A. Vibert Douglas University Lectures McGill 1920-1 15.

1st Course of Lectures
Given at McGill 1920-21 Session
AVD

Ps.

Properties of Matter. 1920-21.

Text Book : Poynting & Thomson's Props. of Matter. Omitting Chap. XV.

Reference Books : Edser's General Physics
Moreley's Strength of Materials
Anderson's Physics for Tech! Students.
Wagstaff's Props. of Matter
Boynton's Kinetic Theory of Matter
Boltzmann's Théorie de Gaz.

Other References : Batho's Torsion in Framed Structures – Quebec Bridge.
J. C. Chamberlain's Origin of Earth
Jeans "Relativity" Proc. R. Soc. March 1920
Langmuir & Harkins: Surface Energy Am. Chem. Journal
Rittinger's Theory of Rock Crushing
Rankines Applied Mechanics (G.W.)

Laboratory Course.

Determination of "g": Kater's Reversible Pendulum
Rolling Oscillator
Falling revolving cylinder & tuning fork

Determination of Moments of Inertia: Inertia Wheel
Loaded Torsion Bal. ~~Maxwell's Needle~~

Determination of Elastic Constants:
E & Deviations from Hooke's law for wire
E by deflection of Cantilever
E " " " Supported or clamped rod
E & N by Searle's Oscillation method for wire
N by oscillations of loaded spring.
N and T (torsion) by Maxwell's Needle

Determination of Surface Tension:
Soap films on simple frameworks (descriptive)
T by Capillary tube & Virtual Work Principle.
T by Jaeger's Method for different temperatures

Determination of Viscosity:
η for water by Syphon method
η for Hydrogen by Lehfeldt's Method.

Ps. 199-27.

5.10.20.

1.

— Properties of Matter —

Text book. Poynting & Thomson – Griffin & Co. Ltd. London.

The following notes are intended merely as a guide in reading the above text-book, summarizing it and where necessary explaining it or amplifying ~~upon~~ it.

Introductory Remarks.

Professor Cox late of McGill Univ. in a lecture on "Matter & Aether" put forward the theory of the oneness of the Universe in the following words " Give the aether certain motions and it forms an "atom" of electricity or electrical energy and it is the gathering together of myriads of these that forms an atom of matter ". This is a somewhat vague statement, but the recent brilliant researches of Sir J. J. Thomson, C.T.R. Wilson, Millikan etc and of Sir E. Rutherford have shown conclusively that matter is built up in definite structure by the aggregation of electrons or ~~electrically~~ negatively charged ultimate particles about a more minute nucleus charged positively. This aggregation forms an atom of matter. One or more atoms form a molecule of matter & this is the smallest possible particle of matter which can exist in the free state. The different elements differ from one another by their different atomic structures ranging from the simple H atom – one electron revolving about its nucleus, and the He atom – two electrons revolving about a nucleus, and so on right up the Mendeleeff table each succeeding one being more complex in structure. This subject is dealt with in other courses as in Molecular Physics – but to understand the general Physical properties of Matter it is essential that these fundamental facts be kept in mind. It is well to form some idea of the relative sizes of electrons, atoms, etc. An electron is $\frac{1}{1700}$ of the ~~size~~ mass of an atom of H, and to borrow a simile from Sir Wm Thomson (1883) an atom or molecule : a football :: football : Earth.

Note 4/10/24 Rutherford : If atom size of large room. nucleus is size of pea.

Three states of matter - solid, liquid, gaseous.

In a <u>gas</u> the molecules are comparatively far apart, and are in rapid erratic motion colliding with one another, rebounding and colliding again. There are about 3×10^{19} molecules per cc of air at S.T.P. The intermolecular forces are comparatively weak. Hence the phenomenon of diffusion.

In a <u>liquid</u> the molecules are much closer together, the velocity of motion is less and the mean free path much less. The intermolecular forces are fairly strong. Tensile str. of water is 72 lbs. per sq. inch.

In a <u>solid</u> the molecules are so closely packed that vibration is their only motion and the intermolecular forces are very strong. The tensile strength of mild steel is 30 tons per sq. inch & this is a measure of the molecular attraction in that substance.

Mechanics formulae & Dimensions.

<u>Space</u> : length L area L^2 volume L^3

<u>Mass</u> : M

<u>Time</u> : T

Velocity = dist/time = v = $\left[\frac{L}{T}\right]$

accel^n = $\frac{dv}{dt}$ = dist/time^2 = $\left[\frac{L}{T^2}\right]$

Force = mass x accel^n = ma = $\left[\frac{ML}{T^2}\right]$

Work = force x dist = fs = $\left[\frac{ML^2}{T^2}\right]$

Potential Energy = mgh = $\left[\frac{ML^2}{T^2}\right]$

Kinetic Energy = $\frac{1}{2}mv^2$ = $\left[\frac{ML^2}{T^2}\right]$

Power = work/time = $f.s/t$ = $\left[\frac{ML^2}{T^3}\right]$

Momentum = mv = $\left[\frac{ML}{T}\right]$

Torque or Couple = force x arm to fulcrum = $f.l$ = $\left[\frac{ML^2}{T^2}\right]$

Chap. I Weight + Mass.

Summary:

Facts observed with regard to weight, which is the attraction of matter on the surface of the Earth towards the centre.

1. Weight of a body is constant at same place.
ii " " " " " " in spite of chem. or phys. change
iii " " " " " " " " " temperature changes
iv. " " " " varies at different places on E's surface
v. " " " " " with altitude.

Note how each fact has been experimentally established.
Hence weight is not an absolute measure.
Analyse it to find the variable element.
Weight is a force, and force is defined as that which produces acceleration in a given mass of matter
i.e. $f = ma$ or $f \propto a$, mass being the proportionality constant, (conversely $a \propto f$.)
Comparing different masses the following facts are deduced or observed.

i. Masses of bodies are propl. to the forces producing equal accelerations in them.
ii Masses of bodies are propl. to their weights - at the same point.
iii The mass of a given portion of matter is constant in spite of change of position, form or of chemical or physical condition.

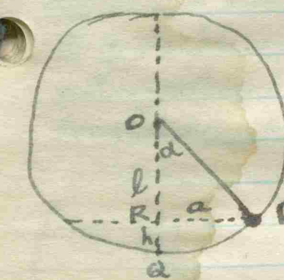
Note Newton's use of pendulum $T = 2\pi\sqrt{\frac{l}{g}}$
From formula for time of swing value of g is measurable. see ~~rough~~ proof on p. 4.

Unit of Mass.
British - 1 pound.
French 1 gm.

1st lecture.

Ideal Simple Pendulum.

Heavy bob of mass m & small volume on a massless string of length l.



Let $OP = l$

$RQ = h$

$RP = a = QP$ approx.

1. P.E. $= mgh$.

$h = a^2/2l = \dfrac{\alpha^2 l}{2}$

$\therefore mgh = mg\dfrac{a^2}{2l}$.

ii. K.E. $= \frac{1}{2} m v^2$

$v = \dfrac{2\pi a}{T}$ where T is time of swing. see Duff

$\therefore \frac{1}{2} m v^2 = \frac{1}{2} m \left(\dfrac{2\pi a}{T}\right)^2$

iii Equate P.E. & K.E. & solve.

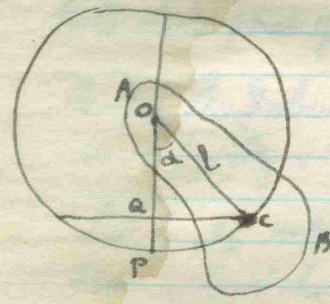
$mg\dfrac{a^2}{2l} = \frac{1}{2} m \left(\dfrac{2\pi a}{T}\right)^2$

$T = 2\pi\sqrt{\dfrac{l}{g}}$

11.10.20.

Compound Pendulum

Let a rigid body be supported so as to be free to rotate about a horizontal axis. If the c.g. of the body does not lie on this axis the body will oscillate to & fro when it is displaced from its position of stable equilibrium & then released. A body supported in this manner constitutes a Compound Pendulum.



Let AB be the comp^d pendulum pivoted at O, c.g. at C.
Let $OC = l$ $\quad P\hat{O}C = \alpha$

1. Consider P.E. ($= mgh$)
C is raised thro. $PQ = h$ and displaced thro $PC = QC = a$

$a = l\alpha$

$a^2 = h(2l - h) = 2lh.$

$\therefore h = \dfrac{a^2}{2l} = \dfrac{l^2\alpha^2}{2l} = l\dfrac{\alpha^2}{2}.$

$\therefore$ P.E. $= mgh = mgl\dfrac{\alpha^2}{2}.$

$= mg\dfrac{a^2}{2l}.$

This shows that motion of c.g. & hence of every element of the body is simple harmonic.

2. Consider K.E. ($= \frac{1}{2} I\omega^2$)

$I = m(K^2 + l^2)$ $\quad$ K = radius of gyration

$\omega = 2\pi\alpha/T$. $\quad$ ~~in Watson~~ Duff - p. 87.

$\therefore$ K.E. $= \frac{1}{2} m(K^2 + l^2) 4\pi^2\alpha^2/T^2$

3. Equate P.E. & K.E.

$\dfrac{mgl\alpha^2}{2} = \frac{1}{2} m(K^2 + l^2)\left(\dfrac{2\pi\alpha}{T}\right)^2$

$\therefore T = 2\pi\sqrt{\dfrac{K^2 + l^2}{lg}}$ $\quad$ //

Reversible Pendulum

To overcome difficulties in measuring length + moment of inertia of a simple or comp^d pendulum, Prony 1800, Bohnenberger 1811 + Capt Kater 1817 independently devised the reversible or convertible pendulum employing the principle of interchangeability of centres of oscillation and suspension.

Note description of Katers Pendulum P+T p. 12.

Comp^d Pendulum formula $T = 2\pi\sqrt{\dfrac{K^2+l^2}{gl}}$

or $\left(\dfrac{T}{2\pi}\right)^2 lg = K^2 + l^2$

Let T_1, l_1 refer to erect position, T_2, l_2 to inverted pos^n

then $\left(\dfrac{T_1}{2\pi}\right)^2 l_1 g = K^2 + l_1^2$

$\left(\dfrac{T_2}{2\pi}\right)^2 l_2 g = K^2 + l_2^2$

Subtracting $\dfrac{g}{(2\pi)^2}(T_1^2 l_1 - T_2^2 l_2) = l_1^2 - l_2^2$
$= (l_1+l_2)(l_1-l_2)$

Rearranging terms

$$\frac{4\pi^2}{g} = \frac{T_1^2 l_1 - T_2^2 l_2}{(l_1+l_2)(l_1-l_2)} = \frac{(T_1^2+T_2^2)(l_1-l_2)+(T_1^2-T_2^2)(l_1+l_2)}{2(l_1+l_2)(l_1-l_2)}$$

$$\frac{8\pi^2}{g} = \frac{T_1^2+T_2^2}{l_1+l_2} + \frac{T_1^2-T_2^2}{l_1-l_2}$$

First term is accurately measurable.

Error in 2nd " is negligible if T_1 + T_2 are almost equal.

Hence difficulty of K and c.g. is eliminated

Find c.g. approximately by graphical method

See Edser p. 99. + lab. work. lab-MS.

Read P. + T pp 13 to 20 for corrections with regard to (1) buoyancy of air

(2) air flow

(3) air drag

(4) yield of support.

Methods of determining variation of gravity over surface of earth

1. Invariable pendulum – hence get earth's ellipticity.

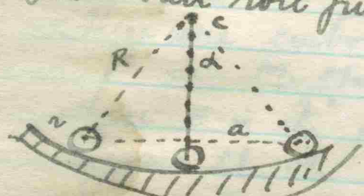
$$\varepsilon = \frac{a-b}{a} = \left[e \ \sqrt{\frac{a^2-b^2}{a^2}} = \right] \frac{1}{283}$$

2. Differential gravity meters – Von Sterneck's Barymeter.
3. Threlfall's + Pollock's Quartz Thread Gravity Balance

2nd lecture

Experimental determination of g by Rolling Oscillator.

Let a ball roll freely on a concave surface



Let C be centre of curvature of surface.

Radius of curvature $R + r$

1. $P.E. = mgh = mg \dfrac{a^2}{2R}$

$= \frac{1}{2} mg \dfrac{a^2}{R}$

2. $K.E. = \frac{1}{2} I \omega^2 + \frac{1}{2} m v^2$

I about diam is $\frac{2}{5} m r^2$

ω at lowest pt is $\dfrac{2\pi a}{Tr}$

$\therefore K.E. = \frac{1}{2} \cdot \frac{2}{5} m r^2 \left(\dfrac{2\pi a}{Tr}\right)^2 + \frac{1}{2} m \left(\dfrac{2\pi a}{T}\right)^2$

$= \frac{1}{2} m \left(\frac{2}{5} + 1\right) \left(\dfrac{2\pi a}{T}\right)^2$

$= \frac{1}{2} m \cdot \frac{7}{5} \left(\dfrac{2\pi a}{T}\right)^2$

3. Equate $P.E. = K.E.$

$\frac{1}{2} mg \dfrac{a^2}{R} = \frac{1}{2} m \frac{7}{5} \left(\dfrac{2\pi a}{T}\right)^2$

$\therefore T = 2\pi \sqrt{\dfrac{\frac{7}{5} R}{g}}$.

Hence length of simple equivalent pendulum is $\frac{7}{5} R$.

See Edser p104

17.10.24

Chap. III. Gravitation

Kepler's 3 laws of motion.

1. The orbit of every planet is an ellipse with the sun in one of the foci.
2. The radius-vector of any planet sweeps out equal areas in equal times.
3. The squares of the periodic times of the planets vary as the cubes of their mean distances from the sun.

These were embodied in the mathematical expression

$F \propto \frac{M_1 M_2}{r^2}$ by Sir Isaac Newton in 1720

where F is the attraction between two bodies of masses M_1, M_2 at a distance r from one another

Then $F = G \frac{M_1 M_2}{r^2}$ where G is the gravitational constant

This is Newton's Inverse Square Law of Gravitation.

Explanatory Note regarding Einstein's modification of Newton's Inverse Sq. Law.

a. The fact that a law is wonderfully simple & beautiful is not a proof of its truth or exactitude.

Compare (1) the statement of early Greek thinkers that circle was perfect curve ∴ orbits were circles. This gives approx. results only, modified to ellipses.

(2) Boyle's Law. $pv = k$ – approx. only

Van der Waals modification $\left(p + \frac{a}{v^2}\right)(v - b) = k$.

Hence it is not astounding to know that Inverse Sq. Law is only an approximation.

b. Two of the causes for suspecting its absolute exactitude were as follows.

(1) The perihelion of Mercury's orbit moved 42" per century more than could be accounted for by Newton's law

(2) Light was deflected by gravitational attraction to a greater extent that Newton's law allowed for.

Read. Jeffreys "The Earth"
Chap IX p. 109
for Isostacy

c. Einstein attempted to meet these and numerous other anomalies by his Theory of Relativity: 1905. This can only be treated mathematically – not physically interpreted – because it deals with a 4-dimensional continuum of coordinates $xyzt$. The equations however predict & explain many physical phenomena. No phenom. is known to be inconsistent with them and several have been proved to be consistent.

d. Sir Frank Dyson Astron. Royal – total Eclipse of May 29/19. Verification of Einstein's prediction of deflection of light in passing Sun through 1.75".

Einstein's first attempt to modify Newton's law of Gravity resulted in 6 times too great a value for Mercury's perihelion motion. He then varied not the law but the continuum and put forward his Ether-Strain theory i.e. kinks & twists in the space around values of $xyzt$ where matter is present. Gravitation is then, not a force but our interpretation of the fact that the continuum is non-Euclidean.

Ref: Jeans in Proc. Royal Soc. March 1920
"Discussion on Theory of Relativity."

$$F = G \frac{M_1 M_2}{r^2}$$

Let M_1 = mass of Earth, r = radius of Earth
M_2 = a mass at surface

then $F = M_2 g = G \frac{M_1 M_2}{r^2}$

$\therefore g = G \frac{M_1}{r^2}$

or $G = \frac{r^2}{M_1} g$

If Δ be mean density of Earth.

$M_1 = \frac{4}{3} \pi r^3 \Delta$

$\therefore G = \frac{r^2}{\frac{4}{3}\pi r^3 \Delta} g = \frac{3 g}{4 \pi r \Delta}$

or $\Delta = \frac{3}{4} \frac{1}{\pi r} \frac{g}{G}$.

Methods of determining G and Δ.

I. Natural Mass.

(a) Mountain :- Bouguer at Chimborazo (1740)
Maskelyne at Scheihallion 1747
also at Arthur's Seat, Mt. Cenis, Fujiyama. P.+T. p.32
(b) Mine pit - Earth's Crust:— Airy - Wolcoath 1826.
Harton Colliery 1854.

II. Prepared Mass

(a) Torsion method :- Cavendish P.+T. p.36
Boys. (Quartz fibre) p.40.
(b) Balance method :- Von Jolly p.41.
Poynting. p.43.

Results. $G = 6.6576 \times 10^{-8}$
$\Delta = 5.5270$

Boys qtz. fibre
torsion bal.

Harton Colliery Experiment. 1854 Airy

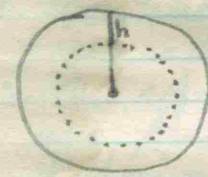
Let Δ = Earth's mean density $= 5.5$

σ = " surface crust density $= 2.5$

h = depth of mine $= 1260$ ft

r = radius of earth $= 4000$ miles

g' = gravitative attraction at base of pit.



At surface attraction is

$$g = G\frac{\frac{4}{3}\pi r^3 \Delta}{r^2} = G\frac{M}{r^2} = g$$

at base of mine pit

$$g' = G\frac{\left(\frac{4}{3}\pi r^3 \Delta - 4\pi r^2 h \sigma\right)}{(r-h)^2} = G\frac{M-m}{(r-h)^2} = g'$$

$$\therefore g' = \frac{G(M-m)}{r^2}\left(1 + \frac{2h}{r} \dots\right) \quad \text{by series expansion.}$$

$$= \frac{GM}{r^2}\left(1 - \frac{m}{M}\right)\left(1 + \frac{2h}{r}\right)$$

$$= \frac{GM}{r^2}\left(1 - \frac{m}{M} + \frac{2h}{r}\right)$$

$$= g\left(1 - \frac{4\pi r^2 h \sigma}{\frac{4}{3}\pi r^3 \Delta} + \frac{2h}{r}\right)$$

$$= g\left(1 - 3\frac{h}{r}\frac{\sigma}{\Delta} + 2\frac{h}{r}\right)$$

$$= g\left(1 + \frac{h}{r}\left[2 - 3\frac{\sigma}{\Delta}\right]\right)$$

$$= g(1 + \varepsilon)$$

corrected & checked 18/10/24.

At surface $T = 2\pi\sqrt{l/g}$

In mine pit $T' = 2\pi\sqrt{l/g'}$

ε is positive only as long as $2 > \frac{3\sigma}{\Delta}$ but as h increases $\sigma \to \Delta$ and $\therefore \varepsilon$ becomes negative & at centre $h = r$, $g' = 0$

$$\therefore \frac{T'}{T} = \sqrt{\frac{g}{g'}} = \sqrt{\frac{1}{1+\varepsilon}}$$

$$= 1 - \frac{1}{2}\varepsilon \quad \text{by series expansion}$$

$$= 1 - \frac{1}{2}\left(\frac{h}{r}\left[2 - 3\frac{\sigma}{\Delta}\right]\right)$$

$$= 1 - \frac{h}{r}\left[1 - \frac{3}{2}\frac{\sigma}{\Delta}\right] \quad \text{3rd Edition}$$

from which Δ is obtained.

[Note. This pit was 1260 ft. deep. In 24 hours the second pendulum only gained 1.64 oscillations by this change in distance from centre of Earth.]

Qualities of Gravitation. P & T. pp. 45 - 52

1. Range - there appears to be no upper limit e.g. Inverse Sq. Law holds approximately for inter planetary distances. The lower limit is not definitely known since when "contact" occurs - i.e. intermolecular distances other forces appear to dominate. *(1)
2. Not Selective :- varies only as the mass + distance not as the nature of the material.
3. Not affected by Medium. proof by spring balance and pendulum not absolutely conclusive. Austin + Thwing's experiments with Pb, Zn, Hg, H_2O, alcohol + glycerine screens - No definite variation detected. *(2)
4. Not Directive - Mackenzie's Calc spar crystals experiment with Boy's arrangement. No definite change found.
   Quartz crystal test for Quadrantal and Semicircular couples gave as upper limits for these couples 1 in 16,000 and 1 in 28,000 which is quite negligible.
   Compare directive influence on light energy through quartz crystal is 1st axes, yellow light changes its rate of travel by 1 in 170.

Summary :- The law $F = G\frac{M_1 M_2}{r^2}$ appears to be unaffected by any change either physically or chemically of the masses concerned or of the intervening medium.

*Note: (1) Recent work with Bohr's Atom shows that the Inv. Sq. Law alone can be considered as operative to within distances of the order of 10^{-10}. (Diam. of nucleus 10^{-15}, diam of atom 10^{-8}) Reference: C. Glo. Darwin on Elect + Magnetic forces within the atom.
(2) Nov 1920 Physical Review - Majorana - detection of variation of order of 10^{-12}, by Balance method.

Note (1) (2) Rutherford + Bieler

then no 4th law of attraction but these are electrical force

25.10.28.

Chap. IV. V. VI. Elasticity; Strain + Stress.
Reference to Morley's Strength of Materials.

Stress. The equal + opposite action and reaction which take place between two bodies, or two parts of the same body, transmitting forces constitute a stress.

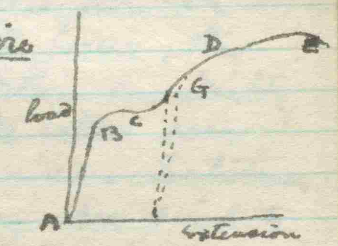
Strain. The alteration of shape or dimensions resulting from stress is termed strain

Stress is dynamical, strain is geometric

Elasticity. A material is said to be perfectly elastic if the whole of the strain produced by a stress disappears when the stress is removed.

Plasticity - A material is said to be perfectly plastic when no strain disappears on the removal of the stress which produced it.

Consider the behavior of a loaded wire
Plot extension against load,
i.e. strain against stress.



By experiment it is found that within what is known as the Elastic Limit the strain is proportional to the load. This is known as Hooke's Law "ut tensio sic vis" (contemporary of Newton.)
If it be written ceiiinosssttuv
Stress = C strain
c is the modulus of elasticity.
If the stress be removed, the strain vanishes.
This is the portion AB of curve. From B onwards the strain is more or less permanent and the wire is said to have acquired permanent set.

at C a very small additional load produces a very large strain. This is known as the Yield point, and the stress at this point is called the Commercial Elastic limit

Beyond C over the portion D of the graph the extension becomes a function of time as well as load, the cross-section is no longer uniform & beyond D the extension increases even with a dimination of load until the Breaking Point is reached.

Metallography or the microscopic study of the structure of metals has shown them to be aggregates of crystals separated by films or membranes of material of different composition. When strained beyond the yield point there is a fracture along the planes of cleavage and a relative slipping of the crystals.

If a ~~glass~~ quartz-fibre and a glass fibre be subjected to tensional & torsional forces, the former recovers its original condition immediately on the removal of the force, the latter does not, considerable time being required. Substances displaying this delay in recovery are said to be subject to Elastic after-effect. (see Curve p.56 P.+T.)

The rate of decay of vibrations of wires of different metals is not constant but varies according to the internal structure of the metal i.e. to its Viscosity, the resistance offered by one layer to the motion of another upon it.

(Note: another use of term Viscosity is that property possessed by some substances to flow like water when subjected to a small force for a long time. (pitch)

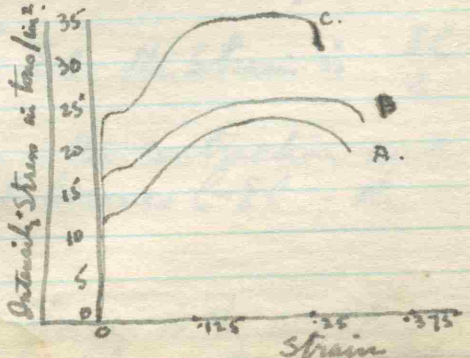
The rate of decay of vibrations of a given wire is not constant but increases if the wire be kept vibrating for some time
This is called Elastic fatigue by Lord Kelvin.
A possible explanation may be that the continued viscous friction generates heat, this causes an expansion and greater molecular activity and hence increases the viscosity & retardation.
No quantitative theory of fatigue. It is a function of past history. Dr Shaw on Bifilar Suspensions. Phil Mag.

Lecture 4.
The ultimate strength of a material under a particular kind of stress is the maximum load necessary to rupture a specimen of that material subjected to that stress, divided by the original area of section at place of fracture.
The working stress of a part of a machine or structure is the greatest calculated stress to which it will ever be subjected.

$$\text{Factor of Safety} = \frac{\text{Ultimate Strength}}{\text{working Stress}}.$$

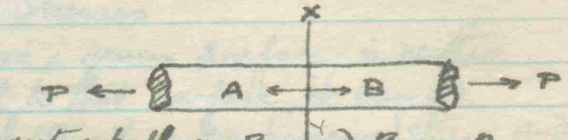
Tenacity is the ultimate strength when the stress is one of pure tension.
Ductility is the property of a material which allows of its being drawn out by tension to a smaller section

Tenacity curves
for A. Wrought Iron
B. Mild Steel
C. Bessemer Steel
See Morley p. 41–43.



Simple Stresses.

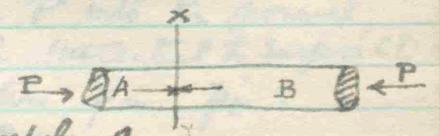
(1) Tensile Stress



Force in one direction only. A exerts a pull on B and B on A.
e.g. tie-bar sustaining a longitudinal pull P if area cross section is a at X then the mean intensity of tensile stress at X is the average force exerted per square inch of section

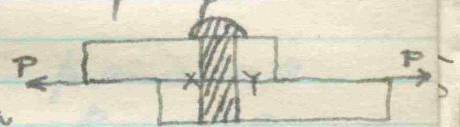
$$p = \frac{P}{a}.$$

(2) Compressive Stress



Force along one axis only, A repels B & B repels A.
Mean intensity of compressive stress $p = \frac{P}{a}$.

(3) Shear Stress exists between two parts of a body in contact when the two parts exert equal and opposite forces on each other laterally in a direction tangential to their surface of contact.
e.g. 2 plates with a rivet



if P tons be the shear stress across section X Y of area a sq. inches then the intensity, q, of shear stress at XY is $q = \frac{P}{a}$.

Strain

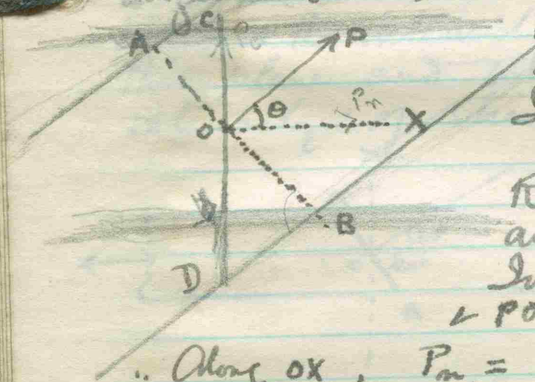
(1) Tensile strain is the stretch in the direction of tensile stress and is measured by the fractional elongation.
If l becomes $l + \delta l$ the strain is $\frac{\delta l}{l}$.

(2) Compressive strain is the contraction under compressive stress. If l becomes $l - \delta l$, the strain is $\frac{\delta l}{l}$.

(3) Distortional or Shear Strain is the angular displacement produced by shear stress in radians

<u>Components of Oblique Stresses</u>.

When the stress across a given surface is neither normal nor tangential to that surface, it can be resolved into normal (tensile or compressive) stress and tangential (shear) stress.



P is the stress $\perp^r$ surface AB of section area a

Intensity normal to AB is

$p = \frac{P}{a}$.

Resolve P into P_n, normal, and P_t tangential to surface CD

Intensities p_n p_t.

$\angle POX = \theta$.

∴ Along OX, $P_n = P\cos\theta$.

Area of CD section is $a\sec\theta$.

hence $p_n = \frac{P_n}{a\sec\theta} = \frac{P\cos\theta}{a\sec\theta} = p\cos^2\theta$.

2. Resolve P into P_t along OC.

$P_t = P\sin\theta$

$p_t = \frac{P_t}{a\sec\theta} = \frac{P\sin\theta}{a\sec\theta} = p\sin\theta\cos\theta$

$= \frac{p}{2}\sin 2\theta$.

Note that p_t is maximum when $\sin 2\theta$ is maxm.

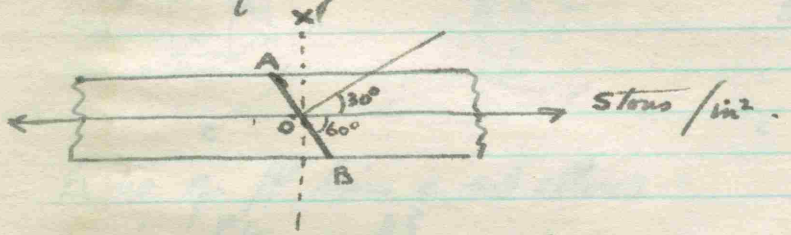
i.e. when $2\theta = 90°$

$\theta = 45°$

Hence fracture often occurs not by the shear of surfaces $\perp^r$ to axis of pull, but inclined to it up to $\frac{\pi}{4}$.

Example.

A tee bar is subjected to uniform tensile stress of 5 tons per sq. inch. What is the intensity of shear stress on a plane the normal to which is inclined 30° to the axis of the bar. What is the intensity of normal stress on this plane and what is the resultant intensity of stress?



Let P ($= 5$ tons/in²) be resolved into P_n, P_t normal to and tangential to AB.

1. $P_n = P\cos 30° = \frac{\sqrt{3}}{2}P = \frac{5\sqrt{3}}{2}$

Intensity of $P_n = \frac{P_n}{\text{area of section AB}}$

$= \frac{P_n}{(\text{area section X})\sec 30°}$

$= P_n \cos 30 = \frac{5\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2} = \frac{15}{4}$

$= 3.75$ tons/in².

2. $P_t = P\sin 30 = 5 \times \frac{1}{2}$

Intensity of $P_t = \frac{P_t}{\sec 30°} = 5 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{5 \times 1.732}{4}$

$= 2.165$ tons/in²

3. Resultant intensity $= \frac{P}{\sec 30} = 5 \times \frac{\sqrt{3}}{2} = 2.5 \times 1.732$ tons/in²

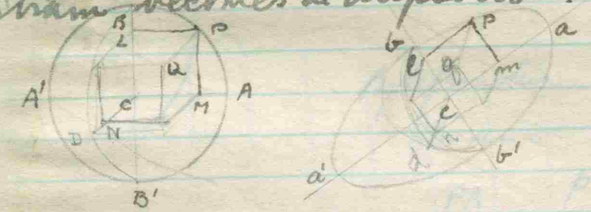
$= 4.330$ tons/in².

28.10.20.

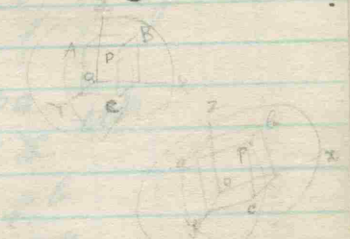
Homogeneous Strain

A body is said to be subjected to homogeneous strain if any two equal & ||^{l} lines in the unstrained state remain equal & ||^{l} after the strain has taken place.

To prove that a sphere subjected to Homogeneous Strain becomes an Ellipsoid.



Consider 3 dimensions.



Ratio of ||^{l} lines is not altered

$$\therefore \frac{PL}{CA} = \frac{pl}{ca}$$

$$\frac{PM}{CB} = \frac{pm}{cb} \quad \text{and} \quad \frac{PQ}{CD} = \frac{pq}{cd}$$

Squaring & adding columns.

$$\frac{PL^2}{AC^2} + \frac{PM^2}{CB^2} + \frac{PQ^2}{CD^2} \quad \frac{pl^2}{ca^2} + \frac{pm^2}{cb^2} + \frac{pq^2}{cd^2}$$

Since P is on circum of ~~circle~~ sphere, by solid analytical geometry. $\frac{PL^2}{AC^2} + \frac{PM^2}{CB^2} + \frac{PQ^2}{CD^2} = 1$.

$$\therefore \text{also } \frac{pl^2}{ca^2} + \frac{pm^2}{cb^2} + \frac{pq^2}{cd^2} = 1$$

∴ a' b p a b' is an ellipsoid.

P.T.O. p.18a

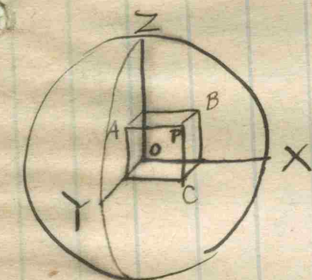
and by analytical geometry. cd, ca, cb are <u>conjugate diameters</u>.

There can only be one set of 3 mutually $\perp^r$ conjugate diameters in an ellipse

∴ only one ~~pair~~ set of $\perp^r$ axes in the unstrained condition will remain mutually $\perp^r$ after straining.

In 3 dimensions there 3 mutually $\perp^r$ axes which remain $\perp^r$ are called the Prime Axes of Strain

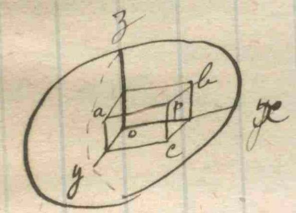
Ellipsoid of Strain



P on surface of Sphere of radius $OP = OX = OY = OZ$.

By definition of homogeneous Strain

$$\frac{PA}{OX} = \frac{pa}{ox} \text{ etc}$$



$$\frac{PB}{OY} = \frac{pb}{oy}$$

$$\frac{PC}{OZ} = \frac{pc}{oz}$$

but Equation of Sphere gives

$$\frac{\overline{PA}^2}{\overline{PX}^2} + \frac{\overline{PB}^2}{\overline{PY}^2} + \frac{\overline{PC}^2}{\overline{PZ}^2} = 1$$

Substituting
$$\frac{\overline{pa}^2}{\overline{px}^2} + \frac{\overline{pb}^2}{\overline{py}^2} + \frac{\overline{pc}^2}{\overline{pz}^2} = 1$$

∴ Strained figure is an ellipsoid and px, py, pz are Conjugate diameters.

A pure strain is one in which the Principal axes of strain do not undergo rotation.

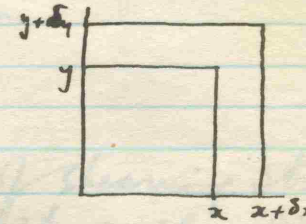
Consider axes of x y z as the princ. axes of strain & let the elongations (or compressions) in these directions be $\pm e$, $\pm f$, $\pm g$.

$e = f = g$ gives uniform dilation or compression.

Let unit cube $(1, 1, 1)$ be strained into a parallelepiped $(1+e, 1+f, 1+g)$
Its volume is $(1+e)(1+f)(1+g)$
$= 1 + e + f + g + \Sigma ef + efg$.
products Σef etc are of 2nd order & may be neglected.
$\therefore$ cubical dilation $= e + f + g = \delta$

If $e = f = g$ then $\delta = 3e$.

Analysis of Homogeneous Strain.



In the xy plane
let δx, δy be the extensions
then $\delta x = ex$
$\delta y = fy$
Write these in the algebraic form
$\delta x = \frac{1}{2}(e+f)x + \frac{1}{2}(e-f)x$
$\delta y = \frac{1}{2}(e+f)y - \frac{1}{2}(e-f)y$.

Hence it is always possible to regard a homogeneous strain as being made up of 2 parts
(1) Uniform dilation $\frac{1}{2}(e+f)$
(2) Shear strain $\frac{1}{2}(e-f)$ elongation on one axis and $\frac{1}{2}(e-f)$ contraction on 2nd axis.

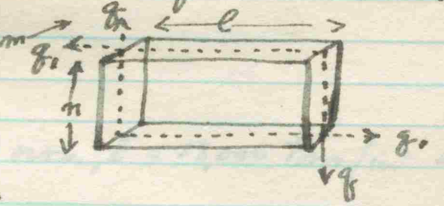
Complementary Shear Stresses.

A shear stress in a given direction cannot exist without a balancing shear stress in a direction at right angles to it.

Consider a rectangular block of sides $l\,m\,n$.

q = shear stress intensity over ends.

q_1 = shear intensity over top & bottom.



By statics there can be no equilibrium if the tangential forces q only are present since they exert a couple

$F.l = q(n\,m).l$

Balancing this there must be shear q_1 giving a couple $F_1.n = q_1(m\,l).n$

$Fl = F_1 n$ for equilibrium.

$\therefore q\,lmn = q_1\,lmn$

$q = q_1$

Hence the intensities of shearing stresses across two planes at rt. angles are equal

Elastic Constants.

Three moduli corresponding to 3 simple states of stress.

(1) Young's Modulus (E)

For pure tension with no other stress acting intensity of tensile stress = E × tensile strain.

$$p = E \times e$$

$$E = \frac{p}{e}.$$

For steel or wrought iron, E = 13,000 tons/in² approx.

(see example 2. p. 7. Morley's Str. of Mats.)

4.11.20.

(2) Modulus of Rigidity. (N)

otherwise called Mod. of Transverse Elasticity; or Shearing Modulus.

If a shear stress of intensity q produces a shear strain of ϕ radians,

$$q = N \times \phi$$

$$N = \frac{\text{shear stress}}{\text{shear strain}} = \frac{q}{\phi}$$

For steel $N = \frac{2}{5} E$ = 5,200 tons/in².

(3) Bulk Modulus (K)

If three mutually perpendicular and equal direct stresses, of intensity p produce a change in size but not in shape so that the volumetric strain is $3\frac{\delta x}{x}$, then $p = K \cdot \frac{3\delta x}{x}$ and K is the bulk modulus.

$(x+\delta x)^3 - x^3$

$$K = \frac{\cancel{\text{change in volume}}\ p}{\text{volumetric strain}}.$$

Look up Young & Poisson

(4) Poisson's Ratio $\left(\frac{1}{m}\right)$ or (σ)

Direct stress produces a strain in its own direction and an opposite kind of strain in every direction perpendicular to its own. A bar under tensile stress contracts laterally and extends longitudinally.

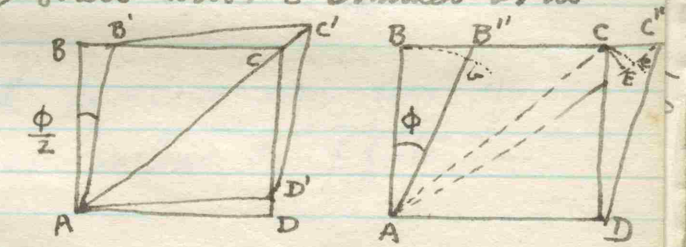
Lateral strain $=$ σ $\times$ longitudinal strain.

$\therefore\ \sigma\ \left(=\frac{1}{m}\right) = \dfrac{\text{lateral strain}}{\text{longitudinal strain}}$

For many metals $\sigma = \frac{1}{4}$ approximately.

Strains in Simple Shear.

A square face of a piece of material under simple shear stress will be strained as in diagram



ABCD becoming AB'C'D'

By a purely geometrical translation swing AB'C'D' into position AB''C''D.

If the strains be very small BB'' may be considered equal to an arc Bb of a $\odot$ centre A, rad. AB and drawing CE $\perp^r$ to AC'', CE approx. equals an arc Ce of $\odot$ centre A rad. AC.

The shear strain $\phi = B\hat{A}B'' = \dfrac{BB''}{BA} = \dfrac{CC''}{CD} = \dfrac{q}{N}$

the linear strain of diagonal $= \dfrac{EC''}{AC}$

$$= \frac{CC''\ \frac{1}{\sqrt{2}}}{CD\ \sqrt{2}} = \frac{1}{2}\ \frac{CC''}{CD} = \frac{1}{2}\ \frac{q}{N} = \frac{1}{2}\phi$$

$\therefore$ angle of shear strain $=$ 2 linear strain of diagonal.

Robt. Hooke 1635 – 1703.

b. Freshwater I. of Wight. educated Westminster School + Christ Ch. Oxford.

Employed by Robt Boyle – made pneumatic pump.
1662 Appointed Curator of Experiments of Roy. Soc.
1663 F.R.S.

Worked upon various problems in surveying, optics, springs, elasticity – "ut tensio sic vis" – use of Spiral springs in watches, clocks &c.

He was mishapen, dishevelled, irritable + quarrelsome, penurious though in his survey work he had become well-off. But reverent + upright of character.

Enc. Brit.

---

quarrelled with Hevelius over use of telescope instead of sights –

Robt Boyle Epitaph

Here lies Hon. Robt Boyle.

Father of Modern Chemistry

+ Brother of the Earl of Cork.

see p. 52 a

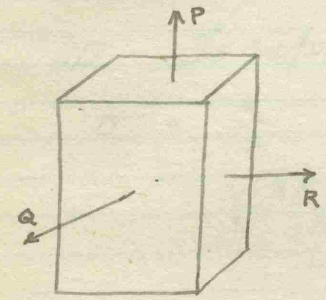
<u>Poisson</u> French Mathematician. b. 1781
Punning quotation from Lafontaine
"Petit Poisson deviendra grand
Pourvu que Dieu lui prête vie."
a gt. prof. of Math & researcher – published 300 works.
His motto "La vie c'est le travail".
Contributions to pure & applied math – paved ~~way~~ for Dirichlet & Riemann
Celestial mechanics, Elect. & magnetism –
In Props. of Matter – Elastic constants – Poisson's Ratio.

<u>Young</u> (Thomas) of Somerset b. 1773 [1823 Wrote of Emmanuel Col. Camb.]
at 14 he knew well or slightly Latin Greek French Italian Hebrew Persian Arabic
Graduated in Medicine; 1801 apptd. prof. of physics in Royal Institution
1803 practised med. & did research – Optics, physiology &c. applied lenses to astigmatic eye
Egyptologist of note – deciphered Rosetta Stone & aided in establishing the hieroglyphic alphabet.

Relations between Elastic Constants E, N, K, σ.

Let $\sigma = \frac{\mu}{\lambda}$ i.e. let the lateral contraction $\perp$ to the direction of unit force be μ and the elongation in the direction of unit force be λ

Consider a rectangular bar under normal stress intensities P, Q, R.



Elongation e produced in dir. of P

$$e = \lambda P - \mu Q - \mu R$$

Similarly

$$f = -\mu P + \lambda Q - \mu R$$
$$g = -\mu P - \mu Q + \lambda R$$

Lecture 6 (12.11.30)

I Express λ, μ in terms of N, K (Rigidity : Bulk)

i. Put $P = Q = R$

then $P = K\frac{dv}{v} = 3Ke$

since $\frac{dv}{v} = 3e$.

But $e = \lambda P - \mu P - \mu P$

$\therefore P = 3K(\lambda P - 2\mu P)$

$\therefore \lambda - 2\mu = \frac{1}{3K}$

ii. Put $Q = -P$ and $R = 0$. i.e. Shear it.

then $P = N\theta = 2Ne$

But $e = \lambda P + \mu P$

$\therefore P = 2NP(\lambda + \mu)$

$\therefore \lambda + \mu = \frac{1}{2N}$

iii. From results of i & ii obtain

$$\lambda = \frac{1}{3}\left(\frac{1}{N} + \frac{1}{3K}\right) = \frac{3K + N}{9NK}$$

$$\mu = \frac{1}{3}\left(\frac{1}{2N} - \frac{1}{3K}\right) = \frac{3K - 2N}{18NK}$$

II To express E in terms of N, K.

Put $Q = R = 0$

$$P = Ee$$
$$= E\lambda P$$
$$\therefore E = \frac{1}{\lambda}$$
$$= \frac{9NK}{3K+N}.$$

III To express Poisson's Ratio in terms of N, K.

$$\sigma = \frac{\mu}{\lambda} = \frac{-f}{e} \text{ when } Q = R = 0$$
$$= \frac{1}{2}\,\frac{3K-2N}{3K+N}$$

Example. For a given material $E = 6000$ tons per sq. in., $N = 2300$ tons/in². Find K and the lateral contraction of a round bar one inch in diam. and 10 ft long when stretched 0.1 inch.

From. I ii $\lambda + \mu = \frac{1}{2N}$

and $\frac{1}{E} = \lambda$

$$\therefore \mu = \frac{1}{4600} - \frac{1}{6000} = \frac{1.4}{27600}$$

From. I i $\lambda - 2\mu = \frac{1}{3K}$

$$\therefore \frac{1}{6000} - \frac{2.8}{27600} = \frac{1}{3K}$$
$$\therefore \frac{1}{K} = \frac{3(4.6-2.8)}{27600}$$
$$K = \frac{46000}{9} = 5111 \text{ tons/in}^2$$

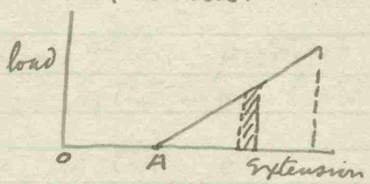
Lat. contraction $= -f = -\mu P = \frac{1.4}{27600} \times Ee$

$$= \frac{1.4}{27600} \times 6000\ \frac{.1}{1200} = \frac{7}{27600}$$
$$= .000254 \text{ in.}$$

Work done in Straining.

$$\text{Work done} = \text{area under Stress-strain curve}$$
$$= \tfrac{1}{2} \text{ load} \times \text{extension}$$
$$= \tfrac{1}{2} (\text{stress} \times \text{area crossection})(\text{strain} \times \text{length})$$
$$= \tfrac{1}{2} \; al \times \text{stress} \times \text{strain}.$$

$= \frac{1}{2}$ stress $\times$ strain per unit vol.



Within the Elastic limit the work done in producing a strain is stored as Strain Energy in the strained material and reappears on the removal of the load. Hence Strain Energy $= \frac{1}{2}$ load $\times$ extension.

The work done during non-elastic Strain is spent in overcoming the Cohesion of the particles of the material + causing them to slide over one another and it appears as heat in the material strained

Resilience is the amount of energy restored by the strained body. Within the Elastic limit

$$\text{Resilience} = \tfrac{1}{2} \text{ load} \times \text{extension}$$
$$= \tfrac{1}{2} \; p(\text{area}) \times \text{strain}(\text{length})$$
$$= \tfrac{1}{2} \; p \times \frac{p}{E} \times \text{volume}$$
$$= \tfrac{1}{2} \; \frac{p^2}{E} \text{ per unit volume}.$$

Proof Resilience is the greatest Strain energy which can be stored in a material without permanent strain i.e. if p_{elim} be the stress intensity at the Elastic limit then Proof Resilience $= \frac{1}{2} \frac{p_{elim}^2}{E}$ per unit volume.

Live Tensile Loads. If a tensile load be suddenly applied to a bar, and does not cause a stress beyond the elastic limit, the bar behaves like a perfect spring & makes oscillations in tension, the amplitude on either side of the equilibrium position being equal to the extension which would be produced by the same load gradually applied.

Hence the maximum instantaneous strain produced is double that which would be produced by the same load gradually applied.

Thus if a "dead" load W produces a strain e
then a "live" load W produces a strain $2e$
or a "live" load $\frac{W}{2}$ produces a strain e.

If a be the area cross section of a bar carrying a dead load W_0 and a live load W be suddenly applied
The maximum stress intensity is $\frac{W_0}{a} + 2\frac{W}{a}$
If the live load be compressive
Maximum stress intensity $= \frac{W_0}{a} - 2\frac{W}{a}$

Example 1. Find the statical or dead load which would produce the same maximum stresses as (a) tensile dead load of 40 tons and tensile live load of 10 tons.
(b) tensile dead load of 20 tons and compressive live load of 30 tons.

(a.) Equivalent static load $= 40 + 2(10) = 60$ tons tension

(b.) " " " $= 20 - 2(30) = 40$ tons Compression.

Example 2. Find proof resilience of a bar of steel $1\frac{1}{2}$" diam 8 ft long, tensile elastic limit being 14 tons/in² and $E = 13{,}500$ tons/in².
Find also proof resilience per cu inch.
Ans. 2760 and 16.26 inch-pounds.

Example 3. Find stress intensity & extension produced in a bar 10 ft. long, 1.5 sq. in. section by a live tensile load of 6 tons. What live load would produce an extension of $\frac{1}{20}$ inch? ($E = 13{,}000$)
Ans. 8 tons/in²; 0.0738 in.; 4.06 tons.

Effect of Temperature on Mechanical Properties of Metals.

The tenacity, ductility & elasticity of most metals do not vary to any serious extent within the limits of ordinary atmospheric temperatures.

At high temps. experiments on wrought iron & steel show
(1) Tenacity: from 60°F to 300°F it decreases by 5% ;
" 300°F to about 600°F " increases to its maximum value by 15%; from 600°F onwards is decreases continuously.

(2) Elastic limit: it falls continuously with increase of temp

(3) Elongation: from 60°F to 300°F it decreases to its minimum;
" 300°F onwards it rises continuously.

(4) Young's Modulus: it decreases with increase of temp
of E = 13,000 approx at 60°F, E = 12,000 at 500°F.

See Morley p 57.

At low temps experiments on mild steel show progressive increase of tenacity with great decrease of temp. The elongation practically vanishes, the material behaving like a very brittle substance.
On return to ordinary temps. no permanent change from the original properties is observed. Morley p57.

[For details of variations with temp see Johnson's "Materials of Construction" with summary of results of Berlin Testing Lab and Watertown U.S. Arsenal.]

Lecture 7 (19/10/20)

Stress due to Change of Temperature.

Where free to do so metals change their dimensions with change of temperature.
If such change of dimensions be resisted and prevented, stress is induced in the material corresponding to the strain prevented

If a bar of length l is heated from t_1 to t_2, its length becomes $l(1 + \alpha\,\overline{t_2 - t_1})$
where α = coefficient of linear expansion.
the extension $\delta l = l\alpha\,\overline{t_2 - t_1}$

$$\therefore \text{ Strain } = \alpha(t_2 - t_1)$$

$$\therefore \text{ Stress } = E\alpha(t_2 - t_1)$$

if the bar ends be clamped and the temp. reduced to t_1

Example 1. A bar of steel 1" diam + 10 ft long is heated to 100°F above temp. of atmosphere, then gripped at ends. Find tension in bar when cooled to temp of atmosphere, if during cooling it pulls end fastenings $\frac{1}{40}$" nearer together. ($E = 13000$ tons/in² and $\alpha = 0{\cdot}0000062$.)

Ans. Stress intensity = 5.33 tons/in². Total pull 4.18 tons.

[Strain = $100\alpha - (\frac{1}{40} \div 120) = {\cdot}00041$
Stress = $E \times {\cdot}00041 = 5.33$.]

Example 2. A short bar of Cu. 1" diam is enclosed centrally in a steel tube $1\frac{3}{8}$" external diam + $\frac{1}{8}$" thick. While at 60°F the ends are rigidly fastened together. Find stress intensity in each metal at 260°F. ($E = 13000$ for steel, $E = 7000$ for Cu
$\alpha = 0{\cdot}0000062$ for steel, $\alpha = {\cdot}000010$ for Cu)

[Excess of free expansion $= {\cdot}00001 - {\cdot}0000062 = {\cdot}0000038$ per degree
$= {\cdot}00076$ for 200°F per unit length.
Let e_s = strain in steel, e_c in Cu per unit length

$e_s + e_c = {\cdot}00076$

Stress intensities are $13000\,e_s$ and $7000\,e_c$

Total stresses are equal $\therefore$ $13000\,e_s \times \frac{\pi}{4}\left(\frac{11^2}{8^2} - \frac{9^2}{8^2}\right) = 7000\,e_c \times \frac{\pi}{4}$

Simplifying $e_c = \frac{65}{56}\,e_s$

$\therefore e_s\left(1 + \frac{65}{56}\right) = {\cdot}00076$ or $e_s = {\cdot}000352$ & $\therefore e_c = {\cdot}000408$

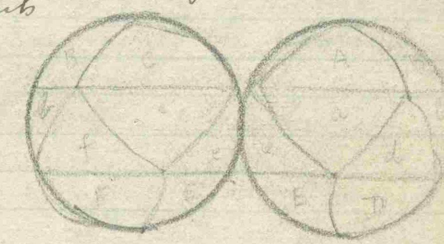
Hence intensity of stress in steel $= E_1 e_s = 4.57$ tons/in².
" " " " Cu $= E_2 e_c = 2.86$ " "]

15.11.20

Application of Theory of Elasticity to Geophysics

Ref: Thos. Crowder Chamberlain - Origin of Earth

Theories of Earth Genesis: (1) Laplaces Nebular Hypothesis
Objections to it.
(2) Planetesimal Theory: Biparental origin
Spiral nebulae, growth due to influx of planetesimals.
Rotation about axis maintained about an equilib. position
Stresses at poles due to ~~slowing down~~ changes of vel. of rotation
Relieved by strains in 3 dirns,
Example of Basaltic columns of Giants Causeway.
Backbones of Continents
Stress-Strain diagram
of Earth



With notation of Maxwell
force diagram

BC = Rocky Mts
fc = Panama + Peruvian height
FE = Andes.
CA = Scandinavian Protaxis
Ce = division of N + S Atlantic
AB = East Asian height
ED = African height
DF = Australia
etc

fb = S. Sea island + suppressed division of N + S Pacific
ad = line from central Africa across S. Arabia, India + joins up with East Asian height
db = belt from Australia thro E. Indies + Malay peninsula.

20/11/20

Critical Features of Planetary System (Solar)

1. Closely appressed disc of revolving bodies centred on invariable plane - with total mass of planets $\frac{1}{745}$ relative to mass of Sun.

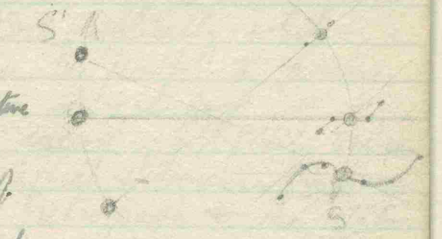
2. Departures from perfect symmetry. Varying eccentricities of orbits. Inclinations of planes of planets from 7° downwards, of planetoids from 38° downwards. These suggest a powerful genetic agency or master force but in addition one or more deviating agencies making the control neither wholly complete nor strictly unified.

3. Invariable plane is inclined to Suns equator.

4. Central controlling body carries $\frac{744}{745}$ of mass of system but only 2% of the revolutionary momentum while $\frac{1}{745}$, the planets carry over 98% of the momentum. On Laplaces hypothesis the Sun should be rotating with 200 times its present rotational velocity.

5. Retrograde satellites, 2 of Jupiter, 1 of Saturn.

Theory of Spiral Nebulae



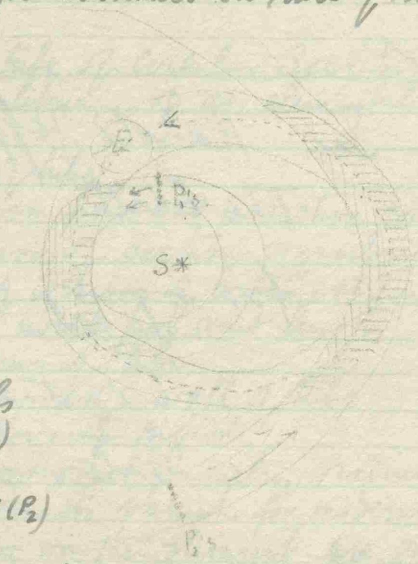
Influence of a large passing star S' on a sun in an eruptive state.

Consider our Sun - 2 belts N + S of its equator of max. eruptive activity. As a star approaches one belt 2 arms of nebula are thrown off - viz. Uranus & Neptune. As it passes over 2nd belt 2 more viz Saturn + Jupiter. Force slightly lessened by 1st pair. ∴ orbits smaller. As star passes its perihelion a multitude of diversified & imperfect eruptions gave rise to planetoids. On its return journey it passes over 2nd area again + Mars + Earth fly off but measurable exhaustion of eruptive potency of Sun is evident. Passing again over 1st belt Venus + Mercury are born.

This is a particular case - not a general theory of spiral nebulae becoming a system.

Each arm of a nebula gathered itself about knots in itself - Possibly these knots contained 30 - 40% of adult mass. Further growth by infall of planetesimals

Effect of infall of planetesimals on rate of rotation of Earth.



When the perihelion of one body's orbit corresponds to the aphelion of another orbit, the body in the larger orbit has the gtr vel. Hence Earth, E, overtakes planetesimals in smaller orbits (P_1) but is overtaken by those in larger orbits (P_2)

If E had no rotation, tendancy to acquire a retrograde rotation is propl. to area shaded vertically while tendancy to a forward rotation is propl. to area shaded horizontally - latter is greater
If E had a forward rotational velocity exceeding a definite value this would increase the retrograde effect & decrease the forward effect until the total effect was retrograde & the rotation was reduced. Thus oscillation about an equilibrium value of rotation would continually take place.

Agencies in Juvenile shaping of earth.
Gravitation tending towards perfect sphere.
Rotation, surface shrinkage & tidal forces tending towards deformation. Of these rotation is greatest tending to flatten polar regions.
Changes of rotation produced swellings & sinkings of polar regions and corresponding compressional & tensional stresses.
→ Hence reciprocating strain triangular wedges across the fulcrum zones. 30° N & S. of equator. forming 6 four sided pyramids swaying N & S. their apecis at centre of earth. This gives an ideal system of flexibility & seesaw action over fulcrum zone. See diagram p. 29.

Lecture 8
26.11.20.

Ch
Ros.
Temp.

1924–25

See Harold Jeffreys
The Earth

for Tidal Theory
Crust of Earth
Internal Temp
etc.

Elasticity Moduli for
crust + core
+ Seismology –

1936 Age of earth. p. 77. H. J. The Earth
p. 78.

Strength of rock with pres. p. 128, 129. F.D.A. etc
Elastic waves. p 155

Forces to build Rockies 10^9 dynes/cm² p. 261.
p. 155 P push, primary, longitudinal $c^2 = (\lambda + 2\mu)/\rho$
S shake, transverse, secondary $c^2 = \mu/\rho$
c = vel. of wave in solid.
Rayleigh wave. p. 157 + 159. } surface layer
Love waves 165 + 167. } only.
velocities p 177, 178.
elastic constants p. 180.

22.11.20.

Chap. VII. Torsion.

1. Thin cylindrical tube of circular Section
Consider the equilibrium of an elemental normal Section.

Let l = length of tube
κ = " from fixed end to section considered
d = thickness of section considered.
Φ = angle of torsion of ~~lower~~ upper extremity.
$\phi = \Phi/l$ = twist per unit length.
a = radius of tube
t = thickness of wall of tube.
N = modulus of rigidity.

From considerations of symmetry radial and longitudinal displacements vanish for moderate torsion.
The shear ~~strains~~ on the element ~~is~~ given by the angle of displacement i.e. $\dfrac{\text{radius} \times \text{angle of displacement}}{\text{thickness of section}}$

$$= \frac{AA'}{d} = \frac{a(\overline{\kappa + d}\,\phi - \kappa\phi)}{d} = \frac{a d\phi}{d} = a\phi$$

$\therefore$ Tangential Stress $= Na\phi$

If external couple $= C$ for equilibrium
C = moment of stress about axis of tube.
= radius × Stress × area of section.
$= a \times Na\phi \times 2\pi a t$
$= 2\pi N a^3 \phi t$
$= \dfrac{2\pi N a^3 t \Phi}{l}$.

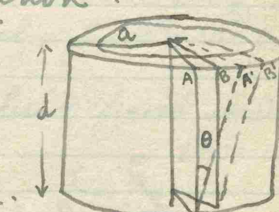


Diagram from Props. of matter by Wapstaff. vol. V. Intermed. Physics.

2. Solid Rod of Circular Section
Consider rod as built up of concentric tubes of varying radii r from $r = 0$ to $r = a$ and thickness dr. Total Couple is sum of couples req^d ~~for~~ each tube.

$$\therefore C = \int_0^a 2\pi N r^3 \phi \, dr = 2\pi N\phi \int_0^a r^3 dr$$
$$= \frac{2\pi}{4} N\phi a^4 = \frac{1}{2}\pi N\phi a^4$$
$$= \frac{1}{2}\pi N \Phi \frac{a^4}{l}.$$

For thick tube a, b $C = \frac{1}{2}\pi n \Phi \dfrac{a^4 - b^4}{l}$.

Work & Energy in Torsion.

The formulae for circular sectioned rods &c. shows that $C \propto \Phi$, hence a str. line curve is obtained and the work done in twisting is equal to area under the $C\Phi$ curve

Work done = energy stored $= \frac{1}{2} C \Phi$

$$= \frac{1}{2} \cdot \frac{1}{2} \pi \cancel{N} \frac{a^4}{\ell} \frac{\Phi}{2} \cdot \Phi$$

$$= \frac{1}{4} \pi \frac{N a^4}{\ell} \Phi^2$$

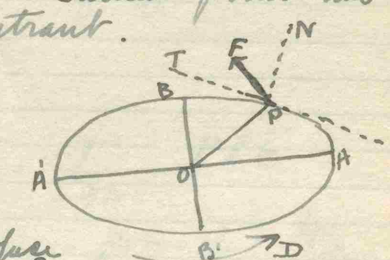
$$= \frac{1}{4} \pi N l a^4 \varphi^2$$

Non-circular Sections

In all cases of the torsion of bars of non-circular section, there is longitudinal displacement both tensional & compressional.

The investigations of the St Venant (see P.T. pp. 80, 81) have led to the following generalizations.

1. Where the component of tangential stress along the normal to the boundary is inwards, the longitudinal displacement is tensional i.e. away from the fixed end.
2. When this normal component is outwards the displacement is compressional, i.e. towards the fixed end.
3. The stress vanishes at a salient point and becomes infinite at a reentrant.



Consider Elliptic Section

Stress at P due to torsion in direction D is PF ⊥r OP.

Resolve along PN normal to surface

Component is outward on an elemental section

∴ for equilibrium there must be an equal inward force.

∴ also a complementary shear stress acting vertically

∴ displacement to or from fixed end) (P.T. notes)

"The Calculation of Torsion Stresses in Framed Structures & Thin-walled cylinders or prisms."

Prof. C. Batho of McGill in a paper on the above subject in "Engineering" Oct. 1915, proves that if a hollow cylinder or prism, either continuous-walled or of framework, and having plane ends perpendicular to its length, be subjected to a twisting moment by couples in the planes of its ends, the total longitudinal shear is everywhere constant and equal to the twisting moment multiplied by the length of the cylinder, and divided by twice the area of one of its ends.

i.e. $S = \frac{Tl}{2A}$. where S = total longitudinal shear
T = twisting moment
A = area bounded by contour of cross section.

1. In a thin-walled cylinder of thickness t, the intensity of shearing stress over a longitudinal section of the wall normal to the contour is

$$q = \frac{S}{area} = \frac{S}{lt} = \frac{Tl}{2A}\cdot\frac{1}{lt} = \frac{T}{2At}.$$

To find N, the modulus of rigidity: consider the work stored. $W = \frac{1}{2}\frac{q^2}{N}$ per unit volume

$= \int \frac{q^2}{2N}\cdot dV$ where dV = element of vol.

Also $W = \frac{1}{2}T\Phi$ where T = total twisting moment & Φ = angle of twist.

$$\therefore \Phi = \frac{2}{T}\int\frac{q^2}{2N}\,dV$$

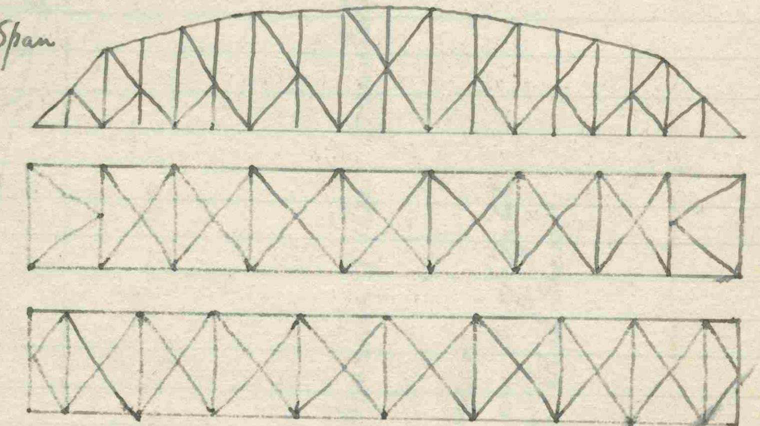
but $q = \frac{T}{2At}$

$$\therefore \Phi = \frac{T}{4A^2Nt^2}\int dV. = \frac{T}{4ANt^2}\,Plt$$

$$= \frac{TPl}{4A^2Nt}$$ P being perimeter of section.

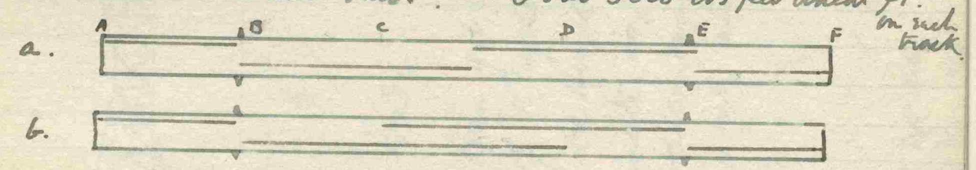
This can be employed in determining torsional shear in solid shafts of circular, elliptical, triangular, L, T and other sections.
"Engineering" Nov. & Dec. 1916.

2. Application to Bridge Design
Torsion stresses in the Suspended Span of a Cantilever Bridge due to Unsymmetrical live loads
In "Engineering" Oct. 1915 Prof. Batho considers the case of the Quebec Bridge



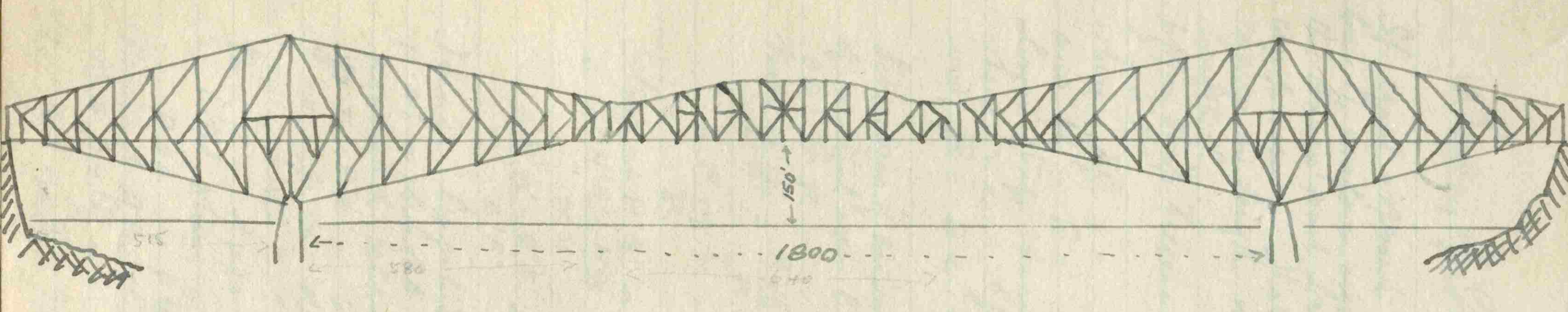
Length 640 ft. : main trusses 88 ft apart.

Plan of Bridge showing (a) loading for maximum torsion in Laterals (b) " " " " Main truss. Load 5000 lbs per lineal ft. on each track.



AB + EF are anchor-arms each 515'
BC, DE " cantilever arms " 580'
CD is suspended span 640'
} Total span 1800 ft.

Theorem. If a framed structure consisting of 2 ||^{l} trusses similar in outline & connected by lateral bracing, be subjected to equal, opposite & ||^{l} couples consisting of unit forces at extremities of base, the Shear S perpendicular to the plane of the trusses is constant throughout the lateral system and equal to the area of the base of the framework divided by twice the area of one of the trusses.



General Elevation of Quebec Bridge.

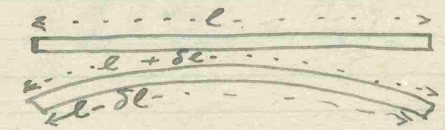
1. Longest & heaviest single span yet built.
2. First important example of "K" system of web bracing. (idea formulated by Phelps Johnson and design developed by G. Herrick Duggan)
3. It is statically determinate as regards stresses.
4. Deflection is uniform & secondary strains are negligible.
5. Each web member carries only about half the total shear.

History: Phoenix Bridge Collapsed 1907.
Dom. Br. Co. suspended span fell Sept 1916.
Bridge completed Sept. 1917
Opened for regular train service Dec 1917

6.12.20.

Bending of Rods. Chap. VIII.

If equal opposite couples be applied at the ends of a bar of uniform cross section it is bent into a bow the convex part being stretched, the concave part being compressed. That axis of the bar which does not change in length is called the Neutral Axis.



Length l

Upper strip becomes $l+\delta l$

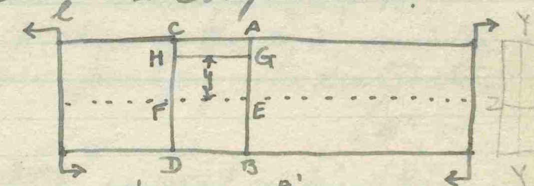
where $\frac{\delta l}{l} = e$

and $p = Ee = E\frac{\delta l}{l}$ = tension

Lower strip becomes $l-\delta l$

where $-e = -\frac{\delta l}{l}$

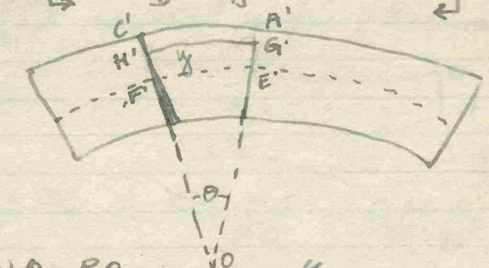
and $p = Ee = -E\frac{\delta l}{l}$ = compression.



Let radius of curvature of neutral axis be $OE' = R$.

Now $\frac{H'G'}{E'F'} = \frac{\overline{R+y}\,\theta}{R\theta}$

$= \frac{R+y}{R}$



Strain at $H'G'$ is

$e = \frac{H'G'-HG}{HG}$

$= \frac{H'G'-E'F'}{E'F'} = \frac{(R+y)\theta - R\theta}{R\theta} = \frac{y}{R}$

Hence longitudinal tensile-stress intensity p at a height y from the neutral surface provided elastic limit is not exceeded is

$p = Ee = Ey/R$. (1)

i.e. $p \propto$ distance from neutral axis.

See Morley p. 105. I is mom. of inertia of area of section about axis.

Now p = stress intensity.

(?) $\therefore$ total stress on element of area δa is

$p\,\delta a = p\,x\,\delta y$ where x is dir^n ~~along~~ axis of bar.

Moment of this stress is $y \times p\,\delta a = p\,y\,x\,\delta y$

But $p = \frac{E}{R}y$ $\therefore$ Moment is $\frac{E}{R}x\,y^2\,dy$

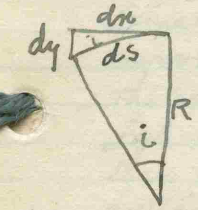
$\therefore$ Total Bending Moment $M = \frac{E}{R}\Sigma\, x\,y^2\,dy$

or $\frac{p}{y} = \frac{E}{R} = \frac{M}{I}$ $= EI/R$ (2)

S. Timoshenko

Value for $\frac{1}{R}$

By definition $\frac{1}{R} = \frac{di}{ds}$

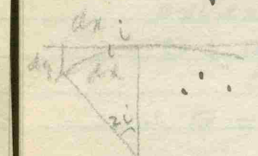


i being small $i = \tan i = \frac{dy}{dx}$

i is angle betw 2 successive tangents

$\therefore \frac{1}{R} = \frac{di}{ds} = \frac{di}{dx}$ approx

$= \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$.



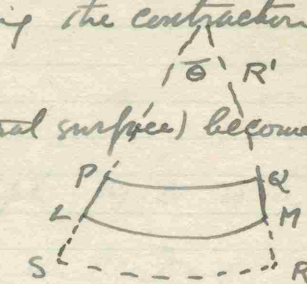
$\therefore \quad \frac{M}{EI} = \frac{d^2y}{dx^2}$. (3)

Consider lateral contraction accompanying the elongation of the upper portion of the rod, and the lateral expansion accompanying the contraction of the lower part.

Let Poisson's Ratio $= \sigma$

Crossection (in plane $\perp^{r}$ to neutral surface) becomes of the form PQRS

LM being line where neutral surface cuts crossection



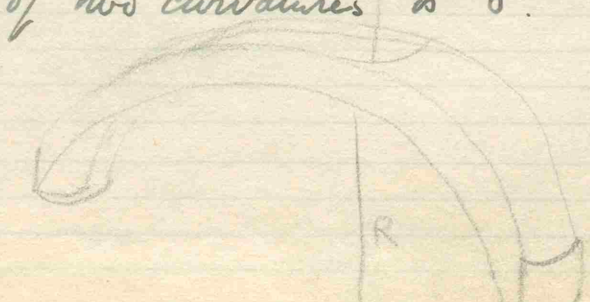
Lateral contraction of PQ $= \frac{LM - PQ}{PQ} = \frac{QM\,\theta}{R'\theta} = \frac{QM}{R'}$

Longitudinal elongation $= \frac{QM}{R} = \frac{y}{R}$ see p. 37.

$\therefore \frac{1}{R'} = \sigma \frac{1}{R}$

or $\frac{R}{R'} = \sigma$

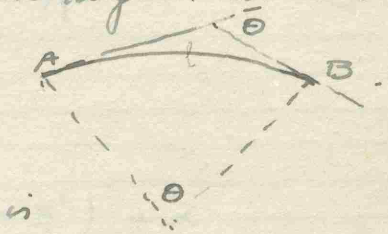
i.e. ratio of two curvatures is σ.



Lecture 10
10/12/20

Angle of deflection

Since under a given set of conditions the radius of curvature is constant, $R = \frac{EI}{M}$, the curve of the rod is the arc of a circle.
Let length of rod be l and radius of curvature R.
then if θ be angle between tangents at the extremities



θ = angle at centre subtended by arc of length l

$\therefore \theta = \frac{l}{R}$

$= \frac{M'}{EI} l$ where M' is

the average Bending Moment from A to B
If $M = 0$ at A and $M = M$ at B.

$M' = M/2$

$\therefore \theta = \frac{1}{2} \frac{Ml}{EI}$.

Energy in bent rod

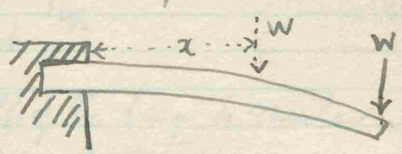
$\theta \propto M$ hence graph M-θ is a str. line

Energy stored = work done in bending
= area under M-θ curve.
$= \frac{1}{2} M \theta$
$= \frac{1}{2} M \frac{l}{R}$.
$= \frac{1}{2} M \; \frac{1}{2} \frac{Ml}{EI} = \frac{1}{4} \frac{M^2 l}{EI}$

Flexure Problems.

I Deflection of a Cantilever arm, simply loaded.

Horizontal axis x
Vertical axis y.
Length of arm l.
If y = depression of end due to load W at a distance x from fixed end.



$$\frac{d^2y}{dx^2} = \frac{M}{EI} = \frac{W(l-x)}{EI}$$

Integrating $\quad \frac{dy}{dx} = \frac{W}{EI}\left(lx - \frac{x^2}{2}\right) + \alpha$

at $x = 0 \quad y = 0 \quad \therefore \frac{dy}{dx} = 0 \quad \therefore \alpha = 0.$

Integrating $\quad y = \frac{W}{EI}\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) + \beta$

at $x = 0 \quad y = 0 \quad \therefore \beta = 0$

$\therefore$ Depression of end is $\frac{W}{EI}\left(\frac{lx^2}{2} - \frac{x^3}{6}\right)$.

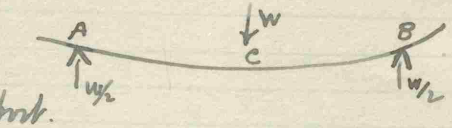
If W be at extremity, $x = l$

then $y = \frac{W}{EI}\left(\frac{l^3}{2} - \frac{l^3}{6}\right)$

$$= \frac{Wl^3}{3EI}. \qquad (I)$$

Note that the deflection at a pt. P due to a wt W at Q is the same as the deflection at Q if the wt. W be a P.

II. Deflection of Beam Supported at Ends, & loaded in middle.



Let l be length between supports.
$\frac{W}{2}$ is reaction on each support.

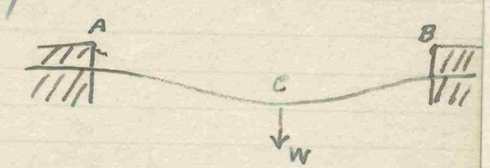
The equilibrium of each half of the beam is similar to that of a cantilever arm inverted.

y = height of A above C

$$= \frac{1}{3EI}\frac{W}{2}\left(\frac{l}{2}\right)^3$$

$$= \frac{Wl^3}{48EI}.$$

III Deflection of Rod clamped at both ends.



Load W at C.
Length $AB = l$
Consider equilibrium of AC.
2 forces acting.

(1) Bending moment $\frac{W}{2}\cdot\frac{l}{2}$ tending to give a deflection $\theta = \frac{1}{2}\frac{M\,l}{EI} = \frac{1}{2}\frac{1}{EI}\frac{W}{2}\frac{l}{2}\frac{l}{2}$

$$= \frac{1}{16}\frac{Wl^2}{EI}$$

and $y_1 = \frac{1}{3EI}\frac{W}{2}\left(\frac{l}{2}\right)^3 = \frac{Wl^3}{48EI}.$

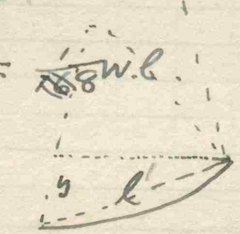
(2) A couple C producing a counterbalancing angular deflection θ since tangents at A & C are both horizontal.

$\theta = \frac{l'}{R} = \frac{1\,lC}{2EI}.$ ← see notes p.39

$\therefore \frac{lC}{2EI} = \frac{1}{16}\frac{Wl^2}{EI} \quad \therefore C = \frac{1}{\cancel{16}\,8}W.l.$

$\therefore R = \frac{EI}{C} = \frac{8EI}{Wl}.$

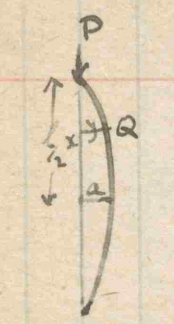
$\therefore y_2 = \frac{l^2}{2R} = \frac{Wl^3}{8\times 8EI} = \frac{Wl^3}{64EI}$



(3) Resultant is (1) − (2) $\quad y = y_1 - y_2 = \frac{Wl^3}{48EI} - \frac{Wl^3}{64EI} = \frac{Wl^3}{192EI}$

Euler's Theorem for Stability of a Loaded Pillar.

(Prof. Brown March 1925
see Morley p. 247



Load P.
Pt. Q (x, y)

$$\frac{d^2y}{dx^2} = \frac{1}{EI}(\text{moment of force at } Q.)$$
$$= \frac{P}{EI}\, y = \alpha^2 y$$

Solution is $y = A\cos\alpha x + B\sin\alpha x$.

Condns are ① $\frac{dy}{dx} = 0$ at $x = 0$

② $y = a$ at $x = 0$

③ $y = 0$ at $x = l/2$.

$$\frac{dy}{dx} = -A\alpha\sin\alpha x - B\alpha\cos\alpha x$$

① gives $0 = 0 - B\alpha$ $\quad \alpha \neq 0$
$\therefore B = 0$.

$\therefore y = A\cos\alpha x$

② gives $a = A$

$\therefore y = a\cos\alpha x$

③ gives $0 = a\cos\alpha\frac{l}{2}$

for stability $a = 0$.
" instability $a \neq 0$ $\quad \therefore \cos\alpha\frac{l}{2} = 0$

Thus $\alpha\frac{l}{2} = \frac{\pi}{2} = \sqrt{\frac{P}{EI}}\,\frac{l}{2}$

$$\therefore P = \frac{\pi^2 EI}{l^2}$$

Read paragraph on Elasticas.
Stability of loaded Pillar - see Lambs Statics.
Laboratory Experiments to find E also
Searle's method of oscillation for N, E.

IX

lecture 11
17/12/20

27.12.20.

Chap. VIII Spiral Springs.

1/12/26

See Morley p 266-8 271

Consider a flat Helix (See Smith's Solid Geom. P. 208)
Axial load W or Equiangular Spiral - Lambs Calc. 366)

for general case of inclined α

Close coiled helix of round wire of ~~radius~~ b about a cylinder of radius a, with an angle of pitch α. Let the no. of complete coils be n, then if l be the length of wire in the spring $l = 2\pi a n$ approx.
Let N be the modulus of rigidity and C the twisting moment
Since the torsional or shear stress is the main force acting on each element of the flat spring

$$C = \tfrac{1}{2}\pi N b^4 \phi \qquad \text{(See Torsion P. 32.)}$$

But $C = Wa$
and $\phi = \Phi/l = \dfrac{\Phi}{2\pi a n}$

$$\therefore \Phi = \frac{4 W a\, \pi a n}{\pi N b^4} = \frac{4 W a^2 n}{N b^4}$$

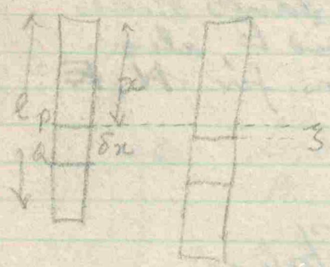
Equate the torsional resilience to the work done in terms of axial load and deflection
Let δ be the total deflection of free end.

$$\tfrac{1}{2} C\Phi = \tfrac{1}{2} W\cdot\delta.$$

$$\therefore \delta = \frac{C\Phi}{W} = \frac{Wa\Phi}{W} = a\Phi = \frac{4Wa^3 n}{b^4 N}.$$

Note $\delta \propto W$ is the principle upon which the Jolly Spring Balance works -

Body stretching under its own wt



Let ξ be amt of stretch at P. then

$\xi = \phi(x)$

$OP' = x + \phi(x)$

& $OQ' = x + dx + \phi(x + dx)$

$= x + dx + \phi x + \delta x\, \phi'(x) + \text{—}$

$\therefore \lim_{dx \to 0} P'Q' = OQ' - OP' = dx + dx \dfrac{d\xi}{dx}$

& tension $\propto \dfrac{P'Q' - PQ}{PQ} = \dfrac{d\xi}{dx}$

But tension $\propto l - x$

$\therefore \dfrac{d\xi}{dx} = K(l - x)$

Integrating $\xi = K\left(lx - \dfrac{x^2}{2}\right) + c$

at $x = 0$ $\xi = 0$ $\therefore$ $c = 0$

$\therefore$ when $x = l$ $\xi = K \dfrac{l^2}{2}$

In this case $\dfrac{\text{Stress}}{\text{Strain}} = \alpha$ experimentally determinable

& the Stress due to its own wt is β (its mass $\times g$)

$\therefore$ K is evaluable in terms of l, α, β

& thus the stretch due to its own wt.

[for an actual case in lab the stretch was equal to that produced by a wt in pan equal to $\frac{1}{4}$ mass of spring)

from notes by A.T.G. Lang[?] '26.

Determination of N for a wire

(1) Searle's Oscillation method. P. 101.

$T = 2\pi \sqrt{\dfrac{Il}{\pi N a}}$ where I = mom. of inertia of bar. l N, a refer to wire.

(2) Oscillations of Loaded Spring. P. 107.

Potential energy $= \frac{1}{2} W \delta$

$= \frac{1}{2} \cdot \dfrac{\pi N b^4}{2 l a^2} \delta \cdot \delta$

Let the extension be x : P.E. $= \dfrac{1}{4} \dfrac{\pi N b^4}{l a^2} x^2$

Kinetic energy is due to vel. $\dfrac{dx}{dt}$ of the weight W and of the spring itself

Former gives $\frac{1}{2} M \left(\dfrac{dx}{dt}\right)^2$

Latter gives :- for element ds at distance s from end vel. is $\dfrac{s}{l}\dfrac{dx}{dt}$ and K.E. is $\frac{1}{2} \rho\, ds \dfrac{s^2}{l^2}\left(\dfrac{dx}{dt}\right)^2$

for whole spring $s = l$ integrate

$\int_0^l \frac{1}{2} \rho \dfrac{s^2}{l^2}\left(\dfrac{dx}{dt}\right)^2 ds = \frac{1}{2}\rho\left(\dfrac{dx}{dt}\right)^2 \left[\dfrac{s^3}{3l^2}\right]_0^l = \frac{1}{2} \dfrac{\rho l}{3}\left(\dfrac{dx}{dt}\right)^2$

$= \frac{1}{2} \dfrac{m}{3}\left(\dfrac{dx}{dt}\right)^2$ where m = total mass of spring. ρ = density of wire.

Thus total K.E. $= \frac{1}{2}\left(M + \dfrac{m}{3}\right)\left(\dfrac{dx}{dt}\right)^2$

P.E. + K.E. = const.

$\therefore \frac{1}{2}\left(M + \dfrac{m}{3}\right)\left(\dfrac{dx}{dt}\right)^2 + \frac{1}{4} \pi \dfrac{N b^4}{l a^2} x^2 = \text{const}$

differentiating with respect to t

$\frac{1}{2}\left(M + \dfrac{m}{3}\right) 2 \dfrac{dx}{dt} \dfrac{d^2x}{dt^2} + \frac{1}{4} \dfrac{\pi N b^4}{l a^2} 2x \dfrac{dx}{dt} = 0$

$\therefore \left(M + \dfrac{m}{3}\right) \dfrac{d^2x}{dt^2} + \frac{1}{2} \dfrac{\pi N b^4}{l a^2} x = 0$

Eqn of type $A \dfrac{d^2x}{dt^2} + Bx = 0$ S.M.H.

$T = 2\pi \sqrt{\dfrac{A}{B}}$

$\therefore T = 2\pi \sqrt{\dfrac{M + m/3}{\pi N b^4 / 2 l a^2}}$.

Scale 12.

2

Seismology.

Modern Seismology – Walker – Longmans, Green. 1913.

The Earth. Jeffreys. Ch. XII.

Longitudinal, compressional — P wave, Primary, PUSH.

$$\text{vel. } v_1 = \left(\frac{\lambda + 2\mu}{\rho}\right)^{\frac{1}{2}}$$

Transverse, distortional — S wave, Secondary, SHAKE.

$$\text{vel } v_2 = \left(\frac{\mu}{\rho}\right)^{1/2}$$

Point of disturbance near surface: interval between P & S waves gives distance ~~or~~ velocity

$v_1 = 7.17$ km/sec

$v_2 = 4.01$ km/sec.

2a

μ & λ are defined as Lamé's notation in Lamb's Statics p. 302. Ex 1.

μ = rigidity modulus

λ is given by.

$$\sigma = \frac{\lambda}{2(\lambda+\mu)}$$

Poisson Ratio

$$E = \frac{(3\lambda + 2\mu)\mu}{\lambda+\mu}$$

See evidence for S cPc S wave & fluid core of Earth by Seismological & Tidal evidence H. Jeffreys & Prof Turner Obsy July 1926 p. 216.

$$\frac{1}{2}\;\frac{3K-2N}{3K+N}$$

$$N=\mu$$

$$2K=3\lambda$$

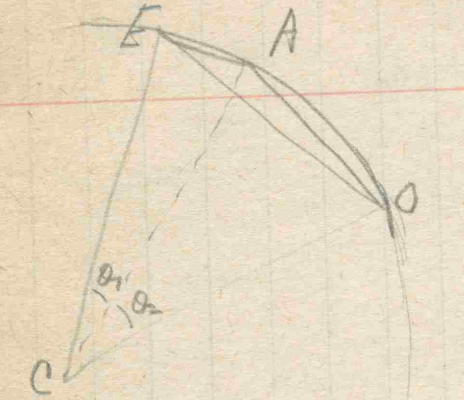
$$K=\frac{3}{2}\lambda$$

$$\sigma=\frac{1}{2}\;\frac{\frac{9}{2}\lambda-2\lambda}{\frac{9}{2}\lambda+\lambda}$$

$$=\frac{1}{2}\;\frac{13}{2}\times\frac{10}{2}$$

Curve

(2)



EO chord

$2R\sin\frac{\theta}{2}$

P wave

EA + AO

longitudinal PR.

When $\theta_1 = \theta_2$ -

EA (P) + AO(S)

EO by S.

+ by reflections.

A'' All these by longer path EA'A''O.

Poissons ratio ~~λ ... $\frac{1}{4}$ ... $\frac{\mu}{\lambda}$~~

~~$\lambda = 4\mu$~~ ~~$\frac{V_1^2}{V_2^2}$~~

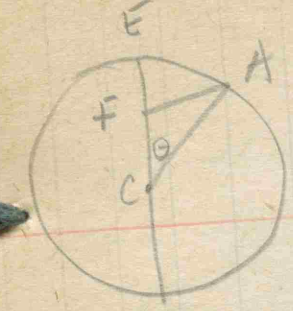
Jeffreys p. 159.

Poissons ratio is $\frac{1}{4}$

then $\lambda = \mu$.

+ $\frac{v_1^2}{v_2^2} = \frac{3\mu}{\mu} = 3$.

3



F = focus

E = Epicentre

average depth 10 km.

Angle of Emergence e

depth L

$$\cos e = \frac{R-L}{R} \sin AFC$$

Example. $L = 10$ Km

$R = 6370$ Km

then $\theta = 3^\circ 12'$

$\Delta = 356$ Km

+ $e = 22^\circ$

but for $\Delta = 1000$ km error in e made by supposing F to coincide with E wd. only be $\frac{1}{2}^\circ$

(4)

If earthquake large, long waves arrive about 2¼ hours after initial via opposite side of earth.

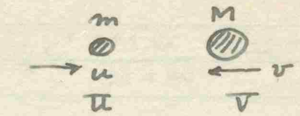
These long waves of Rayleigh type travel round earth with surface vel. 3.53 Km per sec.

Theoretical Rayleigh wave vel is 3.69 Km-sec. $(= 0.92 V_2)$

Complexities only explained by multiple reflection between surface & a deep down surface of discontinuity

Chap. X. Impact.

Newton's Laws of Impact.



1. Relative velocity after impact = e × rel. vel. before impact

$$U - V = e(v - u) \qquad u < v$$

where e is the coefficient of Restitution.
e is independent of the masses
e is dependent on the materials of which the bodies are made and to a slight degree on the initial difference of velocities of the two bodies.

2. Conservation of Momentum

$$mu + Mv = mU + MV$$

In the ideal case of perfectly elastic bodies, as postulated in the Kinetic Theory of Matter, $e = 1$ and the principle of Conservation of Energy holds so that

$$\tfrac{1}{2} mu^2 + \tfrac{1}{2} Mv^2 = \tfrac{1}{2} mU^2 + \tfrac{1}{2} MV^2$$

In all actual cases $e \neq 1$ and there is a loss in K.E.

In Eqns. 1. & 2. solve for U and V

$$mu + Mv - MV = mU = me(v-u) + mV$$
$$(M+m)V = mu + Mv - m\,e(v-u)$$
$$V = \frac{mu+Mv}{M+m} - e\frac{m}{M+m}(v-u)$$

$$mu + Mv - mU = MV = MU - eM(v-u)$$
$$(m+M)U = mu + Mv + eM(v-u)$$
$$U = \frac{mu+Mv}{m+M} + e\frac{M}{m+M}(v-u)$$

Final K.E. $= \frac{1}{2}mU^2 + \frac{1}{2}MV^2 = \frac{1}{2}m\left\{\left(\frac{mu+Mv}{M+m}\right)^2\right.$

$\left. + 2e\frac{M}{M+m}(v-u)\cdot\frac{mu+Mv}{M+m} + \frac{e^2M^2}{(M+m)^2}(v-u)^2\right\} +$

$+ \frac{1}{2}M\left\{\text{ditto} - 2e\frac{m}{M+m}\overline{v-u}\cdot\text{ditto} + \frac{e^2m^2}{(M+m)^2}(v-u)^2\right\}$

$$= \frac{1}{2} \frac{(M+m)(mu^2+Mv)^2}{(M+m)^2} + \frac{1}{2} e^2 \frac{(mM^2+Mm^2)(v-u)^2}{(M+m)^2}$$

$$\doteq \frac{1}{2} \frac{m^2u^2+M^2v^2+2mMuv}{M+m} + \frac{1}{2} e^2 \frac{Mm}{M+m} \overline{v-u}^2$$

$$= \frac{1}{2} \frac{m^2u^2+M^2v^2+Mmu^2+Mmv^2-Mm(v-u)^2}{M+m} + \text{ditto}.$$

$$= \frac{1}{2} \frac{(M+m)(mu^2+Mv^2)}{M+m} - \frac{1}{2} \frac{Mm}{M+m}(v-u)^2 + \text{ditto}.$$

$$= \frac{1}{2} mu^2 + \frac{1}{2} Mv^2 - \frac{1}{2}(1-e^2) \frac{Mm}{M+m}(v-u)^2$$

Hence the loss in K.E. due to impact is

$$\frac{1}{2}(1-e^2) \frac{Mm}{M+m}(v-u)^2.$$

Collision of Railway Carriages with buffer Springs.
(Describe in general terms.

Hertz Theory of Elastic Collision.

Hamburger's Measure of Time of Duration of contact.
Loss in K.E. goes chiefly into heat, very little into vibratory energy.

Note parallel loss in available energy in Elastic fatigue, after effect, hysteresis, &c.

Hodgkinson's comparisons of values of e for different substances + different rel. vels.

Applications of Theory of Impact.

(1) Modern Physics - Collisions of α + β ptcls with nuclei of atoms of gases, metals. &c.

(2) Rock crushing - Rittinger. + Kick.

Rittinger's Theory of Work done in Rock Crushing.

Let A = h.p. reqd per unit area of fracture
Then $3A$ = h.p " " " for the 3 planes of fracture necessary to split a cube into 8 parts.
If there be $n-1$ planes of fracture along one axis there must be n separate pieces
If D be the diam. (edge) of a cube
D^2 is " area of plane of cleavage
and $\frac{1}{D^3}$ " " no. of pieces in unit cube.

If d be the new diam $\frac{D}{d}$ = no. of pieces
and $\frac{D}{d}-1$ = no. of planes of cleavage.

$$\therefore \text{Work} = 3A \cdot D^2 \cdot \left(\frac{D}{d}-1\right)$$

$$= \frac{1}{D^3} \cdot 3AD^2\left(\frac{D}{d}-1\right) \text{ per unit cube}$$

$$= 3A\left(\frac{1}{d}-\frac{1}{D}\right)$$

In general for various kinds of rock

$$\text{Work} = 3AK\left(\frac{1}{d}-\frac{1}{D}\right)$$

where K is a factor depending of the characteristics of the rock — e.g. Sp.gr. &c.

Rittinger "the Father of Ore-Dressing".

6/1/25 This theory is fallacious as shown by S J Speak "Energy Consumed in Crushing" The Mining Magazine Jan 1923. p. 22.
He substitutes Kick's Law.
The energy reqd for producing analogous changes of configuration of geometrically similar bodies of equal technological state varies as the volumes or weights of these bodies.

Lecture 13

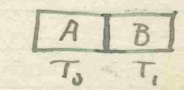
3.2.21.

Chap. XIII. Reversible Thermal Effects accompanying Alterations in Strain.

In the logical development of the subject this chapter should be taken before passing from consideration of the properties of solids to those of liquids & gases.

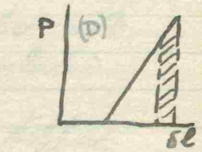
Thus far in considering elastic strains there has been no restriction regarding the inflow or outflow of heat.
If conditions are made to be adiabatic (α δια βατος) not passable.
the values of the elastic constants are found to be slightly altered.

Method of analysis of Lord Kelvin.



(1) Two compartments A, B at const. temps T_0, T_1 absolute.
In A is a stretched wire of length l & area cross-section a, under tension p producing a strain δe where work done on wire $= p a l \delta e$ (D)
Remove it to B. To maintain elongation p becomes p'. Allow it to contract through δe. It does work to the extent of $p' l a \delta e$
Excess work is $(p'-p) l a \delta e = (p'-p)\delta e$ per unit vol.

Compare the reversibility of the Carnot cycle & apply the 2nd Law of Thermodynamics.



$$\frac{\delta H}{T_1} = \frac{\delta h}{T_0} = \frac{\delta H - \delta h}{T_1 - T_0}$$

Mechanical work $= \delta H - \delta h = \frac{T_1 - T_0}{T_0}\delta h$.
in form of heat units

By Joule's Law $W = JH = JK\rho\,\delta\theta \left(\frac{T_1-T_0}{T_0}\right)$

ρ = density
K = spec heat
$\delta\theta$ = change in temp. due to strain δe.

$$\therefore (p'-p)\,\delta e = \frac{T_1 - T_0}{T_0} J\rho K\,\delta\theta.$$

$$\therefore \delta\theta = \frac{T_0}{JK\rho}\left(\frac{\delta p}{\delta T}\right)\delta e \qquad (1)$$

(2) Let wire expand under constant tension as temp increases δT; if ω be coef. of linear expansion
strain $= \omega\,\delta T$
$\therefore$ Stress $= -\delta p = E\omega\,\delta T$ necessary to reduce strain with temp constant.
$\delta p = -E\omega$

Substitute in Eqn. (1)

$$\delta\theta = -\frac{T_0 E \omega}{J K \rho}\,\delta e.$$

But $E\,\delta e = \delta P$

$$\therefore \delta\theta = -\frac{T_0\,\omega\,\delta P}{J K \rho} \qquad (2)$$

Determination of Adiabatic Value of E.

Let tension in a wire increase by δP

If there be no flow of heat the extension δe is made up of 2 parts.

$$\delta e = \frac{\delta P}{E} + \omega\,\delta\theta \qquad \text{where } \delta\theta \text{ is rise in temp due to } \delta P.$$

$$= \frac{\delta P}{E} - \frac{T_0\,\omega^2\,\delta P}{J K \rho} \qquad \text{from (2)}.$$

$$\therefore \frac{\delta e}{\delta P} = \frac{1}{E} - \frac{T_0\,\omega^2}{J K \rho}$$

But $\dfrac{\delta P}{\delta e}$ = Young's Mod under Adiabatic Condns. $= E'$

$$\text{Hence}\quad \frac{1}{E'} = \frac{1}{E} - \frac{T_0\,\omega^2}{J K \rho}$$

i.e. Isothermal E is always less than Adiabatic E.

Note that experimental values for E' do not correspond very precisely with calculated values.

also that E' and E for metals differ by less than 1%.

Lecture 14

see Edser p. 351–358 and 277–283.

Chap. XI Compressibility of Liquids

1. Compressibility diminishes as pressure increases.
2. " increases as temperature " . with a few exceptions
3. " of aqueous solutions is less than that of pure water.

Edser p. 279

Note 1. Compressibility is usually tabulated per megabar (i.e. 10^6 dynes/cm²) To express it per atmosphere increase by $\frac{1}{80}$ of given value.

2. Amagat found compressibility of water a minimum at 50°C.

3. Compressibility of solution diminishes as the concentration increases.

<u>Examples of Value of C per megabar.</u>

Water	25 atmos.	15°C	$C = 48.9 \times 10^{-6}$
"	3000 "	"	25.8×10^{-6}
Hg	———	"	3.7×10^{-6}
Turpentine	$C_{10}H_{13}$	19°C	78.1×10^{-6}
Ethyl Alcohol	C_2H_5OH 200 atmos	310°C	4147.0×10^{-6}

<u>Tensile Strength of Liquids</u>

(a) Mercury (adhesion in Bar. tube)
(b) Water. Prof Osborne Reynolds 72.5 lbs/in²
Alcohol " " 116 " "
H_2SO_4 (strong 173 " "
Principle of centrifugal force. (p. 123)

(c) Berthelot – heating + cooling method.

(d) Worthington. method of c with a Hg bulb and capillary tube stem inserted

<u>Measurements</u> of compressibility depend on change of volume of containing vessel.
Consider alteration in vol. of a tube of length l and int. radius a, ext. rad. b subjected to internal pressure p_0, ext. pres. p_1.
Let ρ be radial displacement and gl the longitudinal extension, e, f, g being the strains in 3 directions at rt.-angles.
Lamé assumes from observation $\rho = Ar + \frac{B}{r}$.
From eqns involving Stresses P, Q, R producing the strains e, f, g, the values of A, B are found (p. 117) [Verify each step]

$$A = g = \frac{1}{3k}\frac{(p_0 a^2 - p_1 b^2)}{b^2 - a^2}$$

$$B = \frac{1}{2n}\frac{a^2 b^2}{b^2 - a^2}(p_0 - p_1)$$

also $e = A - \frac{B}{r^2}$ and $f = A + \frac{B}{r^2}$.

Lecture 15

Hence internal volume of tube becomes

$$\pi\left(a + Aa + \frac{B}{a}\right)^2 l(1+g)$$

$$= \pi a^2 l\left\{1 + \frac{p_0 a^2 - p_1 b^2}{b^2 - a^2}\frac{1}{k} + \frac{b^2}{b^2 - a^2}\left(\frac{p_0 - p_1}{n}\right)\right\}$$

$$\therefore \delta v_1 = \pi a^2 l\left\{ \quad " \quad + \quad " \quad \right\}$$

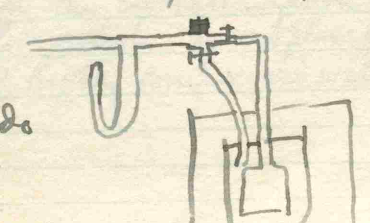
and external $\delta v_2 = \pi b^2 l\left(\quad " \quad + \quad " \quad \right)$

Put $p_1 = p_0$

$\delta v_1 = -\frac{\pi a^2 l}{k} p_0$ k is bulk mod. of tube material.

Let K = bulk mod. for a liquid in the tube.
Change in volume is $\pi a^2 l p_0\left(\frac{1}{K} - \frac{1}{k}\right)$

1. <u>Regnault's</u> method.
Determine $\left(\frac{1}{K} - \frac{1}{k}\right)$ & find k separately.
Piezometer.
See apparatus p. 119.
Pressure p_0 (1) outside only
(2) inside & outside
(3) inside only.



2. <u>Jamin's</u> method

3. Buchanan + Tait's Method for K – measure longitudinal contraction under pressure.

4. Method of water in tubes under tension – change in vol. measured in capillary stem $\frac{\delta v}{v} = \frac{P}{3K}$.

5. Method of comparisons – if K for one liquid can be determined definitely – e.g. Hg. others by direct comparison with piezometer.

omit

This is probably only a temperature effect.

Possible Method for obtaining Compressibility of a liquid based on the observed fact that an adjustably weighted bottle can be made to sink at surface of water but float at a plane of depth 3 ft. or so below surface.
Take a 1 cc wooden or close cork cube + weight it by inserting fine wire pins until it weighs w gms and floats at a depth h below surface of liquid any sample of which has a density ρ.
Since it ~~floa~~ sinks at surface $w > \rho$
" " floats a depth h it follows that the density ρ' of that strata equals w.
Hence $\rho' - \rho$ = compressibility due to excess pressure $h\rho$ or more accurately $h\left(\frac{\rho+\rho'}{2}\right)$

Hence increase in density due to unit increase in pressure wd. be given by

$$\delta\rho = \frac{2(w-\rho)}{h(w+\rho)}$$

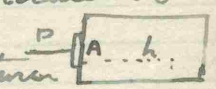
Application of Compressibility of liquids giving accurate transmission of impulses in the synchronization of machine gun fire and propeller action in aeroplanes using an impulse cylinder of highly compressed water.

5.2.21.

Chap XII. Relation between Pressure & Volume of a Gas.

From Kinetic Theory of Gases it is shown that if p be the pressure, n the no. of molecules per c.c. m the mass of each and c the average velocity.
$p = \frac{1}{3} n m c^2$

Proof. Change in momentum of one molecule on collision is $2mc$



If the average distance travelled before return be L The no. of hits in t seconds by one mol is $\frac{ct}{2L}$

$\therefore$ for one mol Total impulse in t sec. is mc^2t/L.

Assume random disposition $\therefore$ $\frac{n}{3}$ are moving in direction $\perp^r$ to face A.

$\therefore$ no. of mols under consideration is $\frac{1}{3}(nAL)$

$\therefore$ Total impulse is $\frac{1}{3} n m c^2 t A$.

Let balancing pres. be $pA = F$

Impulse $Ft = pAt = \frac{1}{3} n m c^2 t A$.

$\therefore \quad p = \frac{1}{3} n m c^2$.

To obtain Boyles Law.

Multiply both sides by v and let $nv = N$.

then $pv = \frac{1}{3} N m c^2 = RT$

where R is the gas constant, T the abs. temp which $\propto c^2$.

If p be ordinate & v abscissa the Boyles Law curve is theoretically a rectangular hyperbola and within limits it is obeyed with great exactitude.

The work done in changing the volume is always. $W = \int p\,dv$.

Consider $pv = RT$

(1) For isothermal variation T is constant
$\therefore \quad pv = \text{const}$.

(2) For adiabatic variation $dH = 0$
$\therefore \quad pv^{c_p/c_v} = \text{const}.$ (see proof)

Proof from $pv = RT$

(a) Vol. const. $\therefore p = kT$

$\therefore \delta p = k\,\delta T$

$$\therefore \frac{\delta p}{p} = \frac{\delta T}{T} \quad (1).$$

Let C_v be the specific heat at const. vol., δH = heat transfer

then $\delta H = C_v \delta T = C_v T \frac{\delta p}{p}$

(b) Pres. const. $v = k_1 T_1$

$\therefore \delta v = k\,\delta T$

$$\therefore \frac{\delta v}{v} = \frac{\delta T}{T} \quad (2)$$

Let C_p be spec. heat at const. pres.

then $\delta H = C_p \delta T = C_p T \frac{\delta v}{v}$.

(c) Vol. & pres. varying.

$$\delta H = T\left(C_v \frac{\delta p}{p} + C_p \frac{\delta v}{v}\right)$$

If the expansion be adiabatic $\delta H = 0$

$$\therefore C_v \frac{\delta p}{p} + C_p \frac{\delta v}{v} = 0$$

$$\therefore \frac{\delta p}{p} + \frac{C_p}{C_v}\frac{\delta v}{v} = 0$$

Integrate

$$\log p + \gamma \log v = \text{const.}$$

$$\therefore \log p v^\gamma = \text{const.}$$

$$p v^\gamma = \text{const} \qquad \gamma = C_p/C_v$$

(Sound waves) — See p. 186 Jeans Dyn. Th. Gases. Laplace's modification of Newton's method

15.2.21.

Elasticity.

Coef. of elasticity = $\frac{\text{Stress}}{\text{Strain}} = \varepsilon$

(a) For isothermal gas. $\varepsilon = -\frac{\delta p}{\frac{\delta v}{v}} = -v\frac{\delta p}{\delta v}$

but pv = const.

$\therefore p\,\delta v + v\,\delta p = 0$.

$v = -p\frac{\delta v}{\delta p} \qquad \therefore \varepsilon = p$.

(b) For adiabatic gas $\varepsilon = -\frac{\delta p}{\frac{\delta v}{v}} = -v\frac{\delta p}{\delta v}$.

but pv^γ = const

$\therefore \log p + \gamma \log v$ = const.

$\frac{\delta p}{p} = -\gamma\frac{\delta v}{v}$; $v = \gamma p \frac{\delta v}{\delta p}$ $\therefore \varepsilon = \gamma p$.

Rob. Boyle. 1627-1691

7th Son of Earl of Cork

Eton, Geneva, Florence (Galileo d. 1642)

With Hooke on return to Eng. 1644 used Pneumatic Engine to study Props. of Air. "Spring of air & its effects" pub. 1660. Criticized by Jesuit Linus & in ans. to him, he enunciated $pv = \text{const.}$ He was really a follower of Baconian method tho' he denied the influence of Novum Organum.

Air, Sound, freezing & expansive force, crystals, colours, hydrostatics but preeminently a Chemist endeavoured to show relation of combustion to respiration & other physiological phenom.

Also a theologian not a controversialist took orders at Oxford towards end of his life & amongst other things published paper on the superiority of Religious philosophy over Natural philosophy.

(Encyclopaedia Britt.)

Torricelli 1608-1647

Deviations from Boyle's Law.

If $pv = \text{const.}$ held rigorously

$$\frac{pv}{p'v'} = 1$$

Regnault's table shows variation of from 0.999 for H to 1.024 for CN (See Kinetic Thy. of Gases – Meyers p. 14–17). 1.099 for carbonic acid.

If $p < p'$ then $\frac{pv}{p'v'} < 1$ is due to the volume of the mols being not negligible in comparison with the change in vol. due to gtr pres. p'.

If $\frac{pv}{p'v'} > 1$ it is due to the effects of Cohesion which cause the volume to decrease more rapidly than the change in P would seem to warrant.

These effects are more marked with vapours.

See P & T. p. 125–6. Despretz, Regnault, Amagat.

Van der Waals Eqn.

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT.$$

Correction $\frac{a}{v^2} \propto a\rho^2$ is a Surface energy effect due to attraction on a mol. by each mol. near it. This is propl. to its mass – i.e. to ρ. Also the attrn. depends on the concentration of mols. near it, i.e. on ρ. Hence on ρ^2.

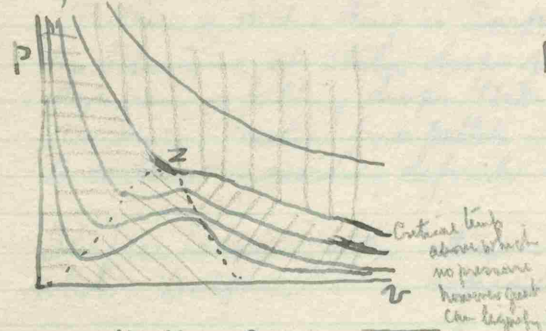
Correction b due to the small but finite size of the molecules themselves as opposed to the volume which they occupy. $b = 4v_0$ approx.

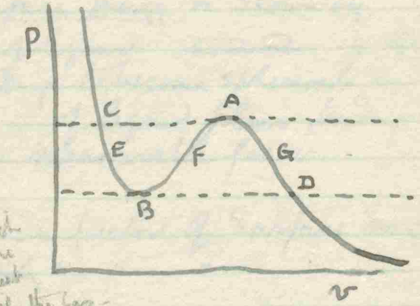
$$\left(p + \frac{a}{v^2}\right)(v - b) = pv + \frac{a}{v} - pb - \frac{ab}{v^2} = RT.$$

Multiply up by v^2 & arrange as a cubic in v.

$$pv^3 - (pb + RT)v^2 + av - ab = 0$$

Plotting this for various temperatures curves are obtained as follows: (Isothermals)





Gas |||||| Liquid ≡

vapour ////// Region of 2 states \\\\\\

This is Boltzmann's diagram. (Théorie de Gaz)

The cubic has three roots $v_1\ v_2\ v_3$
all real as at E F G + distinct
two coinciding as at A or B.
three " as at Z
Z is called the <u>critical point</u> and is the highest temperature at which a gas can be liquified

From Thy. of Eqns.

$$v_1 v_2 v_3 = ab/p$$
$$v_1 v_2 + v_2 v_3 + v_3 v_1 = a/p$$
$$v_1 + v_2 + v_3 = (bp + RT)/p.$$

at Z $\quad v_1 = v_2 = v_3 = v_c$

Hence find
$$v_c = 3b$$
$$p_c = a/27b^2$$
$$T_c = 8a/27bR$$

In any gas put $p = \pi p_c$, $v = \phi v_c$, $T = \vartheta T_c$ + substitute in Van der Waals + get the constant relation $\left(\pi + \frac{3}{\phi^2}\right)(3\phi - 1) = 8\vartheta$

Two substances having the <u>same</u> $\pi\ \phi\ \vartheta$ are said to be in Corresponding States.

25.2.21.

Capillarity & Surface Energy – Chap XIV.

That there is such a thing as Surface Energy is shown by the behaviour of static drops of liquid ; of dynamic or falling drops tending to become spherical – raindrops, molten lead pellets ; of liquid films, tending to contract & exerting definite measurable force.

Surface Tension is the intensive factor of Surface Energy while Surface " capacity " " " .

Consider a mol. in a liquid at rest. By hydrostatics it is subject to equal force in all directions.
A mol. at surface is under differential force – a[n] attraction downwards. This force of attraction acts only over a very small distance, hence the differential force is not an inverse square law but is independent of the mass of liquid below. In this respect it is similar to chemical force.

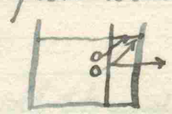
Surface tension (1) acts over small distance.
(2) is selective { compare chem. force; contrast gravitational force
(3) is indept. of p.
(4) is due to downward attraction.

25/2/21 lecture 17.

Measure it by the mechanical pull it exerts $= F$
Let tension be T per unit length of a moveable strip in a frame of width l, thus

There is tension on both sides of the film $\therefore 2Tl = F$
Move the strip outwards through a distance d.
Work done $= Fd = 2Tld = T \times$ new surface created.

This is a measure of the "downward attraction" as can be understood by considering that to enlarge a surface mols have to be brought out of the liquid from within to form the new surface – See figure



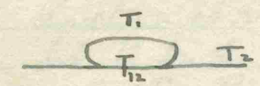
Surface Tension of a liquid varies with the vapour or gas above it
" " " " " with temperature & pressure.
" "s of different liquids differ widely from one another
Standard Values are tabulated at S.T.P. with air above.

There is a corresponding surface energy in gases – giving rise to the correction $\frac{a}{v^2}$ in V. der Waals Eqn.

Also in solids though not measurable except in rare cases e.g. a Cu bar cut in two & surfaces highly polished then put in contact under pressure, they cohere and a wt. of 1 kg has been suspended without rupture.

Where two liquids which do not mix lie in contact as oil on water Langmuir + Harkins have shown that the molecules of the complex often organic compound are orientated (being oblong things) and the Surface Energy is due to certain groups + certain parts of the mol. which are drawn end-on into the water surface. See further of this investigation on p. 60(a).

Consider a drop on a surface (liquid or solid)

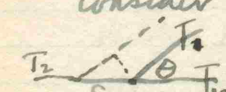


Will it remain as a drop or spread. This depends on the relative values of T_1, T_2, T_{12}, and the law that Pot! Energy tends to a minimum.

If an increase of area ~~dx~~ S by drop – Work done = pot! energy stored or used $= T_1 S + T_{12} S - T_2 S$
$= (T_1 + T_{12} - T_2)\, S$

If this is + the drop will not spread.
" " " − " " " " .

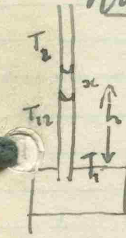
Actually the above is not rigorous due to curved shape of drop. Consider its angle of contact θ.



Increase in energy =
$T_1 S\cos\theta + T_{12} S - T_2 S = 0$ for a very small displacement by the Principle of Virtual Work.

$\therefore \cos\theta = \frac{T_2 - T_{12}}{T_1}$

If $\theta < \frac{\pi}{2}$ $\cos\theta$ is + and liquid will spread or rise in a capillary tube with concave meniscus.
If $\theta > \frac{\pi}{2}$ $\cos\theta$ is − and drop will not spread but liquid will tend to be depressed in a capillary tube with convex meniscus as is case of Hg.



Rise of Liquid in a Capillary Tube.

Equating pot. energy in slight displacement x to 0.

$$2\pi r x (T_{12} - T_2) + (\pi r^2 h g \rho) x + (g \rho v) x = 0$$

v = meniscus

Put v = [meniscus sketch] $= \frac{1}{2}(\pi r^2 \cdot 2r - \frac{4}{3}\pi r^3) = \frac{\pi r^3}{3}$

$$2(T_{12} - T_2) + r g \rho h + \frac{r^2}{3} g \rho = 0$$

$$\therefore 2 T_1 \cos\theta = r g \rho (h + \frac{r}{3})$$

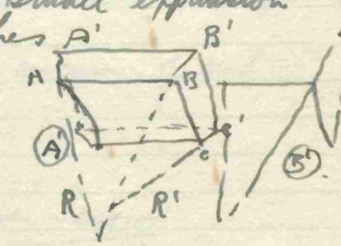
$\cos\theta = 1$ when $\theta \to 0$ as when liquid wets surface

$$\therefore T_1 = \frac{1}{2} r g \rho (h + \frac{r}{3})$$

Use this for determination of T_1.

Relation between Pressure & Curvature of films.

Let the radii of curvature of a film element in two directions at rt. angles be R. R_2.
Let P_0 be the internal pressure and P_1 external and $P_0 - P_1 = p$.
Let surface tension be T.
By Virtual Work Princ. the work done by the press. diff against the surface tension in a small expansion displacement of the element vanishes



Radii R_1, $\pm R_2$

If a small radial displacement d.

Work done by T is $T \times$ change of area $= 2T(A'B' \cdot B'C' - AB \cdot BC)$

$\frac{A'B'}{AB} = \frac{R+d}{R}$ $\therefore A'B' = AB(1 + \frac{d}{R})$

$\frac{B'C'}{BC} = \frac{R' \pm d}{R'}$ $\therefore B'C' = BC(1 \pm \frac{d}{R'})$

Lecture 18. 4/3/20.

$\therefore A'B' \cdot B'C' = AB \cdot BC(1 + \frac{d}{R} \pm \frac{d}{R'})$ $\therefore$ diff of area $= AB \cdot BC(\frac{d}{R} \pm \frac{d}{R'})$

Work done by p is $p \times$ area $\times$ displacement
$= p \times AB.BC \times d$.
These are equal.
$\therefore \quad p = 2T\left(\frac{1}{R} \pm \frac{1}{R'}\right)$

1. Consider the special case of a spherical drop – (eg. ~~rain~~ soap bubble.)

$$p = 2T\left(\frac{1}{R} + \frac{1}{R}\right) = \frac{4T}{R}.$$

2. Consider the special case of a film with equal pressures on both sides.

$$p = 0 = 2T\left(\frac{1}{R} + \frac{1}{R'}\right)$$

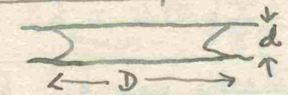
$$\therefore \quad R = -R'$$

Examples. – twisted framework (skew surface) or catenary.

Stability of Cylindrical Films.

See P + T pp. 147–152
Stability if length of film be less than semi-circumference of its end.
Unstability if length exceed semi-circum. of end.
Apparatus of Boys.

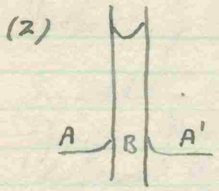
Attractions + Repulsions due to Surface Tension.

(1) 

Pressure difference $= p = T\left(\frac{2}{d} - \frac{2}{D}\right) = \frac{2T}{d}$ approx
where D is large.
Let A be area wet $\therefore$ attractive force is
$2AT/d$ (approx.

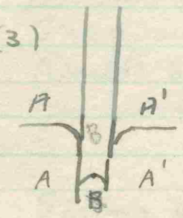
This can be used to determine T.

(2)



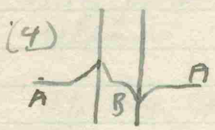
Horizontal pressure on walls the same as though there were no curved meniscus
Prove by equating forces (p. 153)
Hence pressures at A, A' exceed that at B
Since at A, A' it is $\underline{B}$ (barometric)
and at B it is $\underline{B - \overline{h}}$ h is equivalent Hg. column.
∴ plates are attracted.

(3)



Similarly where θ is $> \frac{\pi}{2}$
Pressure at ~~A, A'~~ B is Barometric
at ~~B~~ A A' it is $\underline{A} + h$ ∴ greater outside
∴ plates are ~~repelled~~ attracted.

(4)



In this case it may be either attrⁿ or repulsⁿ depending on distance apart and relative differences in pressure on each side of the 2 plates.

Waves + Ripples.

A wave disturbance in a liquid is propagated both by gravity + by surface tension.
It can be shown Mathematically (p.158.) that
$v = \sqrt{\frac{\lambda}{2\pi}\left(g + \frac{4\pi^2 T}{\lambda^2 \rho}\right)}$; where λ = wave length.
+ ρ = density of liquid
This is minimum when the 2 terms are equal
Then $\lambda_{\min v} = 2\pi\sqrt{\frac{T}{g\rho}}$ and $v_{\min} = \sqrt{2}\,\sqrt[4]{\frac{Tg}{\rho}}$

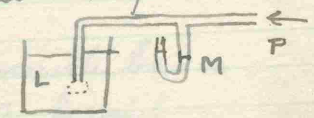
As λ becomes greater, v increases due chiefly to gravity + for very long waves $v = \sqrt{\frac{\lambda g}{2\pi}}$ approx.

As λ becomes smaller v increases, due chiefly to surface tension and the wave is called a ripple.

Lord Rayleigh used this theory to determine T,
See his method described p. 157, 160. (tuning forks + intermittent light)

Methods of Measurement of T.

In addition to those already referred to - See pp. 155-165.
Jaegers Method for comparing values of tension at different temperatures - least pressure to force air bubbles through a liquid L at any reqd temp.
Pressure bottle at P and Manometer M.



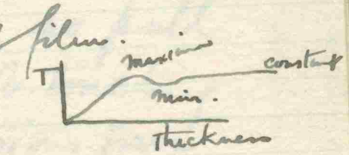
Results show that Surface-tension diminishes as temperature increases.

[illegible] 19
11/3/22

Eötvos attempted to connect Molecular Volume $\left(\frac{\text{Mol wt}}{\rho}\right)$ with temperature + surface tension by the relation $\frac{d}{dt}(T\,v^{2/3}) = \text{const} = -2.1$

If this could be verified + assumed, then work conversely to find molecular structure (ie, for H_2O it points to grouping $2H_2O$ for high temps + nH_2O, $n>2$, for lower temps - possible application to ice formation?)

Variation of T with thickness of film.
Work of Rücker + Reinold



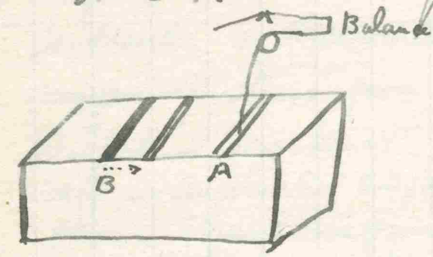
See Questions
Exer p 365
+ 610

Surface tension Equilibrium of Contaminated Surfaces.
Application in Calming troubled waters with oil.

Effect of electrification in causing drops of a pure liquid to coalesce instead of rebounding.

Application of Theory of Capillarity in Geology.
Oil is struck by sinking a shaft in an anticline where and impervious upper layer is followed by porous layers over brine & shales & another impervious layer below. Oil in the Pocano Capillary attrn. draws the oil up - Effect of large + fine pores on the pureness of the oil. Fine pores gtr attrn for H_2O.

Note on Langmuir & Harkins' work.
on Orientation of Molecules.



A is a moveable bar on surface of liquid attached to a balance so that the horizontal pull on the bar due to Surface tension can be measured accurately.
B is a fixed bar.
A known number of mols. of the organic acid under consideration, (several C atoms and a COOH group) is put on surface between A & B.
If B far enough from A, no pull on A hence mols are not touching & cannot produce the S. Tension force. Bring B nearer until the first indication of pull on A. Then the mols. are in contact, one layer deep and $\frac{\text{total area between A, B}}{\text{no. of mols.}}$ = cross-sectional area of each mol. This is found to be almost constant for all oils & acids with a COOH group no matter how long the Carbon chain. ∴ Orientation.

14. 3. 21.

Visited Strength of Materials Testing Lab.

(1) Compressional & Tensional Machines.
Hydraulic & electrically run machines giving ranges of stress from up to 180,000 pounds -
Wickstead Machines.
Self or automatic adjustments for drawing stress strain diagrams within & beyond elastic limit to the breaking pt.
Saw samples of soft iron, mild steel wrought iron & wooden blocks subjected to both tension & compression.
Scaling of metal at the yield pt.
Plane of cleavage often approximately 45°.
showing direction of maximum shearing force
Extensometer for accurate measurement of elongation.

(2) Torsional machine - hand turned - wt. applied on end of a 25" lever arm.
mild steel turned 5 times before cleavage.
Cast iron bar snapped before one complete turn, cleavage was in tension straight ends being joined by a spiral plane about 45° inclination

(3) Impact. 50 lb. hammer with electro-magnet clutch wh. raises it to given height then cur. is automatically reversed & hammer drops.. Wood specimen broke in tension below neutral axis & split horizontally ‖ to neutral axis

(4) Water jet 9 ft head △ slot Vena Contracta

17.3.21

Visited Forest Products Lab & Museum.

(1) Sections of various trees showing yearly growth & spring & summer wood, pith, knots, etc.

(2) Spiral adjustment of fibres to resist undue strain through wind pressure or position on side of a hill. Cracking of knots & horiz. sections on drying due to shrinkage. Hard wood shrinks more than soft.

(3) Imperfections, diseases - fungi, aquatic animals, borers, & beetles (old oak beam from Westminster Hall - 1322 Richard II. from Sussex)

(4) Thin-section-cutter & microscopic examination of cell arrangement & rays (conducting moisture) Projection lamp for studying slides.

(5) Oils by distillation of wood - for floatation of ores.

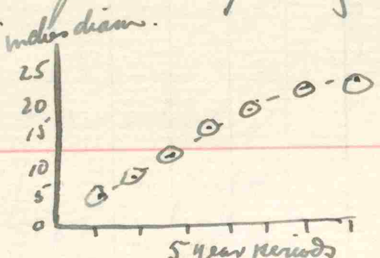
(6) Pulp - Kraft method, sulphide method, grinding up method - cellulose converted into papier-macher, artificial silk, etc.

(7) Experimental pulp machine - stirrer, beater rollers, drying revolving cylinders & large drum for finishing - glossy surface a matter of heating & rolling.

Best equipped pulp lab in Canada. One at Michigan as good.

Experiment during war to produce floating paper.

Curve of growth of a Balsam almost str. line growth for first 20 years.



Surface Tension in Physiology

Dr Du Nouy, Paris.

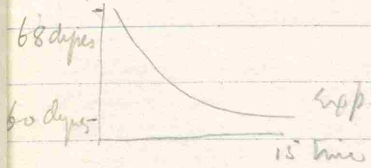
4.3.35

Surface Tension – Static, Dyn.

Physiological importance.

Motion of amoeba in fluid is entirely S.T.

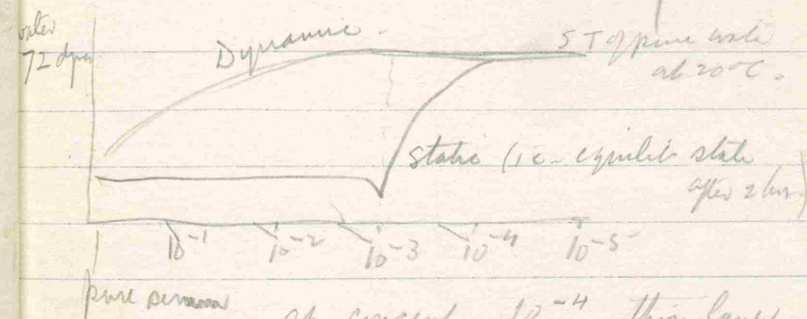
Force to pull a Pt ring from surface of a liquid. Easy method 500 per day.



Exp. curve

Gibbs eqn: Energy of surface tends to minimum.

Dec concentration then increase drop



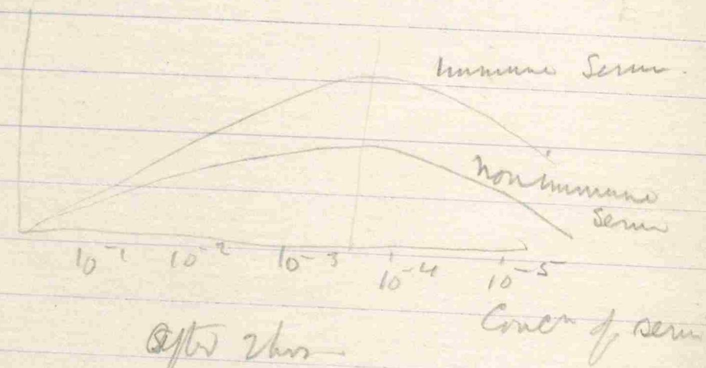
at concent 10^{-4} thin layer of mols on surface.

∴ Kink is due to a mono molecular layer oriented & just forming a perfect film ∴ low s.t. 5 or 6 dynes drop.

Water has higher S.T. than any serum.

Protein mol has mol wt 35 000

Architecture of these mols is so delicate that change of posn of 1 atom alters fields of force & hence S.T.



After 2 hrs.

application to antibodies.

Antigen? no effect till after 8th day

Maxm effect at 12th day – no reaction after 30 days – but some permanent change in circulation is produced; or something remains in the tissues giving immunity.

For about 3 days at max the protein structure changes as shown by crystallization of NaCl in herringbone

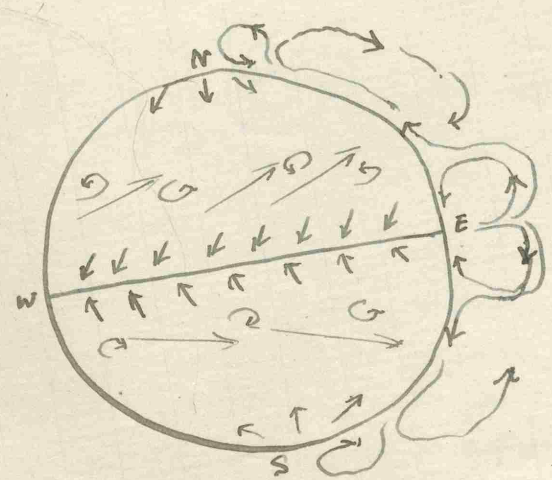
Extra 16.3.21

Circulation of the Atmosphere on the Earth and application of Gas Laws in interpreting Weather Charts.

see R.A.S.C. notes on Stratosphere & Troposphere

Winds

- Planetary (Constant) { Trades, Anti-Trades, Prev. Westerlies (Doldrums, Horse Lats.)
- Periodic
  - Diurnal { Land & Sea Breezes, Mt. & Valley
  - Seasonal { Continental, Monsoons
- Occasional or Intermittent
  - Dust Whirlwinds
  - Tornadoes { Hurricanes, Typhoons
  - Cyclonic Storms.



Directions governed by rotation of Earth & gtr. heating effect over Equatorial Zone.

See also RASC notes
of Pattersons Lecture
on Structure of
Atmosphere –
Stratosphere
Troposphere
+ lines of equal temp.

Cyclonic Storms in Belts of Prevailing Westerlies
are caused by heating over a large area + rising
currents – counter-clockwise in N. Hem. &
reverse in S. Hem. Air rising, expands, cools
+ moisture condenses. Hence this is the Low
pres. area bringing wet weather. Somewhere
the air has to get down again to fill in, hence anti-
cyclones are developed, air descending, being
compressed as it spirals down clockwise in
N. Hem., it warms up ($\frac{PV}{T} = R$) and can hold
more vapour, hence it absorbs moisture and
produces the High Pressure, clear sky, fine
weather area.
Size: up to 1000 miles in diam.
Duration: " " 2 weeks.
Rate of travel: 500 - 1000 mi per day eastward.
Vel. of wind: seldom exceeds 60 m.p.h.

Lecture 20
18/3/21

Deposition of Planetesimal Dust is
controlled by Atmospheric + Ocean Currents.
Ocean surface > land surface ∴ heaviest substances
settling quickly fall most probably into ocean &
form very high density ocean beds.
Lighter dust is carried in suspension for long
time + is only deposited by a low pressure area
where upward current of air is expanding – e.g.
Mt. ridge – fertile soil + plenty of moisture on
windward side.

15.3.21.

Chap. XVI. Diffusion of Liquids.

Chief investigator of phenomena of diffusion of liquids & gases was Graham who showed that the rate of diffusion of solutions depended upon 1. the solvent; 2. the solute; 3. the concentration; 4. temperature; First definite law due to Fick $R\frac{dn}{dx}$ gms of salt pass across unit area containing n gms in its layer at dist. x from a fixed plane, in unit time, where R is the diffusivity of the salt in the solvent considered. See Edser p.585

Measurement of Diffusivity.
1. Kelvin's Sp. gr. beads floating at different levels - not accurate due to crystallization on them.
2. Rotation of plane of polarization.
3. Electric conductivity.
4. Longs Comparison method by passing a stream of pure H_2O through a soln thus 

Amt. of salt carried off by H_2O is propl. to Diffusivity x Concentration.

Diffus. in through membranes is termed Osmosis. Amorphous substances diffuse less easily than crystalline substances - Graham called them Colloids.
The former diminish the vapour pres of the liquid, raise its B.P. lower its F.P. The latter do not show these effects to any marked degree.

Modern interpretation of term Colloid
A colloid is a "dispersion phase" in a "dispersion medium" the arbitrary limits for the degree of dispersion being $\cdot 1\mu$ to $1\mu\mu$. [μ = $\cdot 001$ mm]
Gst authority Wolfgang Ostwald.
Common types of colloid: solid in liquid, liquid in liquid, gas in liquid (foam)
(Pt. in H_2O) (Any solid in limits in a liquid)
(phenol in H_2O) emulsion

The colloidal particles may be crystalline.
The whole question of properties & behaviour turns on the specific surface which gives play to the Surface Energy

30.3.21.

Diffusion of Gases & Vapours. Ch. XVII.

Diffusivity k where $k \frac{\partial \rho}{\partial x}$ gives no of mols. passing across unit area of a plane at height x under a ~~potential~~ density gradient $\frac{\partial \rho}{\partial x}$.

Interdiffusivity increases with temp. increase.
decreases " pressure ".

Diffusion of Vapours regulates rate of evaporation.

A perforated diaphragm does not reduce the diffusion by ratio of area of holes to area of tube but in a less degree since the density gradient is greater at a perforation.

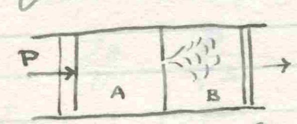
Flow of Gas through Porous Plugs.

(1) Pores large, plug thin: flow as in hydrodynamics. Torricelli's Theorem – $v \propto \sqrt{\frac{\text{pressure}}{\text{density}}}$
Bunsen used this to compare densities of diff. gases.
No change in composition of a mixed gas.

(2) Pores large – Plug thick: flow is governed by viscosity – No change in composition of gas.

(3) Pores fine (meerschaum) [Kadinsehan $H_4Mg_2Si_3O_{10}$ / $2MgO \cdot 3SiO_2 \cdot 2H_2O$] – Diffusion each molecule finds its own way independently of others. ∴ fractional separation of a mixed gas takes place, called by Graham Atmolysis.

Joule-Thomson Effect. Thermal Change with



Turbid state beyond orifice due to "vis viva" of issuing gas under pres. p.

If temp in A be above a critical value t_0 for that gas temp in B is higher, while if temp in A is below t_0 then the stream of gas in B is cooler. Explained by deviations from Boyle's Law – internal energy &c. See Preston p. 771.

Dialysis is the separation of crystalline substances from amorphous ones by passage of former through a membrane.
Atmolysis is the fractional separation of gaseous mixtures by diffusion through porous bodies.

The tendency for a fluid to pass through a membrane by osmosis can be overcome by applying a definite pressure to the solution on the far side of the membrane. This pressure is called Osmotic Pressure P and is of intrinsic importance in determining the properties of the solution.

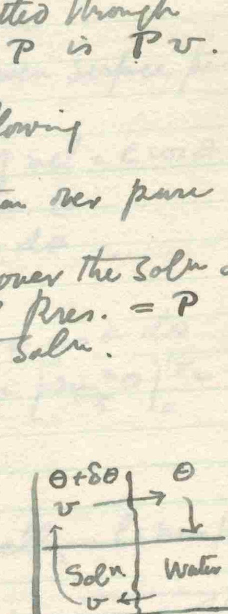
→ Isotonic Solutions are those which have the same P.

Importance in Biology. action through cell tissues

Work done when a volume v is transmitted through a membrane under an osmotic pres. P is Pv.

See P. + T. pp. 190 – 193 for proofs of following proposition.

1. Vapour pres. is lower over a solution than over pure water at same temperature.
2. Vapour pres. will become the same over the solⁿ as over the pure water if an external pres. = P the Osmotic Pres. be applied to the solⁿ.
3. B.P is raised.
4. F.P. " lowered.



Raising of B.P.
Vol v. H_2O passes into Solⁿ under Osmotic pres. P. Pv = work.
$v\rho\lambda$ = heat taken up to turn to steam
∴ " = " given to water after steam passes back & condenses
2nd law Thermodynamics $\frac{v\rho\lambda}{\theta} = \frac{Pv}{\delta\theta}$ = $\frac{\text{Work done.}}{\text{Temp.-diff.}}$

$\therefore \frac{\delta\theta}{\theta} = \frac{P}{\rho\lambda}$.

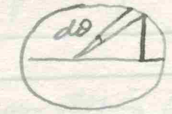
Lowering F.P.
Vol v H_2O goes into Solⁿ. Work done Pv.
Freezes – is put into H_2O & melts ~~giving~~ taking up $\lambda\rho v$
$\therefore \frac{\lambda\rho v}{\theta} = \frac{Pv}{d\theta}$ $\therefore d\theta = \frac{P}{\lambda\rho}\theta$.

Flow of gas through Rubber
depends on the nature of gas (CO_2 fast, air slow) and on the temperature.

Diffusion of metals through Metals - Hg in Pb, Sn, Zn. Au, Ag. Bi.

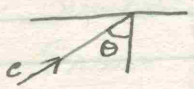
Boltzmann's Treatment of Diffusion in Kinetic Theory.

(1) Molecules moving in all directions with av. vel. c.
Then the probable number going in a given dirⁿ θ
is



$$\frac{2\pi\ r\sin\theta\cdot r\,d\theta}{4\pi\ r^2} = \frac{\sin\theta\,d\theta}{2}$$

(2) No. of mols passing up across a given surface per sec.
Total no. n.



Normal component of vel $= c\cos\theta$.
No. passing along dirⁿ. θ
is $n\,\frac{\sin\theta\,d\theta}{2}$.

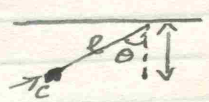
Integrate from 0 to $\frac{\pi}{2}$.

No. crossing per sec. $= \frac{1}{2}\,cn\int_0^{\frac{\pi}{2}}\sin\theta\cos\theta\,d\theta$.

$$= \frac{cn}{2}\int_0^{\frac{\pi}{2}}\sin\theta\,d(\sin\theta) = \frac{cn}{2}\left[\frac{\sin^2\theta}{2}\right]_0^{\pi/2}$$

$$= \frac{1}{4}\,nc.$$

(3) Average distance travelled normally. l = mean free path



Av. dist $= \frac{\Sigma\text{ no.}\times\text{dist}}{\Sigma\text{ no.}}$

$$= \frac{\int_0^{\pi/2}\frac{1}{2}\,nc\cos\theta\sin\theta\cdot l\cos\theta\cdot d\theta}{\int\frac{1}{2}\,nc\cos\theta\sin\theta\,d\theta}.$$

$$= \frac{\frac{1}{2}\,ncl}{\frac{1}{4}\,nc}\int -\cos^2\theta\,d(\cos\theta)$$

$$= 2l\left[\frac{\cos^3\theta}{3}\right]_0^{\pi/2}$$

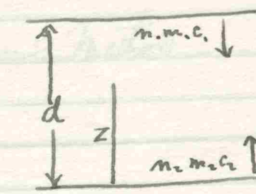
$$= \frac{2}{3}\,l.$$

Consider *diffusion* of 2 gases, $n_1 m_1 c_1$, $n_2 m_2 c_2$.

Let $\frac{\partial n_1}{\partial z}$ be + and $\frac{\partial n_2}{\partial z}$ −

be the density gradients.

& K_1 K_2 the diffusivities.



Consider each gas separately.

at z $\quad n_1$

" $z + \frac{2}{3} l$ $\quad n_1 + K_1 \frac{2}{3} l_1 \frac{dn_1}{dz}$

" $z - \frac{2}{3} l$ $\quad n_1 - K_1 \frac{2}{3} l_1 \frac{dn_1}{dz}$.

∴ no. passing down is $\frac{1}{4}(n_1 + K_1 \frac{2}{3} l_1 \frac{dn_1}{dz})c$

" " up " $\frac{1}{4}(n_1 - K_1 \frac{2}{3} l_1 \frac{dn_1}{dz})c$

Excess going down $= \frac{1}{4} K_1 \frac{4}{3} l_1 \frac{dn_1}{dz} c$

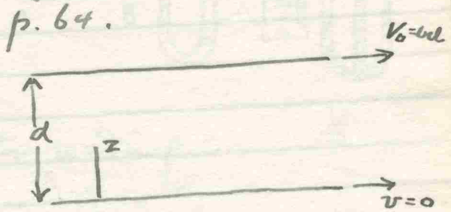
$= \frac{1}{3} K_1 l_1 c \frac{dn_1}{dz}$.

Similarly for gas $n_2 m_2 c_2$.

Excess going up $\quad \frac{1}{3} K_2 l_2 c_2 \frac{dn_2}{dz}$.

Ch. XVIII Viscosity.

Viscosity of gases – Boltzmann's Method.
See lemma's 1, 2, 3 p. 64.



Vel. at a pt. z is
$v = z\dfrac{dV}{dz} = z\dfrac{V}{d}$

Force per unit area F in direction of v
is $F = \eta\dfrac{dV}{dz} = \eta\dfrac{V}{d}$.

At any given plane mol. will arrive with a directed momentum from an average distance $\frac{2}{3}l$
momentum $= m \cdot \dfrac{V}{d}(z \pm \frac{2}{3}l)$

No. of mols. passing thro unit area per sec $= \frac{1}{4}nc$
so total momentum per sec up is $\frac{1}{4}nc\,\dfrac{mV_0}{d}(z - \frac{2}{3}l)$
" " " " " down $\frac{1}{4}nc\,\dfrac{mV_0}{d}(z + \frac{2}{3}l)$

$\therefore \frac{1}{4}nmc\dfrac{V}{d}\,\frac{4}{3}l$ = diff per sec $= \dfrac{mv}{t} = F$

$\therefore \eta\dfrac{V_0}{d} = \frac{1}{3}nmc\dfrac{V}{d}l$

$\eta = \frac{1}{3}nmlc$.

Viscosity of Liquids..
For small values of c this is theoretically true
+ Poiseuille's Eqn. (P + T. p 207 etc)
$Q = \dfrac{(p_1 - p_2)\pi a^4}{8l\eta}$ for motion in a cylindrical capillary tube holds but if c goes beyond a critical limiting value the motion becomes turbulent.
Osborne Reynolds showed experimentally that steady motion ceases to exist if $v > \dfrac{1000\,\eta}{\rho a}$

[1923. Note. See work of G. I. Taylor on behaviour of liquids between rotating cylinders]

Measurement of Viscosity

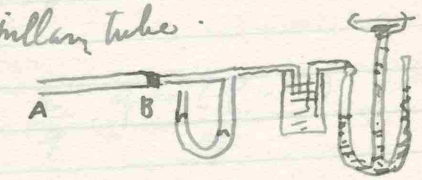
1. Gas. flow through a capillary tube.

(a) Lehfeldts method for H.

H from electrolysis through tube AB.

pres. at A is Barometric

" " B is B + manometer reading.



(b) Rankine's Method. Drop of Hg

2. Liquid. a. Flow down a syphon.

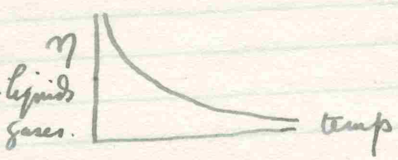
(b) Searles Viscosimeter Rotating cylinder.

(3) Maxwell's. Oscillating disc. log. dec.

Effect of temp.

decrease of η for liquids

increase " " " gases.



Effect of pressure.

Sometimes increases (ether) sometimes decreases (H_2O)

for gases within wide limits no change in η.

down to 1 mm. pres (approx)

Effect of Mixtures

Resultant η is often the arith mean of values for component gases, Sometimes law is not evident.

Effect of Viscosity on the Fall of a small solid or liquid body.

Stokes Law. $V = \frac{2}{9} g \frac{a^2(\rho - \sigma)}{\eta}$.

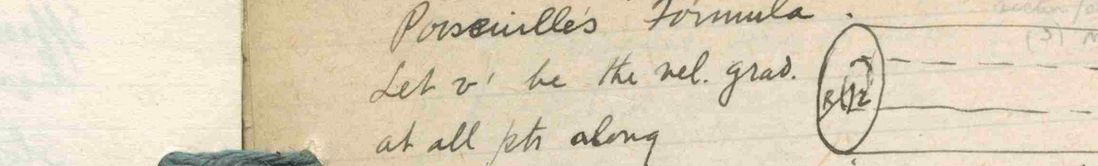
Special Notes on Viscosity of Colloidal Solutions (see Notes from Dr Johnston March 1921)

[1923 Note. Application of Stokes Law in Sven Odén's sedimentation method of obtaining distribution of sizes of particles of effective radii between about .4 μ to 50 μ.
See Proc. R. Soc. Edin. 1915.
" " " 1923]

[illegible] not.
(2) Incompressible
vol passing each cross section/sec is constant
(3) no slip at walls.

(A) Viscosity of Liquids

Poiseuille's Formula.



Let v' be the vel. grad.
at all pts along
surface of an imaginary coaxial cylinder
Then force f exerted on liquid inside cylinder
is $f = \eta v' \times \text{surface} = 2\pi r l \eta v'$
This must equal diff betw. pressures on
both ends $(p_1 - p_2)\pi r^2$.

$$\therefore v' = \frac{r(p_1 - p_2)}{2\eta l}.$$

Thus v' changes from 0 at axis to $\frac{R\, p_1 - p_2}{2\eta l}$
at walls of cylinder.
Av. v' between r & R is $\frac{r+R}{2} \cdot \frac{p_1 - p_2}{2\eta l}$
hence vel v changes by

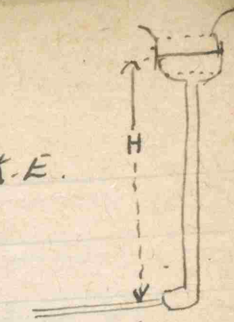
$$(R - r) \times \text{av. } v' = (R^2 - r^2)\frac{(p_1 - p_2)}{4\eta l}.$$

Thus vel at axis $(r=0)$
is given by $\frac{(p_1 - p_2) R^2}{4\eta l}$.

Hence average vel. is $\frac{p_1 - p_2}{8\eta l} R^2$
Area cross section is πR^2
hence vol. Q passing each section per sec.
is $\pi R^2 \times \text{Av. vel.}$

$$\text{or } Q = \frac{(p_1 - p_2)\pi R^4}{8\eta l}.$$

Loss of Pressure due to gain of K.E.

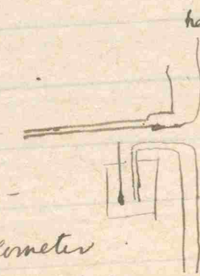


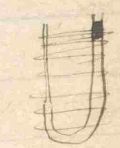
If $(p_1 - p_2)$ is apparently due to a head H then some of this pressure imparts KE to the liquid & some of the pressure overcomes Viscosity.

p.497. ~~Edser~~ gives effective head overcoming Viscosity is $h = H - \frac{2Q^2}{g\pi^2R^4}$

P.&T. etc. give $h = H - \frac{Q^2}{g\pi^2R^4}$.

Measurement of η for Liquid.





(1) Poiseuille's Method.

(2) ~~Rankine's Method~~ Syphon method. Hg drop.

(3) Rotating cylinder. Searles Viscometer

(4) Maxwells Torsion Pendulum.

Stokes Law.

Lubrication. & Fugitive shear Elasticity.

Effect of temp.
Visc. of liquid dec. as temp inc.

Effect of pres.
Visc. of some liquids rise (ether)
" " " " diminish (H_2O)

1925 Notes on Viscosity.

(B) Viscosity of Gases.

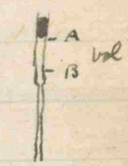
(1) Boltzmann's treatment as on. p. 66.

$$\eta = \tfrac{1}{3}\, n\, m\, l\, c$$

Mean free path l known if η be determined & c calculated from pressure $p = \tfrac{1}{3} n m c^2$

See for air: R+T p. 220. $l = 0.00001$ cm.

(2) Method of Measurement.



1. Rankine's Hg pellet forces air down.
2. Lehfeldts. (p. 67)

(3) Flow thro' a narrow tube $p_1 V_1 = p_2 V_2 = \dfrac{(p_1^2 - p_2^2)}{16 L \eta} \pi r^4$

See Edser p. 515.

Where p = pres. $\underline{v}$ = $\underline{\text{vel}}$. L = length of tube.

Condn. to be satisfied is mass of gas crossing all sections is the same. $\rho\, a\, v$ = const.

or if a is const. ρv & ∴ $p v$ = const.

Take so small an element that its dens. is a const. then Poiseuille's Eqn. holds.

$$V_a = \frac{(p_a - p_b)\, \pi r^4}{8 \eta\, l(a \text{ to } b)}$$

In this element av. pres. is $\dfrac{p_a + p_b}{2}$.

$$\text{Hence } pVl = \Sigma\left(\frac{p_a + p_b}{2} V_a\, l_{a\text{-}b}\right)$$

$$= \Sigma\left(\frac{p_a^2 - p_b^2}{16 \eta\, l_{a\text{-}b}}\, l_{a\text{-}b}\right) \pi r^4$$

$$\therefore\ pV = \frac{p_1^2 - p_2^2}{16 \eta l} \pi r^4$$

(4) <u>Temp effect</u>

Visc. of gases inc. with inc of temp.

<u>Pres. effect.</u>

Visc. of gases indep. of press. throughout wide range. atmospheric to a few mm Hg.
see P & T p. 219 graph.

<u>Visc. of gas mixtures</u>

very erratic, some are A.M. some follow no apparent law.

Extra Topics to be touched upon

(1) Vortex rings – with demonstrations
see Edser.

(2) Extremes of Density

(a) Betelgeuse –

(b) Sirius B.

(3) J.H.J. on Gravitational forces at work on nebulous matter in space – ρ of space. Hubble.

Torricelli 1608-1647
Italian physicist + mathematician studied in Rome, later as amanuensis to Galileo in Florence. Appt^{d} to chair of Math. at Flor. after Galileo's death. Invented barometer 1643. Wrote on Props. of Cycloid; Hydraulics $v = \sqrt{2gh}$ at orifice; Paraboloid is the envelope to trajectories of similar projectiles discharged in all directions with equal velocity from one source (eg. machine gun) Improved mic. + tel.
Died of pleurisy at Florence.

[1923 Note: Engineered the draining of a swampy lake in valley of Tiber between Florence + Rome]

MADISON
FILING FOLDER

Properties of Matter.

Question 1. Durham pit is 1260 ft. deep. If the Earth's mean density be taken as 5.5 and the density of the surface crust as 2.5, how many more complete oscillations will a pendulum make at the bottom of the pit in 24 hours time than it would at the surface, if at the surface it beats seconds? 1.7 oscillations

Question 2. Two similar spheres one of lead weighing 10 lbs. the other of aluminium weighing 1.25 lbs. are projected from a cliff 100 ft. high, the first with a horizontal velocity of 50 ft. per second, the latter with no initial velocity. When will each strike the plane below? What conclusions would you draw as to the nature of gravitation and of the relation of mass to weight if by experiment you found the times to be the same? 2½"

Question 3. A tie bar is subjected to uniform tensile stress of 5 tons per square inch, what is the intensity of shear stress on a plane the normal to which is inclined in 30° to the axis of the bar? What is the intensity of normal stress on this plane? What is the resultant intensity of stress on the plane?

3.75 tons/in²
2.165 tons/in²
4.330 tons/in²

4. A square steel plate of area 36 sq. ft. and thickness 2 in., on the side of a ships hull is subjected to shear stress along its edges of intensity 10 tons per sq. in. Find the nature & value of the stress intensities on the diagonals of the plate. (10 tons/in², tension + compression)

MADISON FILING FOLDER

Properties of Matter.

No. 1. Find the Elongation in a steel tie-bar 10 feet long and 1.5 inches diam. due to a pull of 12 tons. ($E = 13000$ tons/in^2) (Ans .0627 in)

No. 2 For a given material $E = 6000$ tons per sq. in., $N = 2300$ tons /in^2 find K and the lateral con-traction of a round bar 1" in diam. and 10 ft. long when stretched 0.1 inch. ($K = 5111$ tons/in^2; $-f = .000254$ in)

No. 3. Find the proof resilience of a bar of steel $1\frac{1}{2}$

inches diam. 8 ft. long, the tensile elastic limit being 14 tons per sq. in. and $E = 13500$ tons per sq. in. Find also the proof resilience per cubic inch. (2760 and 16.26 inch-pounds)

No. 4. Find the stress intensity and extension produced in a bar 10 ft. long, 1.5 sq. ins. section by a live tensile load of 6 tons. What live load would produce an extension of 1/20 inch? ($E = 13000$)
(Ans - 8 tons/in^2; 0.0738 in; 4.06 tons)

No. 5 A bar of Steel 1 inch diam. and 10 ft. long is heated to 100° F. above temperature of atmosphere, then clamped at ends. Find tension in bar when cooled to temperature of atmosphere, if during cooling it pulls the end fastening 1/40 inch nearer together. ($E = 13000$; $\alpha = .0000062$)
(Ans - 5.33 tons/in^2; 4.18 tons)

No. 6. Ashort bar of Cu. 1 inch diam. is enclosed in a steel tube of 13/8 inch external diameter and 1/8 inch thickness. While at 60°F the ends are rigidly fastened together. Find stress intensity in each metal at 260°F.

$E = 13000$ for Steel
$E = 7000$ for Cu
$\alpha = .0000062$ for Steel
$\alpha = .000010$ for Cu

(Ans In Steel 4.57 tons/in^2
Cu 2.86 " ")

19/11/20.

McGILL UNIVERSITY
MONTREAL

THE MACDONALD PHYSICS BUILDING

Properties of Matter.

Third Year Honours. January 18, 1921.
9-11 am.

(Do questions 7 and 8 and any other ~~four~~ 3.)

1. Write a ~~[illegible]~~ short essay on the subject of Gravitation and its measurement.

2. Define stress, elastic limit, factor of safety, proof resilience.
Explain why fracture often occurs by the shear of surfaces inclined at angles other than 90° to the axis of pull when a body is subjected to intense tension.

3. Define homogeneous strain.
Prove that there can only be one set of three mutually perpendicular axes in the unstrained body which will remain mutually perpendicular after straining.

4. If E be Young's Modulus, N the Modulus of rigidity state [illegible] the relation E = 9NK/[illegible] determined for a specimen [illegible]

5. Write a note on four of the following stating [illegible] each has contributed to the study of Properties of Matter.
Airy, de St Venant, Poisson, Boys, Batho, Kater.

6. In the theory of bending prove that the stress intensity in any strip is proportional to its distance from the neutral axis and to the curvature.
If $M = EI\frac{d^2y}{dx^2}$ find the deflection, y, of the extremity of a cantilever arm of length L when a weight W is applied half way along the arm.

7. Find the value of the couple C required to twist a thin cylindrical tube of circular section through an angle ϕ where l is the length, a the external radius, t the thickness of wall of tube, and N the modulus of rigidity.
How much work is done on a steel tube 20 ft. long when a couple of 10 inch-tons produces a torsion of $\pi/360$ radians per foot?

8. Find the maximum stress intensity and the extension produced in a bar 10 feet long, 1.5 sq. inches in section by a live tensile load of 6 tons.
If the extension due to a dead load be 0.0148 inch find the energy stored in the bar in foot-pounds.
(E = 13000 tons per sq. inch).

6. Contrast the relations between the pressure and the volume of a gas when undergoing isothermal and adiabatic changes.

Show how cohesion and molecular volume necessitated Van der Waal's alteration of Boyle's law, and explain the term "critical point" for a gas both algebraically and diagrammatically.

7. Define Surface Tension, Ripple, Potential Energy of a stretched film, Capillary Repulsion.

Prove that the excess pressure inside a spherical bubble is $4\frac{T}{R}$ where T is the surface tension and R the radius of curvature.

8. Discuss the Stability of Cylindrical Films, illustrating by means of Boys' experiment.

Mention various methods of determining T for a given liquid and describe one method in detail.

9. State Fick's Law of Diffusion. On what does the diffusivity of a salt in a liquid depend?

Explain the terms: Osmosis, isotonic, colloid.

By using the Second Law of Thermodynamics calculate the depression of the freezing point due to the addition of salt to a liquid where P is the osmotic pressure of the solution.

10. Assign to each of the following his contribution to our knowledge of the Properties of Matter:—

Rayleigh, Airy, de St. Venant, Cavendish, Searle, Eötvos, Stokes, Graham, Laplace, Boltzmann, T. C. Chamberlain, Ostwald.

McGILL UNIVERSITY.

PROPERTIES OF MATTER.

THIRD YEAR HONOURS IN MATHEMATICS AND PHYSICS.

April 27th, 1921. Morning—9.00 to 12.00.

(Do not answer more than eight questions.)

1. Describe fully *either* Boys' or Poynting's method of determining the density of the earth. Establish the relation between this quantity and the gravitational constant, G. Mention any three methods of finding "g".

2. Define Modulus of Rigidity, Complementary Shear Stress, Proof Resilience, Elastic Limit, Viscosity of Metals, Live Tensile Load.

   Establish the relation for Poisson's Ratio

   $$\sigma = \frac{3K-2N}{2(3K+N)}$$

   where K, N are the Bulk and Rigidity Moduli respectively.

3. Find the deflection of the middle point of a beam which is freely supported at each end and loaded at the centre.

   Describe briefly any two testing machines that you have seen, either tensional, compressional, torsional or impact; or discuss the various properties of wood as illustrated in the Forest Products Department.

4. State Newton's Laws of Impact.

   Prove that impact is always accompanied by loss in Kinetic Energy, *or* illustrate Hertz's Theory of Elastic Collision by describing each phase in the collision of railway trucks with buffer springs.

5. Discuss the tensile strength and compressibility of liquids with special reference to the investigations and methods of Osborne Reynolds, Worthington, Regnault. What effects would changes in pressure, temperature and concentration have on the compressibility of salt water.

MADISON FILING FOLDER

McGILL UNIVERSITY

FACULTY OF ARTS

PHYSICS 5A PROPERTIES OF MATTER

April 18, 1925. 2 - 5 p.m.

Part I 25%

(Approximately one hour)

Write an essay upon one of the following:
Gravitation
Gas Laws

Part II 50%

(Approximately two hours)

Answer **four** questions

1. Define any five: stress, plasticity, yield point, ultimate strength, homogeneous strain, proof resilience, complementary shear stress, Poisson's Ratio.

Find the energy stored in a bar of steel 1 1/2 inches [illegible] tensile stress of 12 tons per square inch given that E = 13500 tons per sq. in. 1.2 inch tons = 206 ft. lbs.

2. Derive the formula for the deflection of a cantilever loaded at the end

$$y = \frac{Wl^3}{3EI}$$

A plank is supported freely at points 10 feet apart and is 10 inches wide and 1 inch thick. Find Young's Modulus if a weight of 100 lbs. at the centre depresses the plank 5 inches. 1.04 10^8 lbs/ft² = 72 10^4 lbs/in² = 360 tons/in²

3. Define surface tension and discuss its measurement. What is a ripple?

To what height will water (density 1, surface tension 72 dynes/cm.²) rise in a capillary tube of radius 0.02 cm.? 7.3 cm.

4. Write notes on any three: Osmosis, Orientation of molecules, Viscosity, Colloids.

5. Describe two laboratory methods for obtaining the coefficient of rigidity of a wire indicating the theoretical considerations underlying the experiments.

Lab. Report 25%

McGill University.

Department of Physics (111 yr. Honours).

Properties of Matter.

April 1927.
Time - 3 hours.

Answer 6 questions.

(1) What is the gravitational constant? By what methods may it be determined? Describe carefully what you consider the best method.

(2) Define plasticity, stress, elastic limit, ultimate strength, Poisson's Ratio.
Prove that when a body is subjected to homogeneous strain, only one set of perpendicular axes in the unstrained condition will remain mutually perpendicular after straining.

(3) If E be Young's Modulus; N the Modulus of Rigidity; K the bulk Modulus, prove the relation

$$E = 9\,N\,K/\,(3K - N).$$

Find the proof resilience of a bar of steel $1\frac{1}{2}$" diam. 8 ft. long, tensile elastic limit being 14 tons/in^2 and E = 13500 tons/in^2.

(4) Discuss the elastic properties of the earth's crust. What is meant by isostatic compensation? How does seismology throw light on the internal structure of the earth?

(5) Prove that the deflection of the free end of a cantilever of length l under a weight W is $Wl^3/3EI$ where I is the moment of inertia of the cross-section of the beam.
Describe two methods for determining the elastic constants of a wire.

(6) Discuss the significance of the constants in Van der Waals equation. Express the critical temperature, pressure and volume in terms of these constants.

(7) Write notes on Surface energy, ripples, osmosis.
If the surface tension of water is 72 dynes/cm^2 how high will water rise in a capillary tube of radius 0.2mm.

(8) Discuss two of the following:-
Viscosity and its measurement.
The range of density of matter.
Orientation of molecules.