
43. Lockyer and Baxandall—Proc. Roy. Soc. 77, 550, 1906.
44. F. J. M. Stratton—Astronomical Physics, 1925 (Appendix VIII).
45. See Ref. 32 (p. 170).
46. Report Spectr. Class. Comm., I.A.U., 1922.
47. See Ref. 2 (Table V, p. 7).
48. See Ref. 10 (Fig 1, p. 14).
49. See Ref. 2 (Table III, p. 6).

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