11.

A. Vibert Douglas Student Note Books (ii)

RADIATION · THEORY ·

APPLICATION · TO · STELLAR · PHYSICS ·

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Michaelmas Term
1921.

Radiation Theory

Mr. A. E. Milne
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14.10.21.

Texts: Planck: Warmstrahlung. 1919.
Lewis Phys Chem. Vol 2.
Lorentz. Ch. 2.
Richardson El. Thy Matter ch 15. (condensed)
Cambells Mod. El. Thy. ch 10 (non math)
Shuster
Drude
Woods Phys. Optics.
Bryan Thermodynamic pt. of view.

Energy of novae originates from ext. layer.
" " steady stars appears to come from centre. + is brt. to surface by radiation + very little by condn + convec.
Stellar temps 3000°C - 30000°C. + rad. energy is propl. to 4th power of temp.
Convection to small extent as proved by Doppler effect but vel. not > 1 km/sec hence convec. not a major means.

Distrib of energy in continuous spectrum + energy reaching earth gives data from wh. diam of star + its temp may be found
Sampson (Edin) in Oct. Observatory 1921
Agreement of results by these methods + by Radn. Thy confirms two things - the

pressure of light + 2nd law thermodynamics

Radn Thy. concerned with flux of Energy thro' a (possibly) structureless, continuous medium - Electromag. theory of light not essential to thy. only the observable facts of frequency & polarization.

Thermodynamics is not dynamics & only accidentally heat - is applicable not for individual cases but for generalities in macroscopic investigations.
(see Planck on Boltzman - Columbia Lectures)

Complexion is the complete Specification of a field of energy - Many complexions possible for a given field - One may be most probable & to find that macroscopic state is the problem of the Radn. Thy.

Rad. energy comes indept. of temp. gradient e.g. from Sun to Earth in opp. dir. to gradient in atmosphere.

All known cases of transparancy (?) argue from very thin layers. transparent - ∴ thicker opaque layers + ∴ indept. of body.

Rad. heat, Hertzian waves &c are all Rad energy & obey laws of optics — reflection, etc., rigorously.

Use wave theory only if convenient.

Consider plane wave front — energy/cm² sec. no energy transmitted in dirn ⊥r but in nature there is always a transmission in every dirn.

Defn. $\int_{d\sigma}^{\theta} \Pi$ (normal) from every pt. in $d\sigma$

construct cones of angle $d\omega$ in dirn θ to normal Π — all similar. Let E be energy transmitted in t sec.

Then $\mathcal{L}t \dfrac{E}{dt\, d\omega\, d\sigma \cos\theta} \to I =$ intensity of radiation.

Exclude any directions where $\mathcal{L}t$ does not exist or becomes ∞

thus both plane waves & spherical waves are excluded.

N.B. I has a maxm. & falls off rapidly for varying values of θ . . ?

Amt. of energy thro $d\sigma$ in dirn θ is $\mathcal{L}t\, I\, dt\, d\omega\, d\sigma \cos\theta$.

Do not take $d\sigma$ so small or dt

that they wd. become comparable with .... ?

Energy E from $d\sigma$ in time ν to $\nu + d\nu$

then $\frac{E}{d\nu} = I_\nu \, d\nu$.

$\therefore \int_0^\infty I_\nu \, d\nu = I$.

all frequencies

Energy thro $d\sigma$ can be resolved in two $\perp^r$ polarized beams. $I_\nu = \mathcal{J}_\nu + \mathcal{J}_\nu'$

It is possible starting with electro mag. field to deduce the above – (Darwin)

Given E, H, Poyntings' Thm. Flux $= c[E.H]$ vectorprod

Take x compt. of El. Intens

$$X = \iint f\left(t - \frac{x \sin\theta \cos\varphi + y \cos\theta \sin\varphi + y \cos\varphi}{c}\right) \sin\theta \, d\theta \, d\varphi$$

Use Fourier's integral

$f(t)$ interval $(-\frac{1}{2}T, +\frac{1}{2}T)$ resolves into sequence $\pi f(t) = \int_0^a (A_\lambda \cos t + B_\lambda \sin t) \, d\lambda$

Where $A_\lambda = \int_{-\frac{1}{2}T}^{\frac{1}{2}T} f(t) \cos t \, dt$ etc.

$\therefore f(t)$ from above becomes

$$= \iiint A \, \xi(\theta \varphi \nu) \cos(t - \text{—})$$
$$+ \xi(\qquad)$$

hence putting $X = \int_0^\infty d\nu \iint \sin\alpha\, d\alpha\, d\varphi\, \xi_{\alpha\varphi\nu}\, e$

where $e^{-} = e^{2\pi\nu(t - \frac{r}{c})}$

Since $E = 0$, $\frac{dX}{\partial x} + \; + \; = 0$

i.e. $\xi \sin\theta \cos\varphi + \eta \cos\theta \sin\varphi + \zeta \cos\theta = 0$.

Solve for $\xi\ \eta\ \zeta$.

$\xi = A \cos\alpha \cos\varphi - B \sin\varphi$

etc.

From curl formulae get magnetic constants

$\xi'\ \eta'\ \zeta'$

Flux thro small area is

$\iint dy\, dz \int_{-\frac{1}{2}T}^{\frac{1}{2}T} (Y\gamma - Z\beta)$

= net flow in all dirns.

Insert expressions for $X\ Y\ Z$.

& simplify by partial integration

$\frac{1}{2} c^2 \int \frac{d\nu}{\nu} - \int d\alpha \sin\alpha \{(A_\nu)^2 + (B_\nu)^2\} \cdot c$

$= \ldots ? - \ldots \int d\nu \iint \mathfrak{J}_\nu + \mathfrak{J}'$

Identifying terms it follows that

the former eqns are thus established

by this method.

We had. Energy transmitted $\perp^{r}$ x axis in certain time.

$$\mathcal{J}_\nu(\theta\,\varphi) = \frac{c^3}{2\nu}\,|A_\nu|^2 \frac{1}{\text{Sin (time)}\,.\,\sin\theta\cos\varphi}.$$

derived from eqns of type

$$\eta = \int_{-y}^{y} dy \int_{-z}^{z} dz \int_{-\frac{1}{2}T}^{\frac{1}{2}T} dt\ \frac{\gamma}{c^2}\sin\theta\cos\varphi\,\gamma c.$$

Only determinable if λ small & periodic time small compared with corresp. integrals.

Theory of Fourier analysis

$$f(0) = \lim_{T\to\infty} \int_{-\infty}^{\infty} f(\nu)\ \frac{\sin \pi\nu T}{\pi\nu}\ d\nu.$$

$$f(\nu_1) = \lim_{T\to\infty} \int_{-\infty}^{\infty} \left(\int_{-\frac{1}{2}T}^{\frac{1}{2}T} \cos\pi(\nu-\nu_1)t\ dt\right) f\nu\, d\nu$$

This is a sufficiently close approximation the analysis is simplified enough to give results i.e. νT large

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Radiation field - Intensity of radiation I.
I (or I_ν or $\mathcal{J}_\nu$; $\mathcal{J}'_\nu$) is const along the path of any ray in free space.

I along any pencil is const. along that pencil.

$d\sigma \quad \theta \quad r \quad \theta' \quad d\sigma'$

$$d\omega = \frac{d\sigma'\cos\theta'}{r^2}$$

Hence $I \frac{d\sigma' \cos\theta}{r^2}$ r $d\sigma \cos\theta$ is amt of energy passing

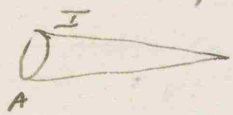
$I' \; d\sigma' \cos\theta' \; d\sigma \frac{\cos\theta}{r^2}$

these are same $\therefore I = I'$.

Consider cones from one pt in $d\sigma$ to every pt in $d\sigma'$ & aggregate of all these from every pt in $d\sigma$ – cones overlap

Flux per unit area at Sun > at Earth

But Intensity of rad^n " " = " "
by def^n.



$\cdot \; \frac{A I}{r^2} = \Omega I$

Solid angle Ω
same

Ω must be so ~~or~~ small as to just include Sun

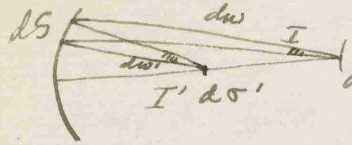
Difficulty for stars that Ω cannot be taken small enough.

Brightness is light rec^d at receiver per unit solid angle.

$I \cos\theta \cdot d\sigma \sec\theta$.

I is unaltered by total reflection from one or more mirrors

Concave Mirror



$I \, d\omega \, d\sigma$

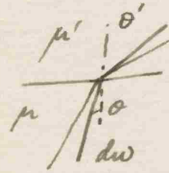
$= I' \, d\omega' \, d\sigma'$

$d\omega = \frac{dS}{r^2}$

$d\omega' = \frac{dS}{r'^2}$

Hence $I = I'$.

Consider Refraction without any reflection.



$I \, d\omega \, d\sigma \cos\theta$

$= I' \, d\omega' \, d\sigma' \cos\theta'$

$d\omega = \sin\theta \, d\theta \, d\phi$

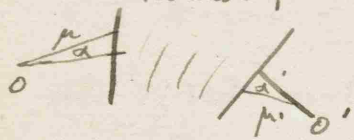
$d\phi = d\phi$.

$\mu \cos\theta = \mu' \cos\theta'$

$\mu \cos\theta \, d\theta = \mu' \cos\theta' \, d\theta'$

$\therefore \frac{I}{\mu^2} = \frac{I'}{\mu'^2}$

Use Helmholtz method.



Let l l' be rel. sizes of images at OO'

Helmholtz says $\mu \alpha l = \mu' \alpha' l'$

$d\sigma \propto l^2$
$d\omega \propto \alpha^2$
$\therefore$ using $I \, d\omega \, d\sigma = I' \, d\omega' \, d\sigma'$

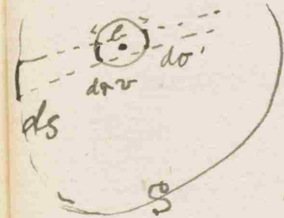
then $\frac{I}{\mu^2} = \frac{I'}{\mu'^2}$ as before.

This is based on Princ. of Least Time.

I is not a vector quantity.

Density of radiant energy in neighbourhood of a certain pt. does not completely specify the field – but is often all the data we have.

Let U be dens.



Take pt + local neighbourhood v + large convex surface.

all radn crossing v pass thro S.

Consider the radn crossing v from dS in one partic dirn.

$$\frac{I\,dS\,d\sigma\cos\theta\cos\theta'}{r^2} \quad ①$$ passes in unit time

time to cross v of dist l is $\frac{l}{c}$.

amt of radn intercepted betw dσ + dσ' is $\frac{l}{c}\times ①$

$$\frac{I\,dS\cos\theta}{r^2}\,\frac{dv}{c} \quad ②$$ where dv is vol. inside v.

$$\therefore \int ② = \frac{I\,dS\cos\theta\; v}{r^2 c}$$

but $d\Omega = dS\cos\theta/r^2$

∴ all radn crossing dS goes thro v

$$= \frac{v}{c} I \, d\Omega$$

∴ from all S.

$$\int \frac{v}{c} I \, d\Omega = \frac{v}{c} \int I \, d\Omega .$$

∴ Energy dens. is determined if I is given.

$$\text{Energy dens } u = \frac{1}{c} \int I \, d\omega .$$

if isotropic i.e. indep of dirn.

$$u = \frac{I}{c} \times 4\pi$$

Hence also $\mathfrak{I}_\nu$ & $\mathfrak{I}_\nu'$ are isotropic so that $\mathfrak{I}_\nu = \mathfrak{I}_\nu'$

$$u_\nu = \frac{8\pi}{c} \mathfrak{I}_\nu .$$

c = vel. of light in vacuum

$\frac{c}{\mu}$ = vel in medium of μ.

$$u = \frac{4\pi \mu I}{c}$$

$$I \frac{d\sigma \, d\sigma' \cos \varepsilon \cos \varepsilon'}{r^2} \cdot \frac{r}{c}$$ = amt passing thro

$\varepsilon \, \varepsilon'$ angle betw normals & rays –

Emission of Radiation.

Isotropic radn from a body emitting radn

$\varepsilon \, d\omega \, dv \, dt$ is amt radiated from solid angle $d\omega$

where ε = coeff of emission

Total emis $= 4\pi \varepsilon \, dv \, dt$.

Mass emission $\varepsilon \rho \, dv \, dm \, dt$.

Energy emitted must be drawn either from around it or from within. It is the phys. state of ether + matter inside the element including the receipt through its boundary.

[consider fluorescence high freq light on a body may cause it to emit low freq. radiation.]

Temp. Radiation.

ε_ν is funct of temp of body only

i.e. uninfluenced by any change wh. does not effect the temp.

This is an arbitrary restriction. + one can conceive

of emission effected by other condns beside temp.

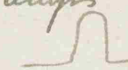
Absorption.

This weakens a pencil, also scattering; former is absorbed for later use, latter lost to it.

$I_\nu \cdot dl \cdot I_\nu + dI_\nu$ $\quad dI_\nu$ is neg.

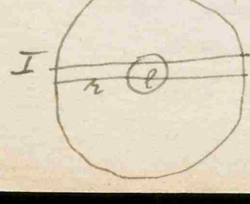
$$k_\nu = \frac{1}{I_\nu} \frac{dI_\nu}{dl}$$

this coef. is indep. of ~~amt~~ intensity of radn unless for high temps

Colour screen  k_ν

$I_\nu e^{-k_\nu l}$ is the reduction in traversing l.

Scattering similar defn for S_ν not constant at a pt. but in isotrop radn. the sum total of the scattering will also be isotropic.



$$I \, k \, l \, d\sigma \, \frac{dS}{r^2} \cos\theta \cos$$

$k \, v \int I \, d\omega$ is absorbed by element.

∴ Total absorption is $4\pi K I$

for scattering $S \int I \, d\omega$

& amount into $d\omega$ is

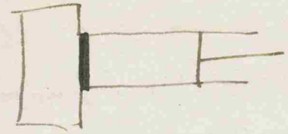
$$\frac{d\omega}{4\pi} \int I \, d\omega = S I \, d\omega$$

for isotropic

Lord Rayleigh showed

$S_\nu \propto \nu^4$.

27/10/21.



T

isotropic black radiation from black radiator

isothermal

reflector in front of black radiator gives adiabatic case.

Both reversible.

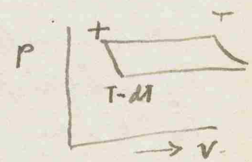
expand slightly

Radiation of small black body may not remain black.

Work done by piston is a reversible process

∴ in adiabatic expans. black radⁿ must remain black.

energy of radⁿ depends only on density of



Carnot's Cycle.

(1) adiabatic compression

(2) Expand isothermally (large compared with (1))

Work done is $p v$

u = energy density

∴ $u v$ is additional energy

∴ Heat is $p v + u v = \frac{4}{3} u v$ taken ~~..~~ from reservoir

3. Adiabatic to $T - dT$

4. Finish by compression isothermally.

$\therefore$ Net external work $= \int_{cycle} p\,dv =$ area

$= v \times dp$

$= \frac{1}{3} v\, du$

By Equivalence of all heat engines + 2nd Law Th. dyn + defn. of temp

$$\frac{dT}{T} = \frac{\frac{1}{3} v\, du}{\frac{4}{3} v u} = \frac{1}{4} \frac{du}{u}$$

$$u = aT^4$$

a = const

This is Stephan's Law.

and Boltzman's proof.

Stephan got at it empirically.

This is evidence against any corpuscular thy of light pressure as it verifies the $\frac{1}{3} v\, du$

[Consider $\frac{dT}{T} = \frac{v\,dp}{vu + vp} = \frac{dp}{u+p}$

$\frac{dp}{dT} - \frac{p}{T} = \frac{u}{T}$ — If by Expt. $u = aT^4$

$d\frac{p}{T} - \frac{u}{T^2} = aT^2$

$\frac{p}{T} = \frac{1}{3}aT^3 + \dots$?

Simpler method by Entropy.

Enclosed area containing black radiation.

Inc. of Entropy $dS = \frac{dQ}{T}$

$dQ = \frac{d(uv)}{\cancel{dt}} + pu = \cancel{\frac{4}{3} u\, dv}$

$= d(uv) + \frac{1}{3} u\, dv$

$$dS = \frac{d(uv) + \frac{1}{3} u\, dv}{T} = \frac{\frac{4}{3} u\, dv + v\frac{du}{dT}dT}{T}$$

$\therefore$ for perfect differential

$$\frac{\partial}{\partial T}\left(\frac{4}{3}\frac{u}{T}\right) = \frac{\partial}{\partial v}\left(\frac{v}{T}\frac{du}{dT}\right)$$

$$\frac{4}{3}\frac{1}{T}\frac{du}{dT} - \frac{4}{3}\frac{u}{T^2} = \frac{1}{T}\frac{du}{dT}$$

$$\therefore \frac{u}{T} = \frac{1}{4}\frac{du}{dT}.$$

Integrate + get T^4 as before.

Put in $u = aT^4$

$dS = \frac{4}{3} aT^3 dv + 4aT^3 v\, dT.$

$= \frac{4}{3} a\, d(vT^3)$

$S = \frac{4}{3} aT^3 v + \text{const}.$

Entropy $\propto$ vol. $\therefore$ const is zero

$\therefore S = \frac{4}{3} aT^3.$

In adiabatic expans. Entropy is const.

$vT^3 = v'T'^3$

$p\,dv + d(uv) = 0$

$p = \frac{1}{3}u$

$\frac{dv}{v} + \frac{3}{4}\frac{du}{u} = \quad vu^{3/4} = \text{const}$

or $vp^{3/4} = \text{const}$ Gas. Eqn.

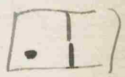
In black body radn.

pres. depends only on temp.

In case of perfect gas energy of a given amt of gas is indept of its pres.

Escape through small hole

$v \cdot aT^4 = v'aT'^4$



$\Delta S > 0$.

Since $u = aT^4$

$u = \frac{4\pi}{c}B$ B = intensity of radn

$B = \frac{ac}{4\pi}T^4 = bT^4$

$B\int d\omega \cos\theta$

$\pi B = \sigma T^4$ Const. of total radn. (Stefans Const)

$\sigma = 5.70 \times 10^{-5}$ ergs sec^{-1} cm^{-2} $degrees^{-4}$.

$a = 7.61 \times 10^{-15}$

$b = 1.816 \times 10^{-5}$.

$\sigma = 8.19 \times 10^{-11}$ cal. min^{-1} cm^{-2} deg^{-4}.

useful in Solar Radiation.

$J = 4.18 \times 10^7$

Example of use – Classic Case

Temp. of Sun.

Solar Const now taken as

1.932 cals/cm²/min.

6.96×10^{10}

1.496×10^{13}

Intensity emerging from Sun in all dirns

$1.932 \times \left(\frac{1.496 \times 10^{13}}{6.96 \times 10^{10}}\right)^2$

div. by σ 8.19×10^{-11}

& take 4th root.

5740° abs.

Lewis & Adams Phys Rev. Vol 3. p 92. 1914 for value of σ.

σ measured by electrical compensation method.

Weid. Ann. 65 p. 406 1896

Wheatstone Bridge

Expose 4th arm to black body and put 100°C. Pt.

5.32×10^{-5}. later 5.45

Gerlach Ann d Phys. 38 p1. 1912.

Angstroms method – thermopile indicator.

2 receivers

5.80 later 5.9 correcting for radiation

100°C – again.

Valentiner 1910 5.36

agreement of order of σ verifies 4^{th} power law.

Thermometric Methods –

5.89, 6.9.

Differential method

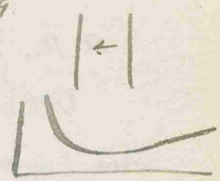
Rate of cooling of plate surfaces bright & black of emissivities E_1 E_2.

find $E_1 - E_2$ and $\frac{E_1}{E_2}$

3 expts.

5.67 Shakespeare P.R.S. 86A 180 1912.

Todd P.R.S. 83A p19. 1909



flow of heat by conduction & radn as plates are moved further apart.

asymptotic curve gives radiation.

5.8.

Lorenty.

black body at 1000°. 2 screens.

T_1 T_2

$$EI = \frac{\sigma}{\pi}\left(\frac{A A_1}{D^2}\right)\left(T_1^4 - T_2^4\right)$$

Solid angle.

Accurate method.

$$\iint d\sigma_1\, \frac{d\sigma_2 \cos^2\theta}{r^2}$$

$$r^2 = D^2 + (x_1 - x_2)^2 + (y_1 - y_2)^2$$

$$\cos\theta = \frac{D^2}{r^2}$$

$$\therefore \iint \text{—} = \iint \frac{d\sigma_1\, d\sigma_2\, D^2}{r^4}$$

$$= \frac{1}{D^2}\iiiint \begin{matrix} dx_1\, dy_1 \\ dx_2\, dy_2 \end{matrix} \left[1 - 2\frac{\quad}{D^2}\right]$$

$$= \frac{A_1 A_2}{D^2}\left[1 - \frac{1}{6}\frac{\quad}{D^2}\right]$$

This simplifies if one small compared with other.

31/10/21

[See Bull. B of Standards Vol 12. 1915–16.
p. 503 & p. 553]

5740° not whole sun – but there are parts where temp is at least that & contrib.
$\sigma(5740)^4$ is the energy. } directly to the beam reaching earth

~~Physting~~ Computing constants

Phil Trans A. 232 p 505. 1903

1. Angstrom 4
2. Langly 3
3. Rosetti 2.5 } Assumptions he worked on

Compare intensity of sunlight & star light
total amt. from Sun at Earth $= 4\times10^6 \times$ total starlight.
sunlight from narrow solid angle.
Star " " whole sphere.

I = intens of sunlight
I' " " star "

angular diam Sun is 32′

Sun $\pi \frac{\pi \times 16}{}(I)$

Energy per unit area is $\int I^2 \cos\theta\; d\omega = \pi I$

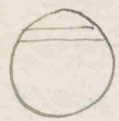
$\pi I' \times 4 \times 10^6 = \pi(\quad)^2 I$

$\frac{I'}{I} = \frac{\quad}{\quad} \therefore I' = \frac{I}{655'}$

Ideal Rotating Planet. 1. orbit ⊙
Sym in plane of ecliptic

2. Assume Temp constant at a given lat.
3. " surface of earth & atmosphere above it have a single effective temp as a radiator
4. Each lat insulated from others ∴ no transfer of heat N or S
5. Albedo of planet is fraction of incident radiation reflected
Assume $\frac{1}{10}$ reflected & $\frac{9}{10}$ absorbed.
6. Assume steady state ∴ radiation = absorption $= \frac{9}{10}$.

Consider zone of lat. λ to $\lambda + \delta\lambda$



If r is radius proj^n in disc ⊥r to sun is
$2r\cos\lambda\, d(r\sin\lambda)$
$= 2r^2\cos^2\lambda\, d\lambda$.

If S – solar const. then energy rec. & absorbed is $\frac{9}{10} S\, 2r^2\cos^2\lambda\, d\lambda$
but actual area is $2\pi r^2\cos\lambda\, d\lambda$
∴ Average emission/unit area =
$\frac{9}{10}\frac{S}{\pi}\cos\lambda$

If Θ_λ is effective temp in lat λ

then $\sigma \Theta_\lambda^4 = \frac{9}{10} \frac{S}{\pi} \cos\lambda$

But $\sigma \cancel{\Theta^4} = S \frac{D^2}{a^2}$

$\therefore$ dw. $\times \sqrt[4]{}$

gives $\frac{\Theta_\lambda}{T} = \left(\frac{9}{10\pi} \frac{a^2}{D^2} \cos\lambda\right)^{\frac{1}{4}}$

$= \frac{1}{20} \cos^2 \lambda$.

Take. $\Theta_\lambda = 300^\circ$ of $T = 600$

$27^\circ C$. is equatorial temp

At poles Θ_{90} $\cos^2\lambda$ wd. $= 0$

wh. is not correct but the atmosphere takes the heat there $\therefore$ of. assumptions

Mean temp $\frac{1}{4\pi} \int_{-\pi}^{\pi} \Theta_\lambda \, 2\pi a^2 \cos\lambda \, d\lambda$

$= \int \Theta_\lambda \cos\lambda \, d\lambda$

$= \Theta_E \int \cos^{3/4} \lambda \, d\lambda = \Theta_3 \frac{1}{2}\sqrt{\pi} \frac{\Gamma(\,)}{\Gamma(\,)}$

$= 0.29$ $\therefore \Theta = 279$

Consider πr^2 $4\pi r^2$

Removing insulation

gives $0.48 \rightarrow \Theta = 287$

not very diff from mean.

What is highest value a black body could reach at dist from sun.

$T \sqrt[4]{\frac{a^2}{D^2}} \frac{1}{146} = 410^\circ$ abs. for black body.

Consider moon as this type with no conduction. Albedo of moon $\frac{1}{8}$.

$410 \sqrt[4]{\frac{9}{8}} = 396$?

D $\leftarrow$ $\frac{\int \cos^2\lambda \, dw}{\int \cos\lambda \, dw}$ is ratio of

zenith radiation reached it over all its surface. $\sqrt[4]{\frac{2}{3}} \times 396 = 368$. or $95^\circ C$.

Upper limit as there is conduction to some extent.

Temp in small highly conducting sphere at dist of sun from earth. T' its temp

$4\pi R^2 \sigma T'^4 = \pi \varpi^2 \sigma T^4 \frac{a^2}{D^2}$

Solar effect.

gives 290°

Emission $\propto$ absorption

$\propto$ intensity of radn.

$\propto \frac{1}{\text{sq. of dist.}}$

$\therefore T \propto \frac{1}{D^2}$

Consider Venus & Mars. – Rotating planets.

	Dist	Eq. Temp	Av. temp
Venus	·72	383	
Earth	1		236
Mars.		3	279
	1.52	242	329
		1	2

If not rotating (Venus not certain)

Albedo $\frac{1}{8}$.

		Rad as black	If highly conducting
Venus	468	438	342
Earth	396	368	290
Mars	321.	298	235
	3	4	5

It seems prob. that Mars is barely if any above freezing pt.

Venus temp. is high hence clouds & difficulty.

→ Mutual repulsion due to radiation pres. is of order of attraction due to gravity.

Radiative Equilibrium of Atmosphere.

Gold's paper.

Prelim. 1. Rate of loss of heat of ∞ plane slab of optical thickness a per unit area when slab is at uniform temp.

Slab is stratified but not homogeneous

$t = \int_0^z k\rho\, dz$

~~k/~~ ρ = dens

$k\rho\, dz\ \ B\, S\, d\omega$ Emission

Radn from element. $B\, dz\, d\omega\, dS$

of this a portion escapes.

$e^{-(a-\tau)\cdot \cos\theta}$

when $\frac{\pi}{2} < \theta < \pi$ put $\pi - \theta = \psi$

$e^{-\tau \sec\psi}$

$B\iint e^{-} \, d\omega\, d\tau + B\iint e^{-}\, d\omega\, d\tau.$

$= 2\pi B\left[\int\int + \int\int\right]$

$=$

$= 4\pi B \int \frac{1 - e^{-a\mu}}{\mu^3}\, d\mu$ put $\sec\theta = \mu$

is total rad^n escaping from both faces.

Put $\int_1^\infty \frac{e^{-a\mu}}{\mu}\, d\mu = \int_a^\infty \frac{d\mu}{\mu} e^{-\mu}$

$= E_i(a)$

notation. $E_{in}(a)$

$\therefore$ above $= 4\pi B\left(\frac{1}{2} - E_{i_3}(a)\right)$

$= 2\pi B\left(1 - e^{a}(1-a) - a^2 E_i(a)\right)$

Total amt emitted is $4\pi B a$.

$\therefore$ diff is amt absorbed.

As $a \to \infty$ $E(a) \to 0$

$\therefore$ absorption $\to 2\pi B$.

If $a \to 0$

$4\pi a B$

i.e. whole energy is again emitted.

Cosine Law of Emission of Radiant Energy.

[but direct law of emission in all directions for radioactive film. e.g. If sun were a sphere of metal plated over with radioactive deposit it would appear as a bright halo whose intensity diminished rapidly toward centre.

E. Rutherford 3.11.21.
]

4.11.21 Book just pub. in U.S. on probability of aqueous condn. of Mars. Mars day = 24 hrs approx.

Thick layer has radn $I(\theta)$ incident on one face.

absorption is $2\pi\int_0^{\pi/2} I(\theta)\sin\theta\cos\theta\,(1-e^{-\alpha\sec\theta})\,d\theta$

$$= 2\pi\int_0^{\infty} I(\mu)\,\frac{1-e^{-\alpha\mu}}{\mu^3}\,d\mu$$

2 ∞ slabs contiguous what is amt absorbed

$dS\,d\tau\,.\cos\theta\,B\,d\tau\,d\omega$ $\uparrow$ τ b absorbing / a emitting

On hitting b reduced to $B\,dS\,d\tau\,d\omega\,e$

" emergence

$\therefore$ Diff = Total Absorptn $= B\int_0^{2\pi} d\omega \int_0^a d\tau \int_0^{\pi} \sin\theta\,(e^{-} - e^{-})\,d\theta$

Put $\sec\theta = \mu$.

$$\frac{d\mu}{\mu}\int_0^a e^{-\mu(a-\tau)} - e^{-\mu(a+b-\tau)}\,d\tau$$

$$= 2\pi B\int_1^{\infty}\frac{d\mu}{\mu^3}\left(1-e^{-\mu a}-e^{-\mu b}+e^{-\mu(a+b)}\right)$$

$$= 2\pi B\int_1^{\infty}\frac{(1-e^{-a\mu})(1-e^{-b\mu})}{\mu^3}\,d\mu.$$

This is symmetrical in a & b.

When a, b both $\to\infty$

$$2\pi B\int_0^{\infty}\frac{d\mu}{\mu^3} = \pi B.$$

b only $\to\infty$ $\quad 2\pi B\int_1^{\infty}$

$$2\pi B\int\frac{(1-e^{-})(1-e^{-})}{\mu^3} = 2\pi B\left[\tfrac{1}{2} - E_{i_3}a - E_{i_3}(b) + E_{i_3}\,a+b\right]$$

Use Eulers const $\gamma = \int$ to evaluate the E_i's.

$$\gamma = L_{n\to\infty} \int_0^n \frac{1-\left(1-\frac{t}{n}\right)^n}{t}\,dt - \int \frac{dt}{t}$$

$E(\ ) = -\gamma - \log x +$ series in x. p.

Diff. $E'_{i_3}(x) = -E_{i_2}(x)$

→ for b small

$$2\pi Bb\left[1 - E_{i_2}(a)\right] + Q\left(b^m \log b\right)$$

some function.

→ for a, b both small

$$2\pi B\Big[1 - e^{-a(1-a)} - e^{-b(1-b)} + e^{-a+b(\ldots)}$$
$$+ a^2 E_1(a) - b^2 E_i(b) + (a+b) \ldots$$

$$=$$

$$+ a^2(\gamma + \log a) + b^2(\gamma + \log b)$$
$$+ (a+b)^2(\gamma + \log a+b$$

$$= \pi B\Big[ab(3-2\gamma) + a^2 \log a + b^2 \log b$$
$$- (a+b)^2 \log a+b)$$

which if a b both small

is $O\{\varepsilon^2 \log \varepsilon\}$.

If $a = b$ $\quad \pi Ba^2\left[(3 - 2\gamma - 4\log 2 - 2\log a)\right]$

Summary of math

Increase coef of absorption – then Emission alters more rapidly than absorption + ∴ the resultant effect is a cooling.

Langley found absorption by 100 m. air was 21% from 100°C.
air 60% rel. Hum.
18°C dew pt.
Find coef. of absorption.

Mauser (German) found layer of air 1 cm thick radiated to another 1°C lower temp. an amt. $= 4\times10^{-5}$ cal./cm²/hr.

Latter is unpardonably inaccurately stated really means layer in a surrounding of medium 1°C lower.

These 2 actually agree.
Langley. k – coef abs.

$$0.79 = e^{-10^4 k} \quad k = 0.236 \times 10^{-4}$$

Rad. intens from both sides of Mauser's layer

$= 4\pi k B \qquad 4\pi kB\ \text{-}\ dB)$

∴ Net amt. radiated $4\pi K \frac{dB}{dt} dt$.

with $dt = 1°C$. $B = \frac{\sigma}{H} T^4$.

273° Abs. $\eta = 16 K \sigma T^3$

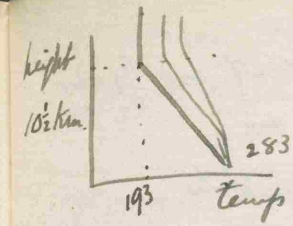
$= 3.8 \times 10^{-3}$

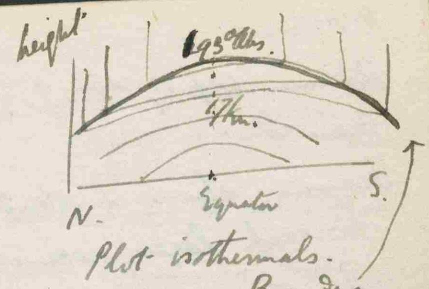
Gold's paper.
R. Soc. Proc. V. 82A p. 43. 1909.
Essay Geophys. Memoirs No. 5. 1913
Vol. 1. p. 65.
Dine's paper Geophys. No. 13 1919.
Jeans Gases Ch. XV (2nd Edn.)

Temp of atmosphere 6°C per km.
? atmosphere tends to be isothermal if no movement. but latter makes it more nearly adiabatic.

10°C per km
30 km $p \propto T^{\frac{\gamma}{\gamma-1}}$ $\rho \propto T^{\frac{1}{\gamma-1}}$

Teisserenc de Bort
found temp become practically stationary above a def. height.





Plot isothermals.
Boundary.
Tropopause

Below boundary
Troposphere.
above boundary
Stratosphere.

~~Coef of~~ Mass motion cannot exist in stratosphere - hence adiabatic state cannot exist there ∴ not to limits of atmosphere.

Each element of air will be
(1) absorbing some solar rad
(2) scattering " "
(3) abs. " earth rad
& " " atmospheric rad. above & below.
(4) it may be gaining & losing by convection, gaining by conduction
5. Radiating

7/11/21.

Assume coef of abs of earth rad by air
Same as " " " " air " " "

Assume atmos in steady state.
∴ each layer receives + loses equal amts.

Earth + air receives + emits same amt.
∴ out current = in current.

net out flow = inward - outward - inward
Solar flow.

∴ net out flow is constant.

this out flux = net radiative flux
+ " convective flux

latter carried by water vapour
or bodily motion of air.

For convective equilib latter is essentially +ve.

∴ net radiative flux ≤ net inward solar flux
function of temp distrib.

Hence complete adiabatic equilib is impos.

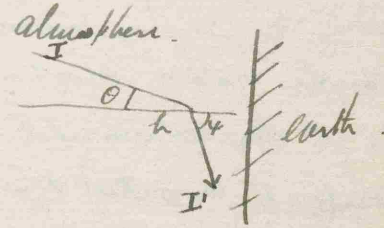
Heat convected up is absorbed by upper layers since it cannot pass boundary

of atmos as convective heat

but for equilib gain by conv. + " " absorption) = loss by emission

former is essentially + for upper layers.
∴ for upper layers Emission > absorption

These 2 statements are equivalent



$I(\theta\, h)$

k : coef of abs.

Put $-k\rho\, dh = dt$

$+\int_h^\infty k\rho\, dh = t$, $\int_0^\infty k\rho\, dh = t_1$

∴ t is optical thickness measured from exterior inwards

∞ means outer limit of atmos

But $+\int_h^\infty g\rho\, dh = p$ = pres

Take g = unity

if p_1 = ground pres
if k indep of h

then $t = kp$. by division above
& $t_1 = kp_1$

T is temp of air near Earth
$B(T_1) = B_1$

Consider radn crossing area horizontal dS in dir. θ inside $d\omega$.
amt. $I \cos\theta \, d\omega \, dS$.
Let it proceed along ds.
It is diminished due to absorption
$k\rho \, ds \, I \cos\theta \, d\omega \, dS$
It is reinforced by emission along its path by $(dS \, ds \cos\theta) k\rho B \, d\omega$
Intensity is now $(I + dI) \cos\theta \, d\omega \, dS$.

$\therefore dI \cos\theta \, d\omega \, dS = -k\rho \, ds \, I \cos\theta \, d\omega \, dS$
$\qquad + k\rho \, dS \, ds \cos\theta \, d\omega$

or $dI = -k\rho(I - B) \, ds$

but $ds = dh \sec\theta$.

$\therefore dI = -k\rho(I - B) \, dh \sec\theta$.

But $dt = -k\rho \, dh$

$\therefore \cos\theta \dfrac{dI}{dt} = I - B$.

Similarly
$-\cos\theta \dfrac{dI'}{dt} = I' - B$.

General eqns for transfer where atmos is stratified in ||l planes.

Solve this eqn keeping θ const.
we have $I e^{-t\sec\theta} = -\int B \sec\theta \, e^{-t\sec\theta} \, dt$ + const.

If earth is black.
at surface $t = t_1$
$B = B_1$
the const is obtained

$$I = e^{+t\sec\theta} \int_T^{t_1} B \sec\theta \, e^{-t\sec\theta} \, dt$$
$$+ B_1 e^{-(t_1 - t)\sec\theta}.$$

Condn at outer boundary is
for all θ $I' = 0$

then $I'(t) = e^{-t\sec\phi} \int_0^t B(t)\sec\phi\, e^{+t\sec\phi}\, dt$.

Absorption in dv is

$k\rho\, dv \iint I\, d\omega = 2\pi k \rho\, dv \Big[\quad \Big]$

$\left[\int_0^{\frac{1}{2}\pi} I(B) \sin\theta\, d\theta + \int I'(B) \sin\phi\, d\phi \right]$

Emission.

$4\pi k\rho\, dv$

Emission > absorption.

$\therefore\ 2B(t) > \int_0^{\frac{1}{2}\pi} I \sin\theta\, d\theta + \int_0^{\frac{1}{2}\pi} I' \sin\phi\, d\phi$

Test Isothermal atmosphere.

then $B = B_1$ everywhere.

Insert for I + I_1 + integrate.

$I = B_1 (1 - e^{-(t_1 - t)\sec\theta})$.

$+ B_1 e^{-(t_1 - t)\sec\theta} = B_1$

i.e. radiation is everywhere black

$I' = B_1 (1 - e^{t\sec\phi})$

$\therefore$ ~~emiss > absorp.~~

radiant equilib holds if

$2B_1 = \int I \sin\theta\, d\theta + \int I' \sin\phi\, d\phi$

or $2 = 1 + \int_1^\infty \frac{1 - e^{-t\mu}}{\mu^2}\, d\mu$.

or $0 = -\int_1^\infty \frac{e^{-t\mu}}{\mu^2}\, d\mu$.

this is always neg.

$\therefore$ ~~rad~~ equilib does not hold

+ emiss is always > absorp.

Test adiabatic equilib.

Ideal $T \propto p^{\frac{\gamma-1}{\gamma}} = p^{\frac{1}{3.5}}$ for $\gamma = 1.4$

Earth's atmos $T \propto p^{\frac{1}{5.67}}$

more nearly.

Take average $T \propto p^{\frac{1}{4}}$

then $T^4 \propto p$

But $p = kt$. for atmos uniformly composed.

then $B \propto t$

or $\dfrac{B}{B_1} = \dfrac{t}{t_1}$.

$$\int_0^{\frac{1}{2}\pi} I \sin\theta \, d\theta = \frac{B_1}{t_1} \int_1^\infty \frac{d\mu}{\mu} \int_t^{t_1} t e^{-t\mu} \, dt + B_1 \int_1^\infty e^{-t_1 - t\mu} \frac{d\mu}{\mu^2}$$

$$= \frac{B_1}{t_1} \int_1^\infty \frac{d\mu}{\mu^2} \left\{ t + \frac{1 - e^{\mu(t_1 - t)}}{\mu} \right.$$

Sim. $\displaystyle\int I' \sin\psi \, d\psi$

$$= \left(t - \frac{1 - e^{-\mu t}}{\mu} \right.$$

$$2B = 2\frac{B_1}{t_1}\tau > \frac{2B_1 t}{\tau_1} + \frac{B_1}{t_1} \int \frac{}{\mu^3}$$

$$\therefore 0 > \int_1^\infty \frac{e^{-\mu t} - e^{-\mu(t_1 - t)}}{\mu^3} \, d\mu.$$

$\therefore$ this is nec. for conv. equilib.

Only true if

$t > t_1 - t$

i.e. $2t > t_1$

$t > \frac{1}{2} t_1$

But $t \propto p$.

$\therefore p > \frac{1}{2} p_1$

$\therefore$ convective equilib. only poss in that part of atmos in wh. press is less than half ground press.

About $p = \frac{1}{2} p_1$ this is reversed & atmos is warming up.

$\therefore$ 6 or 7 km is limit.

Consider net up flux $F(\tau)$

$$F(\tau) = 2\pi \left[\int I \sin\theta \cos\theta \, d\theta - \int I' \sin\psi \cos\psi \, d\psi \right]$$

Insert values for I I'

Subst $B = \dfrac{B_1 t}{\tau_1}$

Consider $F(t')-F(t'')$ when $t'>t''$

$F(t')$ = net flux entering lower boundary of slab. t'' t'

$F(t'')$ .. .. leaving upper boundary

∴ $F(t')-F(t'')$ = excess of abs. over emiss for whole slab.

Result $F(t')-F(t'') = \int_1^\infty \left(e^{-\mu(t'-t'')} - e\right)(--)\frac{d\mu}{\mu^4}$

for conv. equilib this is neg. in up. layers

∴ $t'+t'' < t_1$

i.e. $p'+t'' < p_1$

∴ all layers for wh. sum of press at top & bottom < ground press will gain heat.

Eng. Trans of Planck

Theory of Heat Radn Planck
trans by Morton Masius
Philadelphia
Blackistone 1914.

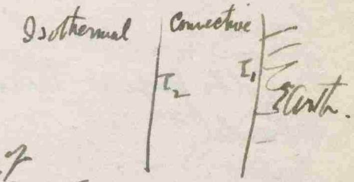
11/11/21.

Convective equilib. $B \propto p \propto \tau$.

Proved isothermal atmos. ~~loses~~ heat in upper layer
adiabatic " gains .. .. ..

Try a mixture of both

2 shells.



Try as in diagram

Calc. gains & losses of heat in upper layer.

Gain in layer $0-\tau_2$ $F(\,)$ = flux

$$F(\tau_2)-F(0) = 2\pi\left[\int_0^{\frac{1}{2}\pi} I\sin\theta\cos\theta\,d\theta - \int_0^{\frac{1}{2}\pi} I'_{\tau_2}\sin\theta\cos\theta\,d\theta - \int_0 I'(0)\sin\theta\cos\theta\,d\theta\right]$$

$[-\ I'(0)=0.]$

$$= \int_1^\infty \frac{d\mu}{\mu^2} e^{\tau_2\mu}\int_{\tau_2}^{\tau_1} B(\tau)e^{-\tau\mu}\,d\tau$$

$$+ B_1\int_1^\infty e^{-(\tau_1-\tau_2)\mu}\frac{d\mu}{\mu^3} - \int_1^\infty \frac{d\mu}{\mu^2}e^{-\tau_2\mu}\int_0^{\tau_2} B\,d\tau$$

$$-\int_1^\infty \frac{d\mu}{\mu^2}\int_0^{\tau_1} B(\tau)e^{-\tau\mu}\,d\tau - B_1\int_1^\infty \frac{e^{-\tau_1\mu}}{\mu^3}\,d\mu.$$

Physical meaning of each.

(1) ~~part 4~~ That prop^n of rad^n emitted by lower layer + abs. in upper

part of 4 + of 3. total emission of rad^n

2 + 3 absorption of earth rad. in upper layer.

In Golds paper these are dealt with separately.

In lower layer $\frac{B}{B_1} = \frac{t}{\tau_1}$ $(t_2 - t)$

upper $\frac{B}{B_1} = \text{const} = \frac{t_2}{\tau_1}$ $(0 - t_2)$

$$F(\tau_2) = \frac{2\pi B_1}{\tau_1}\left[\int_1^\infty \frac{d\mu}{\mu^2} e^{\tau\mu} \int t\, e^{t\mu}\, dt + \tau_1 \int_1^\infty e^{-(t_1 - t_2)\mu} \frac{d\mu}{\mu^3} - \int_1^\infty \frac{d\mu}{\mu^3} e^{-t_2\mu} \int_0^{t_2} t_2 e^{t\mu}\, dt\right]$$

$$= \frac{2\pi B_1}{t_1}\left[\int_1^\infty \frac{1 - e^{-(t_1 - t_2)\mu}}{\mu^4} d\mu + t_2 \int_1^\infty \frac{e^{-t_2\mu}}{\mu^3} d\mu\right]$$

Similarly

$$F(0) = \frac{2\pi B_1}{\tau_1}\left[\tau_2 \int_1^\infty \frac{d\mu}{\mu^3} + \int_1^\infty \frac{e^{-t_2\mu} - e^{-t_1\mu}}{\mu^4} d\mu\right]$$

Subtract for excess.
+ factorize.

$$F(\tau_2) - F(0) = \frac{2\pi B_1}{\tau_1}\left[\int_1^\infty \frac{1 - e^{-t_2\mu}}{\mu^3}\left\{\frac{1 - e^{-(t_1 - t_2)\mu}}{\mu} - \tau_2\right\} d\mu\right]$$

Factor gives thickness of layer.
Other

$t_2 = 0$ + ve   (excess of absorption.

$\tau_2 = \tau_1$ − ve.   (excess of emission

find the value for stationary cond^n of upper layer.

Mul. out.

$$\frac{2\pi B_1}{\tau_1}\left[E_{i4}(\tau_1) - E_{i4}(\tau_2) - E_{i4}(\tau_1 - \tau_2) + \frac{1}{3} - \frac{1}{2}t_2 + t_2 E_{i3}(t_2)\right]$$

Order of mag of τ_1 - optical thickness
$\therefore e^{-\tau_1}$ is net transmission of earth radiation.

Value diff to obtain
Gold tried 2 values.

(1) $e^{-\tau_1} = \frac{1}{100}$ like Langleys for damp air

then $\tau_1 = 4.605$

(2) $e^{-\tau_1} = \frac{1}{2}$.

$\tau_1 = 0.692$

$\boxed{\text{find } \tau_2 \text{ approx}}$ $\tau_2 < 1$ since $1-e^{-(\tau_1-\tau_2)\mu}$

Also $\tau_2 < \tau_1$ $\quad \mu \quad -\tau_2$

τ_1 large neglecting terms above. $\quad \therefore < 1$ if not neg term

$$\tau_2 = \frac{\frac{1}{3} - E i_4 \tau_2}{\frac{1}{2} - E i_3 \tau_2}$$

$\therefore \tau_2 < 2/3$.

τ_1 small $\quad \frac{1-e^{-(\tau_2-\tau_1)\mu}}{\mu} - \tau_2$

$\therefore \tau_2 = \frac{1}{2}\tau_1$ $\quad \tau_1 - \tau_2 - \tau_2$

$2\tau_2 = \tau_1$

(1) Actual $\tau_2 = 0.60$.

approx was .67

(2) $\tau_2 = 0.30$

approx was .5.

$\therefore \frac{p_2}{p_1} = \frac{\tau_2}{t_1} = \frac{.6}{4.6} = 0.13$

or $\quad$ " $\quad$ " $= \frac{.3}{.692} = .433$

real observed height $10\frac{1}{2}$ Km.

pres. is $p_2 = 242$ millebars

$p_1 = 1014$ "

$\therefore \frac{p_2}{p_1} = .0.224$

lying between limits obtained.

If $e^{-\tau_1} = \frac{1}{10}$

then $t_2 = 0.56$.

$+ \frac{p_2}{p_1} = 0.224$

Over Equator atmos is damper & $\therefore$ absorbs more $\therefore \frac{p_2}{p_1}$ is smaller.

hence 17 Km. up to isothermal layer

$\frac{p_2}{p_1} = .1$ approx there.

[Take $K = \frac{\alpha}{q-p}$.

$d\tau = -K\rho\, dh = \tau K\, dp = \frac{\alpha}{q-p}\, dp$

$\therefore \tau = -\alpha \log\left(1 - \frac{p}{q}\right)$

$e^{-\tau/\alpha} = 1 - \frac{p}{q}$ $\quad B \alpha p \alpha 1 - e^{\tau/\alpha}$

$\& \frac{B}{B_1} = \frac{1 - e^{-\tau/\alpha}}{1 - e^{-\tau_1/\alpha}}$

$\mu = \sec\theta$.

$\cos\theta \frac{dI}{dt} = I - B.$

$-\cos\phi \frac{dI'}{dt} = I' - B$

Fundamental Eqns.

mult. by $d\omega$ + integrate over $\frac{1}{2}$ sphere

$2\pi \int_0^{\frac{1}{2}\pi} \frac{dI}{d\tau} \cos\theta \sin\theta \, d\theta.$

$= 2\pi \int (I - B) \sin\theta \, d\theta.$

$= 2\pi \left[\int I \sin\theta \, d\theta - \cancel{B} \right]$

$-2\pi \int \frac{dI'}{d\tau} \cos\phi \sin\phi \, d\phi$

$= 2\pi \left[\int (I' \sin\phi) d\phi - B \right]$

Write these $2\pi \frac{d}{dt} \left[\int_0^{\frac{1}{2}\pi} I \sin\theta\cos\theta \, d\theta - I' \sin\phi\cos\phi \, d\phi \right]$

$= 2\pi \left[\int I \sin\theta \, d\theta - \int I' \sin\phi \, d\phi - 2B. \right.$

$\therefore \frac{d}{dt} F(\tau) = 2\pi \int_0^{\pi} I \sin\theta \, d\theta - 4\pi B$

Rate of inc. of net flux = absorption − emission

τ measured down
F " up

14/11/21

Tables of E_i functions
Glaisher Phil Trans Vol 160 − 1870.

$\int_x^{\infty} \frac{e^{-t}}{t} dt.$

Gold gives Ei_2 Ei_3 also in his paper.

Good short paper in Jahnke und Emde
Funktionentafeln Leipzig 1906.
(graphs &c. of everything one needs.)

$\cos\theta \frac{dI}{dt} = I - B.$ t measured in I outward radn.

$-\cos\phi \frac{dI}{dt} = +I' - B.$

giving results at foot of last page

Approximate for 2nd eqn.

Put $\int_0^{\frac{1}{2}\pi} I \sin\theta \, d\theta = \bar{I} \int_0^{\frac{\pi}{2}} \sin\theta \, d\theta = \bar{I}$

also $\int I \sin\theta\cos\theta \, d\theta = \bar{\bar{I}} \int \sin\theta\cos\theta \, d\theta$
$= \frac{1}{2} \bar{\bar{I}}$

Substitute these mean values

$\frac{d}{dt}\left(\frac{1}{2}\bar{\bar{I}}\right) = \bar{I} - B$

Assume $\bar{I} = \bar{\bar{I}}$ approx.

then $\frac{1}{2}\frac{d\bar{I}}{d\tau} = \bar{\bar{I}} - B$.

This is what would be got if radiation only moved in ‖l cylinders along axis of T but coeff of absorption now twice what it was before.

∴ take new defn of optical mass

$2\,d\tau = 2k\rho\,dl \quad = dt$.

$\therefore k = 2k$.

Call this linear or tubular flow of radiation.

In a layer find Mean coef of abs. for radn coming in all dirns.

Total abs. per unit area $= 2\pi\int I(\theta)\,d\omega\cos\theta \times k\,dl\sec\theta$

Incident radn $= 2\pi\int I(\theta)\,d\omega\cos\theta$

∴ Mean coef of abs = ratio of above

$$= \frac{\int_0^{\pi/2} I(\theta)\sin\theta\,d\theta}{\int I(\theta)\sin\theta\cos\theta\,d\theta} = \frac{\bar{I}}{\frac{1}{2}\bar{\bar{I}}} = 2$$

if $\bar{I} = \bar{\bar{I}}$.

cos θ replaced by ½ i.e. making angle of 60° to axis.

Consider $\frac{1}{2}\frac{dI}{dt} = I - B$ — drop ½ & put dt for $d\tau$.

$-\frac{dI'}{dt} = I' - B$.

Condn. $t = 0 \quad I' = 0$

$t = t_1 \quad B = B_1$ at boundary of earth.

then $I = e^{t}\int B e^{-t}\,dt + B_1 e^{-(t_1 - t)}$

$I' = e^{-t}\int_0^t B e^{t}\,dt$

Net flow $F(t) = 2\pi\left[\int I\sin\theta\cos\theta\,d\theta - \int I'\cos\theta\,d\theta \text{ etc.}\right]$

$= \pi\left[I(t) - I'(t)\right]$

Net excess of abs. over radn in layer 0, t.

$F(t) - F(0) = \pi\{I(t) - I(t) - I(0)$

For isothermal layer abs = emis.

∴ equate above to zero.

For radiant equilib. $t_2 = 1 - e^{-(t_1 - t_2)}$

Excess abs. in an element $d\tau$ near lower boundary

$$2\pi k\rho\, d\tau \left[\int I \sin\theta\, d\theta + \int I' \sin\theta\, d\theta - 2B\right]$$

$$= 2\pi k\rho\, d\tau \left[I + I' - 2B\right]$$

$$I = e^{t}\int_t^{t_1} B e^{-t}\, dt + B_1 e^{-(t_1 - t)}$$

$$I' = e^{-t}\int_0^t B e^{t}\, dt$$

$B \propto t$ in lower half.
B const " upper " .

$$2\pi k\rho\, d\tau\, \frac{B_1}{t_1}\left[1 - e^{-(t_1 - t_2)} - t_2 e^{-t_2}\right]$$

$$2\pi k\rho\, d\tau\, \frac{B_1}{t_1}\, t_2(1 - e^{-t_2})$$ essentially positive

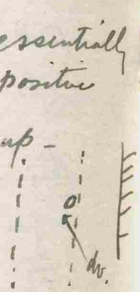
∴ elements are warming up.

At upper boundary.

$$2\pi k\rho\, d\tau\, \frac{B_1}{t_1}\left[-t_2 + e^{-t_2} - e^{-t_1}\right]$$

$$= 2\pi k\rho\, d\tau\, \frac{B_1 t_2}{t_1}\left[-1 + e^{-t_2}\right]$$

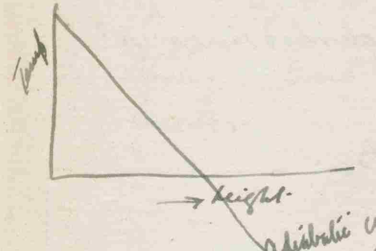
∴ essentially negative
∴ elements are losing heat.

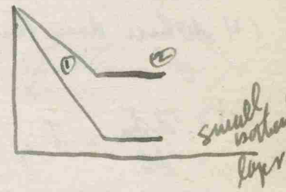


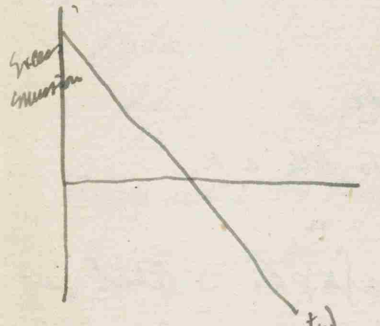
Total result is a balance.

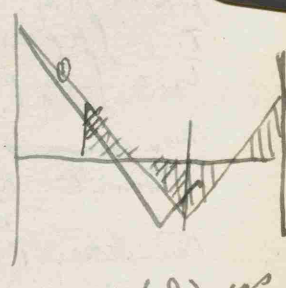
Also in upper part of convective region heat is being gained. wh. is contrary to hypothesis of convective equilibrium.

Explain by curve of temp grad.









Ratio $\frac{\text{excess emiss}}{\text{total emiss}} = \frac{1}{2}(1 - e^{-t_2})$
$=$ from .32 to .22

acc. to value of $t_2 = 1$ or $= .6$

i.e. 25%.

Make shaded areas equal.
Then on whole no excess –
though excess emiss at top & excess abs. at bottom.

18/11/21

Absorption lines

Apply Kirchoffs law vapour absorbs what it emits.

(1) why dark lines where a cloud of gas having a temp grad. is in front of the black rad^n source?

(2) Where does the absorbed energy go?

(1) $\frac{dI_\lambda}{dt} = I_\lambda - B_\lambda$

Obtained taking k indept of λ.

If $k \propto f(\lambda)$

$\tau = \int k \rho \, dx$

$\tau_\lambda = \int k_\lambda \rho \, dx$

More convenient to take a k & write $\frac{k_\lambda}{k} = n_\lambda$

then define $\tau = \int k \rho \, dx$ optical mass.

Strict mean value $\overline{k} = \frac{1}{B}\int k_\lambda B_\lambda \, d\lambda$

or $B = \int_0^\infty \frac{k_\lambda}{\overline{k}} B_\lambda \, d\lambda$

if k is $\overline{k}$

mean value of n_λ is unity.

$B_\lambda (t)$

B_λ is a diff function of λ for diff temps

$\therefore \overline{k}$ varies with temp

& also n_λ

$\therefore$ take a value of k near $\overline{k}$

& $\overline{n}_\lambda$ is approx unity:

In stronger parts of spectrum $n > 1$

" weaker " " " $n < 1$

$\frac{1}{2}\frac{dI_\lambda}{k\rho \, dx} = \frac{k_\lambda}{k}$

or $\frac{dI_\lambda}{dt} = n_\lambda (I_\lambda - B_\lambda)$ where $t = 2\tau$

If temp distrib be known this can be solved

$\frac{d}{dt}(I_\lambda e^{-n_\lambda t}) = -n_\lambda B_\lambda e^{-n_\lambda t}$

$I_\lambda e^{-nt} = -\int^t B_\lambda e^{-n_\lambda t} n_\lambda \, dt$

Suppose medium unbounded in dir. of temp increasing.

then I_λ is derived entirely from matter deeper than pt considered

$$I_\lambda e^{-n_\lambda t} = +\int_t^\infty B_\lambda e^{-n_\lambda t} n_\lambda dt.$$

$\frac{B_\lambda}{I_\lambda} \to 1$ is condn for black radn at ∞

$$I_\lambda = e^{n_\lambda t}\int_t^\infty$$

$$I_\lambda(t_1) = \int_{t_1}^\infty B_\lambda e^{-n_\lambda(t-t_1)} n_\lambda dt.$$

If B_λ const.

$I_\lambda = B_\lambda \times 1$ which is as above.

Suppose temp increases.

Emergent radn

$$I_\lambda(0) = \int_t^\infty B_\lambda(t) e^{-n_\lambda t} n_\lambda dt.$$

2 neighbouring parts of spectrum. having $n_1 > n_2$

$$I_1 = \int_0^\infty B_\lambda(t) e^{-n_1 t} n_1 dt = \int_0^\infty B_\lambda\left(\frac{t}{n_1}\right) e^{-t} dt$$

$$I_2 = \int_0^\infty B_\lambda\left(\frac{t}{n_2}\right) e^{-t} dt$$

new variable. t.

then $B_\lambda(T_1) > B_\lambda(T_2)$ if $T_1 > T_2$ always.

Make up thermodynamic argument as proof

$n_1 > n_2$

$\frac{t}{n_1} < \frac{t}{n_2}$

$\therefore B_\lambda(\ ) < B_\lambda(\ )$

$\therefore I_1 < I_2$

$\therefore$ that part of spectrum is less bright $\therefore$ if n_1 be large enough it appears as a dark line.

Intensity inside darkest dark line is never less than that due to temp of boundary.

i.e. in opaque places one does not see far in
in bright " " sees far in towards centre.

of large bright lines ?

Occasionally towards red end a line shows up bright wh. otherwise wd. have been dark — prob. due to scattering in the star atmosphere.

Gouy Mesures Spectrophotometrique
Annales de Physique 13 p 180
1921

Consider gas flames coloured by a salt
very few metallic mols compared to no of gas mols.

If δ = partial dens. of metallic vapour
l = thickness of flame

total no of radiating metallic ptcls
$\propto l\delta$

If δ small the optic props depend on the prod. $l\delta$ & not on each separately

Consider some particular spectrum flame with bright line due to const temp
Put another flame behind

Let r = ratio $\dfrac{\text{intens 2 flames}}{\text{intens 1 "}}$

from very faint flames
i.e. $l\delta$ small — $r = 2$ almost

as $l\delta$ inc. r diminishes towards unity as thickness increases & radn becomes black.
He found lowest r obtainable was 1.3.

If you take line as a interval (λ_1, λ_2) it must tend to unity:
but if you follow 1 line wh. is really a peak

it gets broader.
& hence Gouy's results —

Suppose p is absorptive power
$1-p$ = transmission
If p_1 corresponds to $l_1 \delta_1$

then $1-p = (1-p_1)^{l\delta/l_1\delta_1} = (1-p_1)^{q/q_1}$

Intens in line $\propto \int p\, d\lambda$ $\quad q = l\delta$

If q is doubled
$p = 1-(1-p_1)^{q/q_1}$

Double q ⇒ $2p_1 - p_1^2$.

Intensity $\propto 2\int p\,dx - \int p^2\,dx$

$r = 2 - \dfrac{\int p^2\,dx}{\int p\,dx} = 2$ app. for thin flame

$1 - \frac{1}{2} r = \dfrac{\frac{1}{2}\int p^2\,dx}{\int p\,dx} = h$

Adopt as standard flame
q_1 ~~such~~ when $r = 1.9$.
Consider Δ

$1 - \frac{r}{2} = h$
$h = \cdot 05$
$p_{max} = 0.15$ $= \frac{1}{7}$ of black body emission for that temp.

$\dfrac{i}{i_1} = \dfrac{p}{p_1}$. & $1-p = (1-p_1)^{q/q_1}$
$= 1 + \dfrac{q}{q_1}\log(1-p_1)$
when q/q_1 small.

∴ $\dfrac{p}{p_1} = \dfrac{q}{q_1}\,\dfrac{\log(1-p_1)}{-p_1}$

Consider rectangle

$h = \frac{1}{2} p_1 = 1 - \frac{r}{2}$
$p_1 = 0.10$ then $\dfrac{p}{p_1} = 1.05\,\dfrac{q}{q_1}$

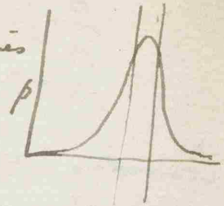
Standard flame $q = q_1$
$r = 1.9$
$q = 2q_1$ $r = 1.82$ $i = 1$.
$q = 4q_1$ $r = 1.65$ $i = 5.83\, i_1$
$q = 16q_1$ $r = 1.43$ $i = 35 \times i_1$

Gouy found $q_1 = 1.22 \times 10^{-11}$

Why is there a sharp edge to 3 un & absorption dark line spect just inside & bright line just outside.

Gouy tries with flames.

Rel. intens p of Sodium lines



$i = 190 \quad \frac{q}{q_1} = 8000$

$q_1 = 10^{-7}$.

∴ thickness 2 cm & $5\ 10^{-8}$

outside p & q.

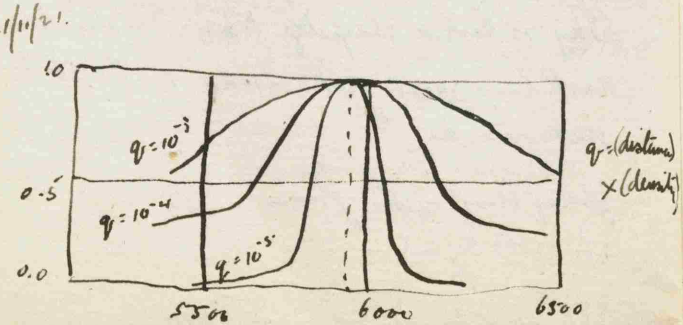
D lines 5890

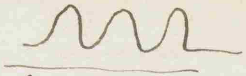
λ angstrom	p
5190	$5\ 10^{-5}$
5490	$1\ 10^{-4}$
5790	$7.5\ 10^{-4}$
5860	$4\ 10^{-3}$
5920	$6\ 10^{-3}$
5990	$1\ 10^{-3}$
6290	$1\ 10^{-4}$
6590	$5\ 10^{-5}$

Inside flame

p = unity almost.

21/11/21.



Spectrum is 

it is continuous when the minima have a definite value.

q	10^{-7}	10^{-6}	10^{-5}	3.10^{-5}	$5\cdot10^{-5}$
p	.007	.07	.5	.88	.94

If lines are 100 angstroms apart the minimum is 50 angstroms from base line.

Apply to Solar Problems.

Old view. bright glowing cloud outside sun a condensation crust & gaseous interior.

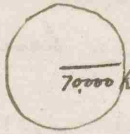
Modern theory – Sun entirely gaseous temp 6000 °C (b.p. Carbon is 3000)

Possibly compounds form on outer fringes of sunspots

both dark Fraunhofer spectrum & bright line spectrum of chromosphere can be accounted for by entire gaseous theory

The solid sun theory has to account for sharp edge. The gaseous theory has to face the problem why is it not sharper?

to 100 000



1000' km from limb there is matter in rare atmosphere or 16' arc from earth. radius Sun

In total eclipse bright boundary crescent.

no slit reqd as so thin

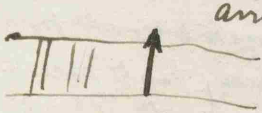
Lines black on bright ground all curved ((((((((

Beyond this on chromosphere the lines are continued top & bottom on the same curve but are bright against dark.

From geometry the height of the diff. elements above disc is obtained Ca Mg &c extend

3000 km most only 5000

Some

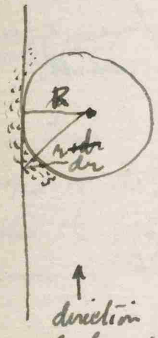


arrow point bright ag. dark over the ordinary Fraunhofer line

Arrow is the chromospheric bright cap & its projection slightly downwards is due to blanket of absorbing gas –

Prominences – difficulty due to gt. velocity & shifting of lines –

Corona – still further out. during eclipse.



Consider ray passing thro zone of gas length of path in layer dr

is $\frac{dr \times r}{\sqrt{r^2-R^2}}$

$\frac{\delta r\, dr}{\sqrt{r^2-R^2}}$ is contribution to q.
δ – density.

$\frac{\delta r\, dr}{\sqrt{(r+R)(r-R)}} = \frac{r \delta\, dr}{\sqrt{2R(r-R)}}$ approx

this at each end ∴ mult by 2.

$q = \int_R^\infty \frac{2 r\, dr}{\sqrt{2R(r-R)}} \cdot \delta$

Assume δ determined by gravitational equilib as usual

$\frac{dp}{dr} = -g\rho$ & $\frac{p}{\rho} = \frac{R_1 T}{m}$

R_1 = gas const.

$\therefore \frac{R_1 T}{m} \frac{1}{\rho} \frac{d\rho}{dr} = -g$

$\rho = C e^{-\frac{mg}{R_1 T} r}$.

$g = 2.75 \times 10^4$ (27 times g on earth)

put $a = \frac{mg}{R_1 T}$

Hence

$$q = C \int_R^\infty \frac{2r e^{-ar} dr}{\sqrt{2R(r-R)}}$$

Put $x^2 = (r-R)$

$$q = C \int_0^\infty \sqrt{\frac{2}{R}} \frac{(x^2+R) e^{-ax^2-aR}}{x} \cdot 2x\,dx$$

$$= C\sqrt{\frac{8}{R}} \left[\int_0^\infty x^2 e^{-ax^2} dx + R \int_0^\infty e^{-ax^2} dx \right] e^{-aR}$$

$$= C\sqrt{\frac{8}{R}} e^{-aR} \left[\frac{1}{4}\sqrt{\pi}\, a^{-\frac{3}{2}} + \frac{1}{2} R \sqrt{\frac{\pi}{a}} \right]$$

Show 1st term neg. comp^d to 2^nd.

for if not sun wd. have no atmosphere

Shows R large comp^d to $\frac{1}{a}$.

ie R " " " $\frac{R_1 T}{mg}$

or $\frac{R_1 T}{mgR}$ small comp^l to Unity.

but $g = \gamma \frac{M}{R^2}$ M = mass of Sun.

$\therefore mgR = \gamma Mm/R$.

= potl. energy in suns grav. field

but $R_1 T = \frac{2}{3}$ K.E of mol. due to its temp

$\therefore$ term is small if $\frac{K.E.}{P.E.}$ is small

& this is the condn for having an atmosphere

else the gas mols wd all escape from suns influence.

$\therefore$ proved.

$$q = C\sqrt{\frac{8}{R}} e^{-aR} \frac{1}{2} R \sqrt{\frac{\pi}{a}}$$

$$= C\sqrt{\frac{2\pi R}{a}} e^{-aR}.$$

Let q_1 be q for a ray which goes thro boundary of chromosphere.

q_2 " " " extreme limit of continuous bright sun.

q_1 is value of q for a barely visible bright line

q_2 is value for a just forming continuous spectrum.

We found $q/2 = 10^{-5}$

Gouy gives $q_1 = 10^{-12}$ is order of mag of faintest line affecting a photo plate.

$$q_1 = c\sqrt{\frac{2\pi R_1}{a}}\; e^{aR_1}$$

$$q_2 = c\sqrt{\frac{2\pi R_2}{a}}\; e^{-aR_2}$$

R_1 in R_2 $\perp^r$ dist onto ray.

$$\frac{q_1}{q_2} = e^{-a(R_1 - R_2)}$$

or $R_1 - R_2 = \frac{1}{a} \log_e \frac{q_2}{q_1}$

$= \frac{1}{a} \log_e 10^7$

$R_1 - R_2$ is height of chromosphere

a is $\frac{mg}{R_1 T}$ find m

mol-weight 2	$R_1 - R_2 = 967$ km.			
" " 20	97		421	1544
50	39		42	154
100	19		17	62
200	10		8.	21
			4	15

this depends on $\frac{q_2}{q_1}$ 10^7

if $\frac{q_2}{q_1} = 10^3$

& 10^{11}

so that except for H $(m=2)$ wrong order

in order to get $R_2 - R_1 = 1000$ km for heavy mols Gouy shows $\frac{q_2}{q_1} = 10^{100}$ wd. be necessary –

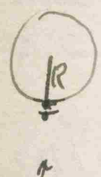
∴ gravitational theory gives too sudden a condensation.

Flash Spectrum of limb

found He.

Looking at centre of Sun thro minimum thickness of chromosphere line only comes out in a flash spectrum.

q then $= \int_R^\infty \delta\, dr \therefore \int_R^\infty C e^{-ar} dr$



$$\frac{q''}{q'} = \frac{1}{\sqrt{2\pi R a}}$$

$a = \frac{mg}{R_1 T}$ gives layer far too thin

So take $a = 1.6 \times 10^{-7}$

& get $\frac{q''}{q'} = \frac{1}{250}$

∴ $\frac{1}{250}$ no. of absorbing ptcls in layer that are radiating from within.

$g = c\sqrt{\frac{2\pi R}{a}}\, e^{-aR}$ = $\frac{\text{length}}{\text{dens.}}$

Calculate length. 75000 km.

$g = 10^{-5}$

∴ average density is $\frac{10^{-5}}{7.5\ 10^{9}}$

$= 10^{-15}$

∴ Solar density is 10^{-11} atmospheres

Very low pressure - hence less matter outside than expected.

2°

Possible error due to law of force not being entirely gravitational.

See Eddington M.N., R.A.S. 80 p 723 June 1920.

Assume whole radⁿ is absorbed

∴ all radⁿ is leaving normally & find upper limit of pressure.

6.2×10^{10} ergs/sec. - Solar const

1.93 cal cm^{2} sec^{-1}

∴ Pres $= \frac{6.2\ 10^{10}}{3\ 10^{10}}$ = 2 dynes/cm^{2}.

Eddington calls it an upward wind of 2 dynes supporting upper layers.

Actually want radⁿ incident on outer layer - $\left(\frac{12000}{6000}\right)^{4} = 2^{4} = 16$

∴ 30 dynes cm^{-2}

But 1 gm solar surface weighs 2.67×10^{4} dynes = "g" on face of sun.

∴ 30 dynes = wt. of about 1 mg.

This is utmost mass that can be supported by radⁿ press. per sq cm of suns surface.

∴ Hydrogen ptcles could stick out 10000 km $= 10^{7}$ cm

Av. density $= \frac{10^{-3}}{10^{7}} = 10^{-12}$ gms cm^{-3}

(H. prominence supported by radⁿ press)

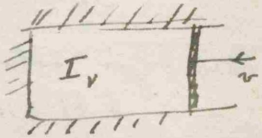
Other types of argument bring results of same order.

Eddington says gtr masses of matter appear to be involved & ∴ radⁿ press does not seem to explain the supporting of these prominences - Elect. forces -?

Radn Theory contd. Planck's law must be made available to get further progress towards colours temps &c.

Wien's Law.

$$B_\lambda = \frac{1}{\lambda^5}\,\phi(\lambda T)$$



Piston - 3 walls diffusely ~~reflecting~~ reflecting

Piston reflective regularly.

Compress the radiation.

Stefans Law gives total result

Wiens " sorts out effect on each λ.

I_ν is radiation intensity & ν any gradually changing frequency.

Compress

Work is done on radn & will appear in Radn density. Energy is increased & frequencies are transformed on reflection from moving pistons

For each train incident

(1) Freq. altered by reflection

(2) Angle of reflection altered by the motion

(3) Energy is altered.

(1) means energy betw ν & $\nu + d\nu$ now in interval $\nu', \nu' + d\nu'$

(3) Energy altered by work done on the radn. by moving piston.

If energy isotropic before not necessarily isotropic after compression if regular reflection geometrically. Radn of freq. ν wd suffer different alterations according to angle of incidence on mirror piston & this is accumulative. Hence make walls diffusely ~~reflective~~ reflective & get homogeneous effect in long run.

Assumed that $\frac{v}{c}$ is small.

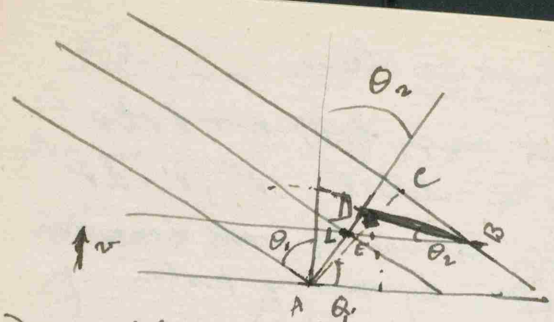
Vague assumptions not very satisfactory

Hence radn remains isotropic & a definite distribution remains.

assume Huygens wave front representation.

see next page.

2 posns. of mirror for v



Draw circle of
rad. CD & draw tan DB & draw ⊥r AC
Mirror has moved forward dist $v\,\delta t$
& $AD = c\,\delta t$

$CB = c\,\delta t$ also

$DCBA$ being cyclic.

$\therefore D\hat{A}C = D\hat{B}C$

$\therefore \Delta DEA = \Delta CEB$.

$\therefore AE = EB = l$

Project ⊥r to mirror ~~AB, EB~~

$\therefore l \sin\theta_1 - l \sin\theta_2 = v\,\delta t$

Now $D\hat{A}E = \frac{\pi}{2} - \theta_1 - \theta_2$

$\therefore l = DA \sec\left(\frac{\pi}{2} - \theta_1 - \theta_2\right)$

$= c\,\delta t \operatorname{cosec}(\theta_1 + \theta_2)$

$\therefore$ dividing $\sin\theta_1 - \sin\theta_2 = \dfrac{v\,\delta t}{c\,\delta t \operatorname{cosec}(\theta_1+\theta_2)}$

or $\dfrac{v}{c} = \dfrac{\sin\theta_1 - \sin\theta_2}{\sin(\theta_1+\theta_2)}$, or readjusting,

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{1 + \frac{v}{c}\cos\theta_1}{1 - \frac{v}{c}\cos\theta_2}$$

Length LD of reflected train has been produced or formed whilst an amt. of the incident train has been annihilated equal to the lag of L behind B.

Think this out.

This lag $LB \sin\theta_1$.

If new freq. is λ_2

$$\frac{LD}{\lambda_2} = \frac{LB \sin\theta_1}{\lambda_1}$$

but $\dfrac{LD}{LB} = \sin\theta_2$

$$\therefore \frac{\sin\theta_1}{\lambda_1} = \frac{\sin\theta_2}{\lambda_2}$$

or $\dfrac{\nu_2}{\nu_1} = \dfrac{\sin\theta_1}{\sin\theta_2} = \dfrac{1 + \frac{v}{c}\cos\theta_1}{1 - \frac{v}{c}\cos\theta_2}$

Plancks method is to count now entering & no emerging
Formula not so symmetrical
Quantum remains same quantum

after reflection

for $h\nu_2 \sin\theta_2 = h\nu_1 \sin\theta_1$

this is momentum

If E_1 is energy incident

& E_2 comes off

then $E_1 \sin\theta_1 = E_2 \sin\theta_2$

by momentum theory

$\therefore \frac{E_1}{E_2} = \frac{\nu_1}{\nu_2}$

This explains why in $\frac{v}{c}$ very small

Planck gets factor $1+\frac{2v}{c}\cos\theta$.

Richardson derives this from relativity considerations.

If $\frac{v}{c}$ small.

$$\frac{\nu_2}{\nu_1} = \frac{1+\frac{v}{c}\sin\theta}{1-\quad} = 1+2\frac{v}{c}\cos\theta.$$

$\nu_2 = \nu_1(\quad)$

$\therefore d\nu_2 = d\nu_1(\quad \frac{2v}{c})$

$E_2\,\delta t = E_1\,\delta t\left(1+\frac{2v}{c}\cos\theta\right)$

$E_1\,\delta t \vdots I_\nu \cos\theta\, d\sigma\, d\omega\, d\nu\left(1+\frac{v\sec\theta}{c}\right)$

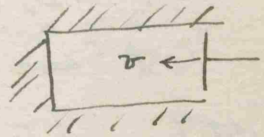
factor accounts for extra by reflection

due to motion.

Coming off.

$I_{\nu'}\, dt \cos\theta'\, d\sigma\, d\omega'\, d\nu'\left(1+\frac{v\sec\theta}{c}\right)$

28/11/11



Formula obtained for compression.

Freq. is changed thus

ν_1 before $\qquad \nu_2 = \nu_1\left(1+\frac{2v}{c}\cos\theta\right)$

ν_2 after. $\qquad d\nu_2 = d\nu_1\left(1+\frac{2v}{c}\cos\theta\right)$

for energy. $E'dt = E\,dt\left(1+\frac{2v\cos\theta}{c}\right)$

$E\,dt$ incident

$E'dt$ reflected.

& $E_1\,dt = I_\nu \cos\theta\, d\sigma\, d\omega\, d\nu\left(1+\frac{v\sec\theta}{c}\right)$

Now calculate energy in a small chamber V

Total energy inside interval is $V u_\nu\, d\nu$.

In time δt vol increased by δV

Energy is $\left[V d\nu + \delta(V u_\nu)\right] d\nu$

Increase in energy must equal the energy communicated as reflected light less that lost as incident light

Spectral interval $(\nu, \nu+d\nu)$ is converted into another interval not overlapping if

$\frac{2v}{c}$ is large compared with $d\nu$

Hence assume this

Loss = all energy incident = $E\,dt$

$$= A I_\nu \cos\theta\, d\theta\, d\omega\, d\nu\, dt \left(1+\right)$$

A = area of piston mirror

Total Loss = $\int E\, dt\, d\omega = 2\pi A I_\nu\, d\nu\, dt \left(\frac{1}{2} + \frac{v}{c}\right)$

Gain = what is projected into & has the freq. changed into the interval considered.

Let $(\nu', \nu' + d\nu')$ be that interval wh. after reflection θ becomes interval $(\nu, \nu + d\nu)$

Then $$\nu = \nu'\left(1 + \frac{2v}{c}\cos\theta\right)$$

$$d\nu = d\nu'\left(1 + \frac{2v}{c}\cos\theta\right.$$

{for every θ there is a different $d\nu'$ & ν'

∴ Gain is

$$A I_{\nu'} \cos\theta'\, d\omega'\, d\nu'\, \delta t \left(1 + \frac{2v}{c}\cos\theta\right)\left(1 + \frac{v \sec\theta}{c}\right)$$

Connect $I_{\nu'}$ with I_ν

Assume not monochromatic

$$I_{\nu'} = I_\nu + (\nu' - \nu)\frac{\partial I_\nu}{d\nu}$$

$$= I_\nu - \frac{2v}{c}\cos\theta\, \nu \frac{dI_\nu}{d\nu}$$

Substitute in former.

$$A\left(I_\nu - \frac{2v}{c}\cos\theta\, \nu \frac{dI_\nu}{d\nu}\right)\cos\theta'\, d\omega'\, d\nu \left(1 + \frac{v\sec\theta}{c}\right)$$

Integrating over whole half sphere θ' or θ is merely variable of integration

∴ Total Gain = $\iint$ ———

$$= 2\pi A\, d\nu\, \delta t \left[\frac{1}{2} I_\nu + \frac{v}{c} I_\nu - \frac{1}{3}\frac{2v}{c}\nu\frac{\partial I_\nu}{\partial \nu}\right]$$

$\iint \cos^2\theta \sin\theta\, d\theta = \frac{1}{3}$

Thus $\delta(V u_\nu) = -2\pi A\, d\nu\, \delta t \left(\frac{2}{3}\frac{v}{c}\nu\frac{\partial I_\nu}{\partial \nu}\right)$

now $A\, v\, \delta t = -\delta V$

$$\therefore \delta(V u_\nu) = \frac{4\pi}{3}\frac{1}{c}\nu\frac{dI_\nu}{d\nu}\delta V$$

also $u_\nu = \frac{1}{c}\int I_\nu\, d\omega = \frac{4\pi}{c} I_\nu$

$\therefore \quad \delta(V u_\nu) = \frac{1}{3}\nu \frac{\partial u_\nu}{\partial \nu}\,\delta V.$

$\therefore \quad V\,\delta u_\nu = \left(\frac{1}{3}\nu\frac{\partial u_\nu}{\partial\nu} - u_\nu\right)\delta V.$

Regard u_ν as function of V only

$\therefore \quad V\frac{\partial u_\nu}{\partial V} = \frac{1}{3}\nu\frac{\partial u_\nu}{\partial\nu} - u_\nu$

This gives whole answer to question
What is structural change in radiation.

Check by integrating with respect to $d\nu$
Change of energy of the whole radn
is $\int_0^\infty \delta(V u_\nu)\,d\nu = \int_0^\infty(\delta V u_\nu + V\delta u_\nu)\,d\nu$

$= \delta V\int_0^\infty \frac{1}{3}\nu\frac{\partial u_\nu}{\partial\nu}\,d\nu$

$= \frac{1}{3}\delta V\left[\left[\nu u_\nu\right]_0^\infty - \int_0^\infty u_\nu\,d\nu\right]$

$= -\frac{1}{3}\delta V\,u = -p\,\delta V$

as it should be —

To solve write
$u(V\cdot\nu) = z \qquad V = x \qquad \nu = y.$

$x\frac{\partial z}{\partial x} = \frac{1}{3}y\frac{\partial z}{\partial y} - z$

$xp - \frac{1}{3}yq = -z$ Lagrangian form.

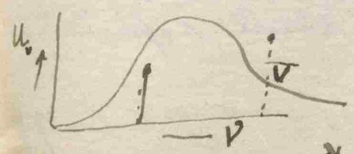
$\frac{dx}{x} = \frac{dy}{-\frac{1}{3}y} = \frac{dz}{-z}$

1 & 3 give xz = const.
1 & 2 " xy^3 = const.

Put $xz = f(xy^3)$ gives general Soln.

Hence $u_\nu = \frac{1}{V}\phi(V\nu^3)$
$= \nu^3 f(V\nu^3)$

Consider vols V, V'
& freqs ν ν'
So that $V'\nu'^3 = V\nu^3$
then $\frac{u_\nu}{\nu^3} = \frac{u'_{\nu'}}{\nu'^3}$.



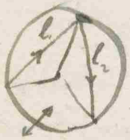
Given for any V it can be found for any other V

Avoiding non-monochromatic assumption use Westphal's Proof

(Verhandl. d. Deut. Phys. Gesell. XVI p. 93 1914.

On deduction of Wien's Law.

Sphere capable of contraction or expansion regularly.



Diffusely reflecting because spherical surface made up of an infinite no. of small plane mirrors regularly reflecting

Radius R $\qquad v = \frac{dR}{dt}$

Consider element mirror & ray incident & reflected.

Let small plane reflecting element make angle ψ with surface of sphere & let η be true angle of incidence on plane element

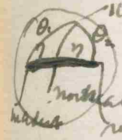
Vel. of element normal to itself due to motion of sphere expanding is $v\cos\psi$

$$\therefore \nu' = \nu\left(1 - \frac{2v\cos\psi}{c}\cos\eta\right)$$

freq. smaller since expansion $\therefore$ − sign.

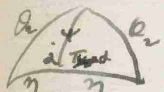
Let θ_1 be apparent incidence &

θ_2 .. .. reflection angle.



$$\cos\theta_1 = \cos\eta\cos\psi - \sin\eta\sin\psi\cos\alpha$$

$$\cos\theta_2 = \cos\eta\cos\psi - \sin\eta\sin\psi\cos(\pi-\alpha)$$



adding

$$\therefore \cos\theta_1 + \cos\theta_2 =$$

$$2\cos\eta\cos\psi$$

$$\therefore \nu' = \nu\left(1 - \frac{2v\cos\psi}{c}\cos\eta\right)$$

$$= \nu\left(1 - \frac{v}{c}(\cos\theta_1 + \cos\theta_2\right)$$

$$\cos\theta_1 = \frac{\frac{1}{2}l}{R} \qquad \cos\theta_2 = \frac{\frac{1}{2}l}{R}$$

$$\therefore \nu' = \nu\left(1 - \frac{v}{c}\frac{l_1 + l_2}{2R}\right)$$

$$\therefore \nu' = \nu\left(1 - \frac{v}{c}\frac{l_1 + l_2}{2R}\right)\left(1 - \frac{v}{c}\frac{l_2 + l_3}{2R}\right) \text{etc}$$

for n reflections

gives

$$\nu' = \nu\left(1 - \frac{v}{c}\,\frac{l_1 + 2l_2 + \dots\dots l_{n+1}}{2R}\right)$$

$$= \nu\left(1 - \frac{v}{c}\,\frac{\sum l}{R}\right)$$

$\sum l$ = total path $= c\,\delta t$.

$$\therefore \nu' = \nu\left(1 - \frac{v}{c}\,\frac{c\,\delta t}{R}\right)$$

$$= \nu\left(1 - \frac{v\,\delta t}{R}\right)$$

But $v = \dfrac{\delta R}{\delta t}$.

$$\therefore \nu' = \nu\left(1 - \frac{\delta R}{R}\right) = \nu\left(1 - \frac{1}{3}\,\frac{\delta V}{V}\right)$$

$$\therefore d\nu = \frac{1}{3}\,\frac{dV}{V}$$

or $V\nu^3 = $ const. (1)

2/12/21.

u = Actual energy per unit vol. $[u_\nu d\nu]$

$$d(Vu) = -\frac{1}{3}u\,dV$$

$$V\,du - \frac{2}{3}u\,\delta V = 0$$

$$u^{\frac{3}{4}}V = \text{const.}$$

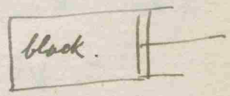
u by $u_\nu d\nu$ $\qquad \dfrac{u}{\nu^4} = $ const.

$$\frac{d\nu}{\nu} = \frac{d\nu'}{\nu'} \qquad \frac{u_\nu}{\nu^3} = \text{const.} \quad (2)$$

Since states (1) is a function of (2).

Assume radn = black.

Compress.



As proved earlier
radn remains black.

(for suppose not black – insert small radiating body – it becomes converted black ∴ entropy increases.

Expand back again & recover exactly same work since adiabatic & vol. & pres are same.

∴ Net work is zero.

but radn may or may not be black
If not insert body again & another irrev. process takes place & entropy increases

but state of universe has not changed not state of radn ∴ inc. of entropy is impossible.

∴ black radn on compression is black

Increase of energy means a net loss of availability:

Compress black rad^n &

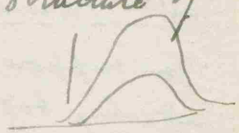
$T^3 V$ is const by Stephan's Law

In general $u_\nu = \nu^3 f(\nu V)$ ∴ for black

$u_\nu = \nu^3 \phi \frac{\nu^3}{T^3}$

$= \nu^3 \phi_\nu\left(\frac{\nu}{T}\right)$

This gives structure of



black radiation

One curve deducible from another.

Since $u_\nu\, d\nu = u_\nu\, d\lambda$

$u_\nu = \frac{1}{\lambda^5}\phi'(\lambda T)$

(since $\nu\lambda = c$

$\frac{d\nu}{\nu} = + \frac{d\lambda}{\lambda}$ + by def^n)

Another form of Wien's Law.

$d\lambda = \frac{1}{\lambda^5}\phi(\lambda T)$ Put $\lambda T = W$

$-\frac{5}{\lambda} + \frac{T\phi'(\lambda T)}{\phi\,\lambda T} = 0$

$\lambda_{max} T = W$ gives max^m pt. 0.2885

Entropy of Radiation

u = energy dens of black radn. expand by δV.

then $\frac{1}{3} u\,\delta V$ = work done at const temp.

gain of rad. energy $u\,\delta V$.

Total energy communicated from source is $\frac{4}{3} u\, dV$.

∴ $\frac{\frac{4}{3} u\,\delta V}{T}$ = loss of Entropy from source.

This appears in the new rad^n created, ∴ no loss or gain in universe.

∴ $\frac{\frac{4}{3} u}{T}$ is entropy of black rad^n.

$u = aT^4$

∴ $S = \frac{4}{3} a T^3$

Def^n of Entropy from Energy pt. of view

of energy of amt. u possesses

entropy S. & if T' is the lowest temp. that is going to be considered then the max^{m} amt of external work that can be obtained by turning u partly into work and partly into heat energy at temp T' is $u - ST'$.

(Fundamental defn of entropy)

$S = \frac{4}{3}\frac{u}{T}$ Take $T' = T$.

amt of work $= u - \frac{4}{3}u = -\frac{1}{3}u$.

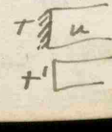
i.e. an amt of radn energy u at temp T has no work value – but work $\frac{1}{3}u$ has to be performed ~~on~~ on it to turn it into heat.

Take $T' \neq T$

$S = \frac{4}{3}\frac{u}{T}$

$u - \frac{4}{3}u\frac{T'-T}{T}$ or $u - \frac{4}{3}u\frac{T'}{T}$ if T' much larger than T

Work a perfect heat engine – betw T' & T



Cycles – gain of heat $\frac{4}{3}u$

∴ useful work is $\frac{4}{3}u\left(1 - \frac{T'}{T}\right)$

work used $\frac{1}{3}u$.

∴ net gain $= u - \frac{4}{3}u\frac{T'}{T} =$

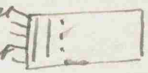
Similar proof on Carnot Cycle gives same formula

Entropy of Separate frequencies separately.

Energies of sep. frequencies exist independently ∴ each should have its own entropy whether radn be black or not.

This cannot be calculated directly, for this wd. be tantamount to solving B_ν

Take chamber (1) Source of heat



(2) Layer of absorbing material inside

(3) Screen only transmitting interval ν $\nu + d\nu$

(4) Selectively transmitting Shutter.

(5) Shutter moveable –

(1) close shutter till density in chamber is equal to black body dens at temp T for interval $(\nu\ \nu+d\nu)$

(2) Shutter removed nothing happens since pressures on both sides of 3 are equal for $\nu, \nu+d\nu$

(3) Increase temp by dT. then u_ν will increase to $u_\nu + du_\nu$ in the chamber.

Hence energy transmitted thro screen is $d u_\nu$ per unit vol.

& this energy is leaving a source at temp T

$\therefore$ Inc. in entropy $dS_\nu = \dfrac{du_\nu}{T}$

$\therefore \dfrac{dS_\nu}{du_\nu} = \dfrac{1}{T}$ is ~~u_ν~~ ν is const

Variables for S to be function of Energy constant

$\therefore$ if ν const

$S = f(u_\nu)$ $\quad S = f(\nu, u)$

$\therefore \dfrac{\partial S}{\partial u} = \dfrac{1}{T(u,\nu)}$ T is itself a function of u, ν.

Planck's proof.

Call T temp. of u_ν

Take chamber enclosing all frequencies but not neces. black.

Introduce abs. & rad. ptcls. & leave it to itself. By irrev. processes it becomes black & entropy increases till total entropy of all constituents is a max^{m}.

S = total entropy $= \int_0^\infty s\, d\nu$

$= \int_0^\infty s(\nu\ u)\, d\nu$

$\delta S = 0$ subject to $\delta u = 0$.

$\delta u = 0$ means $d \int u\, d\nu = 0$

$\int \delta u_\nu\, \delta\nu = 0$

$\delta S = \int_0^\infty \delta s(\nu . u)\, \delta\nu$

$= \int_0^\infty \dfrac{\partial s}{\partial u}\, \delta u\, d\nu$

$\therefore \dfrac{\delta s}{\delta u}$ = const (by Calc. of variations)

But $\dfrac{\delta S}{\delta u} = \dfrac{1}{T(u,\nu)}$

$\therefore T(u,\nu)$ must be const indep of ν.

which is satisfied by defn.

T same for all values of ν.

$\therefore$ Entropy is max for all possible interchanges of energy wh. dont alter the total energy

Since $\frac{\partial S}{\partial u} = \frac{1}{T}$ in posn of maxm entropy

then soln would give all we require regarding distribution.

$$\frac{dS}{du} = \frac{1}{T}$$

Chamber with 2 intervals of freq. present.

$\nu, \nu+d\nu$

$\nu', \nu'+\delta\nu'$

Separate them without doing any work.

Slide in a telescope with one end opaque to ν only.

& replace orig. vessel with screen at one end opaque to ν' only

Then pull out telescope & get only ν in one & only ν' in other. & no work has been done.

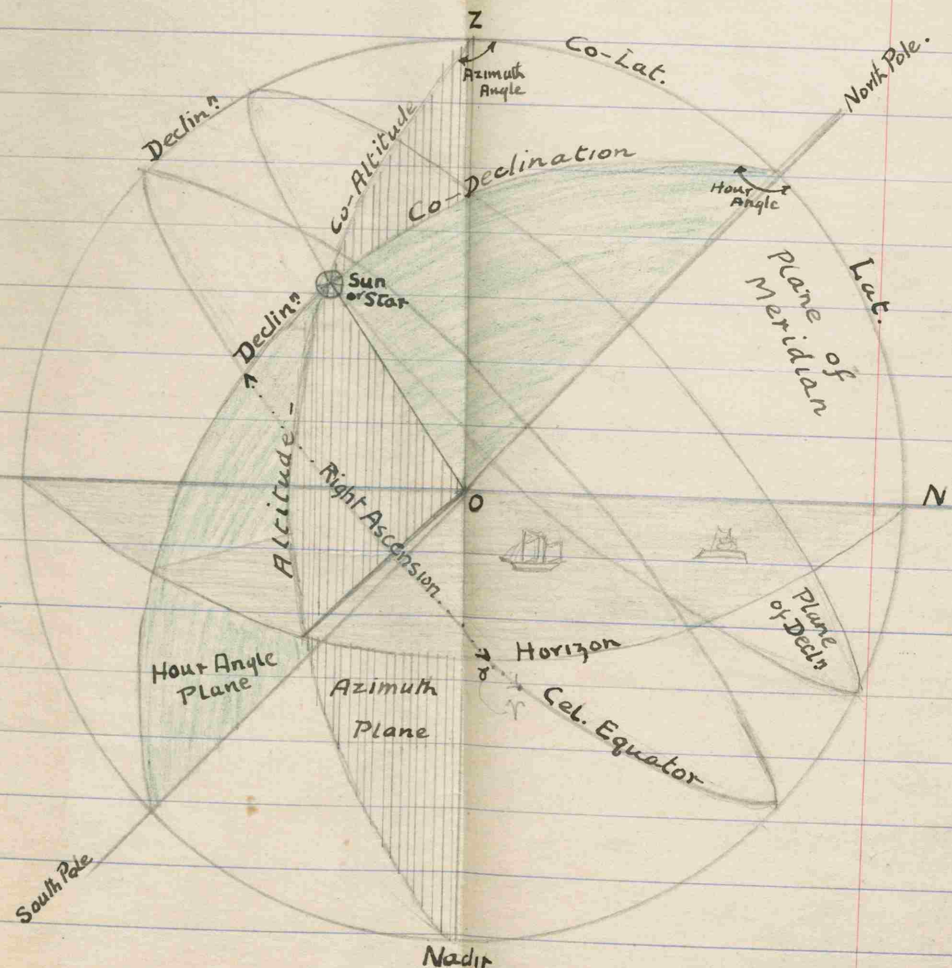
To filter away ν from black radn. is not possible due to Doppler effects.

PRACTICAL · ASTRONOMY ·

A. V. Douglas.

Prof. A.S. Eddington.

15/10/21.



image

tion cone, etc.

no definition

&c. instead

t. of light.

examine it

emons +

ed up on the

cale. That photo

oint $\propto D^2$.

is proptl to $\frac{D^2}{F^2}$

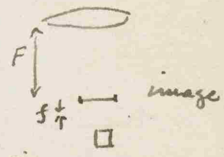
t.

Prof. A. S. Eddington.

15/10/21.

Practical Astronomy.

Telescopes.



magnifying eye piece

Smaller the eyepiece the gtr the magnifying

Since F/f gives magnification.

Limits to magnification due to aberration cone, etc. and to physical optics. no definition diffraction rings &c. instead of pt. of light.

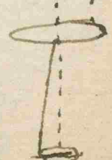
Adopt 2 stages.
Put photographic plate under & examine it with eyepiece.

Oscillations due to atmospheric tremours & clockwork vibrations are integrated up on the photo plate.

For lunar work eye more delicate than photo.

Faint stars - point source.
Aperture D gives intensity of point $\propto D^2$.
Light on any one grain of photo plate is propl. to $\frac{D^2}{F^2}$

Similarly for nebulae & tail of comet.



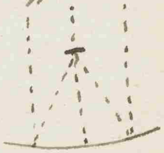
Thus for magnification F large
for light intensity of surface brightness F small.

In practice $F/D = 12$ in order to get good curve

Cambridge refractor is 18 : 1.

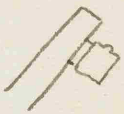
A reflecting tel. gives possibility of much smaller diff: $\frac{F}{D}$ is from 6 to 3 giving better condns. for faint star work.

Difficulty - Elic mirror does not always accurately focus to pt - coma effect.



To get $\frac{F}{D}$ small a doublet or triplet eyeglass can be used

To get very little mag.ⁿ but large field - attach special camera to tel.



"portrait lens"

Impressive results - whole comet tail + myriads of adjacent stars &c.

For star parallaxes - high magnification reqd.

Camb. Sheepshank tel is a triplet 19 ft focal length.

<u>Resolving power</u> for separating doublet star or bi planetary object.

angular diam of image $1.2 \frac{\lambda}{D}$ where λ

λ = wavelength of light.

D = aperture

$1.2 \frac{\lambda}{D} F$.

angular separation known for tel. + ∴ images will be separable if not $< 1.2 \frac{\lambda}{D} F$.

Phys optics - Hence value of large aperture.

<u>Retinae</u> if both images fall on same element of retina then indistinguishable. if on two or several elements by gt enough magnification then 2 distinct images -

Geom. Optics. D magnification $= \frac{D}{d}$

$= \frac{F}{f}$.

diam of pupil of eye δ d $\parallel$ δ

the only part of D is effective. $\therefore$ adjust d to eye.

$m = \frac{D}{d}$ $\therefore$ $D = m \times$ diam of pupil of eye.

$\therefore$ To take max^m advantage of big tel make m such that whole aperture D is effective.

28 in object glass (largest at Greenwich)
pupil of eye $\frac{1}{10}$ in.

then $m = 280$.
used up to 600 on very clear nights.

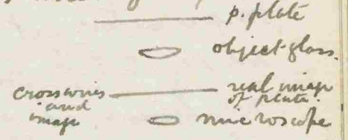
for doublets 2 condns - ang. dist $1.2 \frac{\lambda}{D}$
must fall on > 1 element of retina

These are almost coincident as result of evolution of eye - possibly.

Measurement of distances - real image in an accessible place - put scale there + get no parallax
Galileo's tel had a virtual image - no good for measurement

In practice use Ramsden eyepiece giving real image with cross wires at same place.

For photography cross wires give parallax
∴ use microscope



Visual & photo tels.

17.10.21. At Observatory.

focal length 9 ft.
Saw Transit. 1870 app. 12 ft. centrally pivoted on east-west axis. Errors :- declination i.e. not exactly east west to fraction of second of arc, 2. Level - axis not quite horizontal 3. Collimation - middle cross hair of eyepiece not absolutely on geom. axis of telescope. Find value of (1) by observing time of transit of a well known star, (2) by turning telescope vertically down over a bowl of Hg. + noting deviation of image of cross hair. Eliminate (3) by getting two small telescopes - one N + one S. of Transit + the cross hair in eyepiece all in alignment with a distant object due S. (Grantchester Ch tower) Time of transit of a star is supposed

to be correct to $\frac{1}{1000}$" but this one is not certain beyond $\frac{1}{100}$" - Sidereal time thus obtained can be converted into ~~M.S.T.~~ G.M.T. & thus chronometer set. Now, however, chronometers are regulated from Eiffle Tower - time being wirelessed at 9.45 to 10.3 daily.

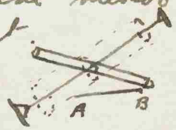
Electrical adjustment for recording time of coincidence of Star with the various cross-hairs by pressing button - also automatic record of declination reading at any moment required. Supports are solid cement piers from 10 ft below surface of Earth. The obsns for Star catalogue - 25°-30° N. Dec. were made with this instrument.

Saw small early form of transit & a portable telescope - app. 5 ft. for observing Solar Spots &c.

Saw great 18~~46~~35 telescope used by Airy. It should have discovered Neptune. Airy had diagrams of that region of sky but had not looked for motion of planet as he was very incredulous at Adams's prediction from mathematical considerations - 1845. Then Le Verrier obtained similar theoretical evidence & informed Berlin & planet was located & reported. Heavy rigid wooden frame work at inclination of Pole Star. & telescope 19 1/3 ft. focal length 15 or 20 ft long mounted centrally with an altitude movement & a rotation of whole framework of axis giving Rt. Ascension aperture 11 1/2 inches.

Northumberland Telescope.

Rather crude (but fairly accurate for those days) clock work & ingenious geometrical method of getting ~~hour angle~~ declination adjustment



graduated rod clamped from eye piece end to rigid frame - B A. / Triple ~~object~~ glass 12 1/2" diam.

Sheepshanks Telescope 1899

~~Saw the Cordic (?)~~ telescope - very good for photographic work, elaborate electrical device for keeping clock work at 120 rev either more or less being automatically compensated & prevented. in the Sheepshanks set up by Sir R.K. Ball. ... 19 1/2 ft focal length

Made in two parts like V with plane mirror at angle 20 attached as always to bisect angle between stationary eyepiece end & moveable object glass end which can be adjusted in the regd. 2 1r ~~directions~~ planes. the axis being towards Pole & fixed - Working end in room above, with all controls & special arrangement to read declination scale at angle of V down below.

Saw eclipse of moon through this telescope at 10.50 p.m. 16th Oct. 20 min. before maximum of 93.8% darkness.

Present use chiefly for photographs of parts of sky of which plates were taken 20 years ago - By taking the present ones on reverse side of a plate it is possible to superimpose the two & work out proper motions of stars - up to 13th magnitude Parallax plates are also taken. 7

Sextant

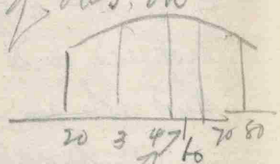
Altitude of Sun - Correct for refraction
58" tan $z\,\overline{\odot}$ (neg.)

for parallax (added)

Semidiam (added) Obs. lower limb

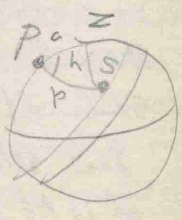
Sun on meridian ± 14' due to change in Suns declination

Longitude from curve of obs. on alt - Time of crossing of meridian.



Chronometer Errors -

Find Hour Angle of Sun



SPZ - HA

get longitude on land

& hence Eq. of Time & G.M.T.

Mod. method - 10 o'c GMT by wireless

$\cos z = \cos c \cos p + \sin p \sin c \cos h$

$\sin z \frac{dz}{dh} =$

$\frac{dz}{dh} = \sin c \sin A.$

Hence alt. is changing at max rate at due E or due W i.e. 8 or 9 am at Aden - good time.

Note $dh = \frac{dz}{\sin c \sin A}$

error in time $\propto$ error in alt observation & this is least when A biggest

22.10.21 (from notes by C. Payne)

Irregularity of Moon's motion

(1.) Secular accel. due (with little doubt) to tidal friction this changes place of eclipse

(2.) Long period irregularity - 270 yrs. Sine curve.

(3) Approx. 40 yr. irregularity

Transfer of angular momentum betw E & Moon? Possibly explicable on lines of slip of one layer of earth on another.

These errors cause change in place of eclipses of about 10" In such cases GMT must be determined & this if not telegraphable must be derived from some astronomical obsn.

Telescopes

Refractor. Advantage in large scale for accurate measurements - e.g. parallax

Disadvantage is imperfect achromatism.

Reflector: small field but perfect achromatism

Cambridge telescope gives equally good visual & photog. work, but involves thick lens wh. is of flint-glass absorbing blue rays & hence slow for photog. work.

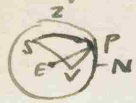
Adjustment of Telescope

Optic axis difficult to determine in finished instrument – optic axis gives region of best definition. Hence cannot be used as fiducial line. wh. is furnished by crosswires. This joined to optical centre of objective gives accurately determined line.

Collimation Error

1. Adjust collimators 1 & 2 ∥ to one another (not necessarily in same line) The 2 wires will thus be coincident. This gives ∥ N & S directions.
2. Put in main telescope & adjust wire coincident with 1. Then turn & examine it thro 2. The wires will again coincide if there be no collimation error. Otherwise it must be shifted thro twice the distance it is out & half way between the two points is the point sought which eliminates the coll. error.

Chief errors of Transit.



Axis shd. point E. Let it actually point to pt. V.

Suppose a star S is observed

… where

… m.

… time of

… P sin P V cos SPV

… (SPZ − m)

… sec δ

…)

… instrument

… mined daily

…) & n is thus determined. For stars near the pole tan δ is great & thus best chosen there. (tan δ for Polaris is approx 60)

9

Reflector : small field but perfect achromatism

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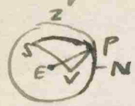
is out & half way between the two points is the point sought which eliminates the coll. error.

Chief errors of Transit.

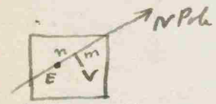


Axis shd. point E. Let it actually point to pt. V.

Suppose a star S is Observed.

VS should be 90° but is $90 - c$ where c is coll. error.

Then PV is $90^\circ - n$ and EPV = m.



Effect of above on time of transit

Differentiate $\cos SV = \cos SP \cos PV + \sin SP \sin PV \cos SPV$

$\therefore \sin c = \sin \delta \sin n + \cos \delta \cos n \sin(SPZ - m)$

If these errors are small

$$c = n \sin \delta + \cos \delta (SPZ - m)$$

$\therefore c \sec \delta = n \tan \delta + (SPZ - m)$

$\therefore SPZ = c \sec \delta - n \tan \delta + m$

Tabular time − obsvd time $= m + n \tan \delta + c \sec \delta$

(altering convention of measuring n)

c is eliminated on some stable instrument but usually this col. error is determined daily

Determination of n

2 stars δ_1 & δ_2

$T_1 - t_1 = m + n \tan \delta_1$

$T_2 - t_2 = m + n \tan \delta_2$

or allowing for clock error H

$T_1 - t_1 = H + m + n \tan \delta_1$

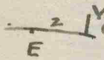
$T_2 - t_2 = H + m + n \tan \delta_2$

$\therefore (T_2 - t_2) - (T_1 - t_1) = n(\tan \delta_2 - \tan \delta_1)$ & n is thus determined. For stars near the pole $\tan \delta$ is great & thus best chosen there.

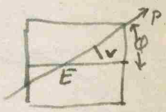
($\tan \delta$ for Polaris is approx 60)

Determination of m.

Inseparable from clock error + ∴ cannot be done astronomically. Often unnecessary for this very reason – But for obs. of Rt Ascension it is reqd. Done by obs. of level. Vertical error thus given



l thus known but Z unknown



But $Z = n \cos\phi + m \sin\phi$

$l = n \sin\phi - m \cos\phi$

Everything here except m is known Hence m determinate.

n is often called the azimuth error which Z really is

Level error

Collimation line (where c is zero) is 1r level line. Point telescope into trough Hg. If level error exist beam is turned thro double angle + image of fiducial wire displaced. Thus angle = 2 l where l = level error. Distance from wire to its image + focal length of telescope are all that is reqd. to give level error – Special eyepiece is used.

With small instruments a spirit level is made to suffice

29.10.21.

Photographic work with transit.
to locate dec. + rt as. of stars

Use plate 2° square + have at least 3 known reference stars – better to have 10 per 4 □°

Plate 5° square wd. be better if feasible

2° square is 6¼ × 6¼" inches –

There are 40000 □° in sky

hence 100,000 ref. stars are reqd.

Necessary to observe to 9 magnitude

but gain of pos. to stop at 8m.5

Differential + Fundamental Observations

Errors less difficult to detect if observations are differential.

2 stars of same dec. closely following each other.

E.g. If 4000 stars over sky found fundamentally observe in zones + compare differentially the 96000 others.

Pole star + 1st pt of Aries got by oscillation of sun + planets orbits to get ecliptic.

Azimuth error – n – often varies diurnally. At Cambridge imperceptible – but at Greenwich on a hill it is noticeable due to heating up of diff. sides of hill

Get n by comparing a polar star n tan δ large with a horizon star n tan δ small

There are 7 azimuth stars for big telescope & fewer for a small instrument. Use Polaris as one visible by day. Each of 7 crosses twice once above & once below pole hence 14 chances in 24 hours.

The transit takes approx 5 or 10 min. & no other obs. is poss. at same time.

Obs. placed where it is in camb. so that mark on Grantchester Ch is due South. Check ap azimuth error weekly or monthly.

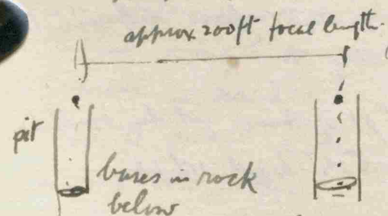
At Greenwich a mark due N in Epping forest. not always visible.

Method inaccurate.

If 3 miles off i.e. 190,000 inches

1 inch sway of steeple or other mark brings in error of 1" of arc.

Sir David Gill at Cape Obs. revived this method by digging deep down into rock bed

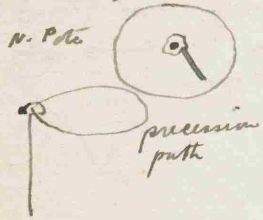


Collimation line adjusted with ref. to bases in rock.

trough of Hg in bottom of pit reflecting light ray - set pointer above on its own image & thus get 2 centres adjusted.

Gill has 4 pits - 2 more South to serve as checks

Slight variation remains due to change of a line fixed in Earth. Since earth's axis moves at pole ± 20 ft from its mean posn. in 1230 days - due to nutation.



line ∴ swings slightly

At Cape the E. & W. polar movements were detectable just as N & S. movements are detectable by Obs. on Zenith distances.

Hamburg tried to copy Gill's method & set one end on a brick pier entirely undoing the effect.

Time errors -

Faint stars appear to cross later.

Magnitude Eqn. (Greenwich observer)

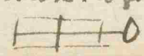
	sec	sec. (other observer)
5 - 5.9	+ 019	+ 002
6 - 6.9	+ 001	- 012
7 - 7.9	- 010	+ 003
8 - 8.4	- 005	+ 007
8.5 - 8.9	- 030	- 031
9.0 -	- 058	- 005 -

It is possible to reduce magnitude of a star by placing special screens thus e.g. make a 6.5m look like a 8.5 & compare readings with & without to find systematic personal error.

Reversion Prism

Micrometer screw method with chronograph. adjustment for apparent inversion to get magnitude error

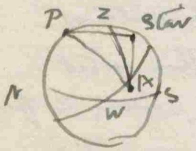
keep crosswires on star.



Clock gear to keep eye piece moving with star & hand gear to adjust crosswires on star

Personal Eqn then eliminated. Sq? error of fastest to slowest may amt. to ½". It gets elim. in Rt ascen. as same for all stars reference & faint ones –

At Observatory 1/11/21



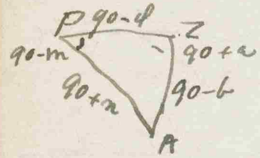
$PZ = 90 - \phi$ = co-lat
1. axis not horizontal
West end up is + error.
a = azimuth error
a pos. if W. end is South
b = level error.
A is posn. of W. end.

$ZA = 90 - b$
$PZA = 90 + a$.
Put star on middle wire. X
PX is $90 - \delta$ $\quad \delta$ = declination
Middle wire not in optical axis of tel.
Collimation error – c.
AX is not 90 but $90 + c$

△ PXA
Call $WPA = m$
& dist of A above meridian n
$APZ = 90 - m$
$AZP = 90 + a$.

Diff in times when X is on mer. & appears to be on it is "T" & is angle ZPX. In diagram as here is negative.

Find T in terms of a, b, c.



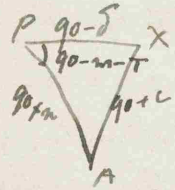
Thus get m & n in terms of a & b.

$\cos PA =$

$-\sin n = \sin b \sin\phi - \cos b \cos\phi \sin a$.
$\therefore -n = b \sin\phi - a\cos\phi$. approx.

Try $\sin(90+n) \times \cos(90-m) =$
$\cos 90{-}b \sin 90{-}\phi - \sin\phi\, b \cos 90{-}\phi \sin 90{+}a$
$\therefore m = b\cos\phi + a\sin\phi$.

△ PXA



$\cos XA = \cos 90+c$
$= \cos 90+m \cos 90-\delta$
$+\sin$ " $\sin$ "
$\cos 90-m-T$.
$\therefore -c = -n\sin\delta + (m+T)\cos\delta$.

Now $-T = m - n\tan\delta + c\sec\delta$
after passage over meridian T is neg
call $-T$ τ
then $\tau = m + n'\tan\delta + c\sec\delta$
$(n' = -n)$ This is Bessels formula

Insert a & b for m & n & get. A B are functions of $n+m$ & ...
$\tau = aA + bB + cC$
$C = \sec\delta$.

a further correction must be subtracted for aberration constant (order of $\frac{1}{30}''$)

$T = aA + bB + cC -$ Aber. Const.

Adjust wire to get rid of col. error c.
i.e. axis of telescope exactly $\perp^r$ to line joing ~~axis of~~ optical axis to object glass.

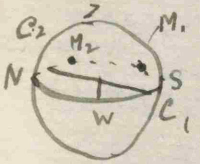
a. only obtained by observing a slow moving star cross meridian.

b. see later.
sec δ is always + ∴ T is always + for star east of meridian but to right or west side in eyepiece.
c is +

Determination of c.

i.e. C pos. when middle wire is to W of true meridian.

To measure c. see p. 9.



Adjust collimators $C_1\ C_2$ ∥ to one another + in a line not necessarily N + S.
Say $C_2\ C_1$

Call $C_1S = \alpha$

M_1 is middle wire in S. pos. + $M_1S = C$
Since angle betw. axis + middle wire is $90 + C$.
Read posn of M_1 move middle wire until it coincides with C_1 ($\angle M_1SC_1 = D_1$
$D_1 = \alpha + c$

Now look at N. coll. + $M_2N = c$
+ $C_2N = \alpha$.
move mid. wire until it coincides with C_2 $\quad D_2 = C_2M_2$
$= C - \alpha$

$\therefore C = \frac{1}{2}(D_1 + D_2)$

Sign of c.
Find signs of D_1 + D_2.
In diagram D_2 is neg. i.e. star hits mid. wire after true merid. in but D_1 is pos. as it is to west or right of true mer.
In diagram M_1 is actually east of S but on inversion it appears west. ∴ reading is positive.

5/11/21. Zenith Distances.

Select very small zone — with given fundamental stars
Differential declinations. errors not insurmountable.
" rt. as. more difficult since near 6 hrs
+ ∴ various errors creep in.
Fundamental star declination very difficult.
Exact posn of pole result of a yr. or 10 yrs obsns
not convenient to find every night. ∴ find zenith + diff of alt. of Z + P is co-lat.
Zenith determined by pointing tel down on Hg + getting image + wire to coincide.

Scale divisions often bring in errors. i.e. 45° pt not midway betw. 0 + 90.

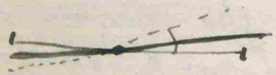
Flexure errors

Centre of obj glass to middle of crosswire

Errors thro necessary unequal weighting

flexure ∝ sin Z. Dist. from considerations of Bending moment

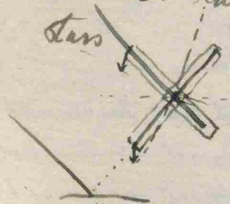
Put tel. horizontal onto S. collimator



Turn thro 180° onto N. coll & get line of sight

∴ move thro angle) to get rid of flexure.

Actually flexure does not follow this sin Z.D. law as found out by obs. of stars



angle betw is double alt. of star.

Hg

direct obs. of posn of star R

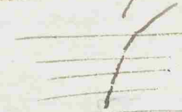
$\frac{1}{2}(D+R)$ should be

This strange discrepancy from sin law is found in all extensively used instruments:

Correction for refraction. 57" tan δ.

remarkable that corrections are as satisfactory as they are.

depending on assn. of equal stratification of atmosphere.



$\mu_1 \sin i_1 + \mu_2 \sin i_2$

.... $\mu_n \sin i_n$

Knowledge of $\mu_n \sin i_n$ for layer at telescope is sufficient.

Accuracy in thermometer suice.

1° Abs gives change $\frac{1}{270}$ in correction

i.e. $\frac{1}{2}$" arc.

Zenith distance of Sun - determines Posn. of ecliptic.

Obs. Suns R.A. + Dec. each day & compare with tabular place.

& get 2 quantities $\Delta_\alpha, \Delta_\delta$

from [Obs - Tabulated result].

Error of lat means Ecliptic is not in true place.

Error of long .. Sun is not in rt. part of ecliptic.

∴ change these into Δ_ρ Δ_β error of lat.

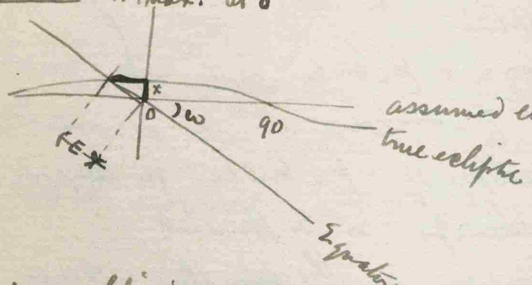
Examine for systematic error

& then for periodic error by formula

$\Delta\beta = x \cos l + y \sin l + z$.

Obtain about 12 or 20 eqns thus & expand by fourier series. for x, y, z

<u>x term</u> is max. at 0

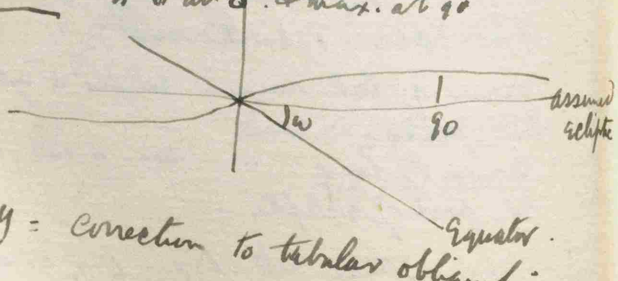


ω = obliquity of ecliptic

$E = \dfrac{x}{\sin\omega}$ = diff. between assumed 1st pt of Aries (♈) & true Aries (♈) along equator.

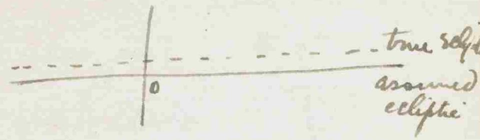
E. This is corr. for all rt. Ascensions & is called equinox correction.

<u>y term</u> is 0 at 0 & max. at 90



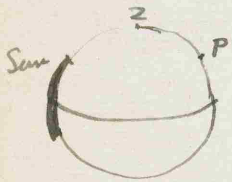
y = correction to tabular obliquity.

<u>Z term</u>



This means ecliptic is a small ⊙ ∴ not attributable to Sun but is an instrumental error for all obsns taken near eq. or ecliptic.

E.g. a scale div. or flexure systematic error.



Further error is Personal

2 obsn tangential to each limb - use reversion prism to reduce error.

Check on Z by obsn on planets.

Greenwich Obs 1904

$x = +''.284$ $\quad \times \dfrac{1}{\sin\omega \times 15} = 0^s.046$ sec. of time

$y = -''.131$

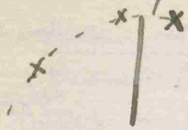
$z = -''.031$

correction for old nautical Almanac

<u>Zenith telescope</u> (for getting latitude) better than transit circle.

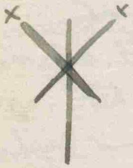
In latter large arcs have to be measured " former this is obviated.

Consider field work for detn of lat.



Star near Z to known th.

better to use pair of stars on either side of Z.



set wire on S star & reverse & N star is just off wire - screw up Dist & then get mean of 2 stars ~~& that~~ gives Zenith

& thus

At Observatory 8/11/21

Evaluation of wire distances Mag 3 stars in daylight

Wire intervals in seconds of time -

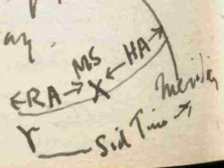
Conversion from G.M.T. to Sidereal Time

H.A.M.S. + R.A.M.S = Sidereal Time -

Error at 3 pm. 1500 or $3^h 00^m$ G.H.A.M.S.

1924 G.M.T will begin at 12 midnight

At pres. measured from midday.



HAMS 3^h (after noon)

RAMS increases at 9" per hour - & is tabulated for noon.

Transit ☉

S Col. 89.5

Tel. on S. Col.

A.O.L.	29.151	29.208 CHP.
	29.347	29.309
	30.066	30.151
	30.368	30.267
4	118.932	4 118.935
	29.733	29.734
	32.173	31.~~817~~ 507
	31.476	31.~~744~~ 623
	31.629	32.013
	31.460	32.131
error +	31.809	31.818
	61.542	61.552
Error	30.771	30.776.

+ 30.77^{35} Reading for no Colln error.

Clock errors. Sid. Chron. Graham S.C.

RAMS. $15.8.30^s.95$ + $4^h 22^m$ + $43^s.4$ = $19^h 31' 14.05''$ 1.9

18. 15 35.2 43.4

1 18 38.85"

4-20 - [4 3] 18.13.350

4.21 18.14.350

4.22 18.15.35~~2~~

4.23 .353

24 35.4 18

Special lecture in St. John's Hall 12th Nov 1921.
by Prof. Sampson F.R.S. Astron. Royal for Scotland.

The Modern Drift in Astronomy.

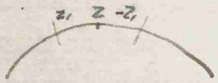
Lecturer entered St John's 37 years ago & found that Astron. was a most "repulsive" subject as taught then all theoretical math. not even beautiful math. in fact what Sir Geo. Darwin might have called degraded math. He however chose Astron. as his life study. He proceeded to speak in general terms of the attempts to determine whether Universe is finite or infinite, of the result of many years of counting the no. of stars of each magnitude & trying to prove the series thus obtained convergent to a sum of 30,000,00000; of the result of finding the proper motions of stars pointing to 2 main drifts as far as the 6000 plotted stars are concerned; of the wonderful results of spectral study of stars made especially at Potsdam in giving not only the vel. of approach or recession but also the apparent & abs. luminosity & hence the distance & diam; of the laborious & careful repetition of math analyses of the Newtonian Gravitation law in its bearing on the times & motions of the planets - over & over again until the small discrepancies were fully established e.g. the few seconds of arc over estimate of the motion of Mercury's perihelion - giving the foothold on which Einstein planted his beautiful generalizations; of the great outstanding problems calling for someone to give them his life-study such as the relation of comets in the Solar System, why Halley's comet was two days late last time, the life history of stars from nebulae to cooling & disintegrating orbs, the origin of the Solar system & so on, not least but of primary importance to Physics as to Astronomy - what is light. [Here he diverged to gently hit the Physicist who lives from day to day - cannot prophesy with the extraordinary accuracy of the Astron. for 300 years to come & holds onto 2 mutually contradictory hypotheses of light the undulatory & the quantum thy. - in fact is no better than a politician!] The modern drift in Astronomy is to accumulate more & more facts - many often of doubtful accuracy & apparent uselessness individually but by the application of non-rigorous math (i.e. thy of probabilities applied in an essentially modern statistical method) to deduce results whose value is beyond all doubt." The drift of modern Astron. is towards the Light & since the stars are our main source of light it may well be that by continued study of them the astronomer will some day be enabled to reveal the real nature of Light."

"Einstein is undoubtedly a genius of the very first water" - It is a mistake to try to interpret verbally an essentially mathematical expression. Einstein has showed that time & space are inter-dependent but it is not a logical deduction to say that they are interchangeable

19/11/21 Measurement of Zenith Dist & Lat.

Avoid using long axes by using Zenith telescope.

Talcott Method



Rotation about vertical axis - reading equal distances east & west.

use spirit level of kind  Conical spirit level

reading to $\frac{1}{10}$" arc. theoretically

errors due to heat - (draughts - etc.)

Observe transit of star z_1 reverse to $-z_1$ & watch for another star & shift wire to $-z_2$

difference is $-z_2 - (-z_1) = z_1 - z_2$

then $\frac{1}{2}(z_1 - z_2)$ gives mean.

In field select stars whose decl^n is known.

$\frac{1}{2}(\delta_1 + \delta_2)$ to be added to get latitude

these are continually changing due to nutation & precession

A definite Talcott pair are used for any specific latitude

6 stations on same lat

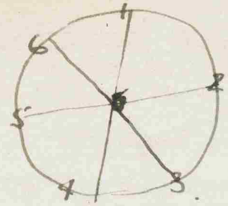
Mozusawa Japan.

Tschardjui Siberia

Carloforte Italy

3 in U.S.

approx. all 60° apart Longitude.



coordinates of motion of pole obtained by 6 readings.

Take 12 groups

observe 2 simultaneously

1,2 ; 2,3 ; 3,4 ; ...... 11,12 ; 12,1

each month.

↑ closing error always present tho small.

Analysis should give change of lat

$d\lambda = x \cos l + y \sin l$

x = displacement of pole towards Greenwich

y = " " " 90°.

a constant was found necessary but its value is not obvious

$d\lambda = x \cos l + y \sin l + Z$

Z = Kimura term & was found by him at Japan Station. It is periodic

& seems not real but due to observational pecularities & closing error.

If c. gravity goes north in winter vertical is swung & ∴ co-lats in N. hemisphere wd all be increased.

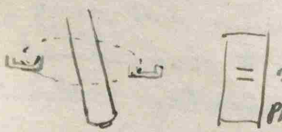
2 stations were set up in S hemisphere & found corresponding effect.

There are theories to account for this on melting of ice & meteorological condns but Eddington considers them no suff. & prefers to regard it as unreal due to refraction or something affecting observations.
Gen. opinion Kimura-term not physical.

Floating Telescope of Cookson

made & used at Camb by Cookson & after his death taken to Greenwich 1912 & has given good results
Obviate bubble troubles by floating iron annulus in Hg trough. It carries a telescope & whole freely floats & is not ~~is~~ clamped

Photographic trails of N. star & S. star



measure 1/2 dist between trails
Photo plate

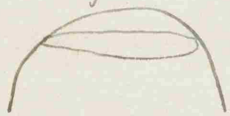
Results more accurate than old method.
Not wide choice of stars
Rather slow - 4 pairs per hour.
Sometimes wavy trail if system is oscillating in the Hg. - can be accurately measured nevertheless.

6.5 visual magnitude
not determinable within limits 0.5 - 2.0

Hanging telescope - Durham
depended on visual work.

Almucantar

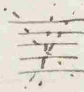
Constant altitude circle - Take 3 transits of stars across this ○ & thence find latitude
See formula in a book



float a telescope in Hg & set at any angle & observe transits of 3 known stars.

Not very accurate as angles of approach are not same

cross wires



← path of transit
personal errors & all errors hopefully mixed up in final solution.

Airy's Method

to get absolute Z. dist by rotating whole thing about vertical gives effect of stars rotated about vertical & new light ray follows dotted path to Hg & back to wire in front of eyepiece

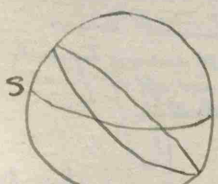
Main drawback is 2nd passage thro glass near its edge giving discrepancies due to non homogeneity of glass. Possible to obviate this

Airy was so certain of its accuracy & that stability of frame not necessary that he mounted his one at Greenwich on boards. Hg trough must be free from vibration.

other disadvantage is paucity of stars in field

Constant of Aberration K

displacement of star toward apex of earth's way.



Observation of star on meridian at Sunrise ab. factor is max^m N

at Sunset ab. factor is max^m in dir^n S.

∴ take obs. at sunrise & sunset & get measure of ab. factor — few months interval

Must know declin. of star if Ab. factor be got from two obs. on

Same night.

unknown star at sunrise $Z_1 + K + \delta\phi_a$ lat change.

another at sunset. $Z_2 - K + \delta\phi_a$

$Z_2 - Z_1 - 2K = —$

few months later — one years observations

$Z_4 + K + \delta\phi_b$

$Z_1 - K + \delta\phi_b$

$\therefore Z_1 - Z_4 - 2K = —$

get. $Z_4 - Z_3 - 2K = —$

$\therefore Z_3 - Z_2 - 2K = —$

$- 8K = —$

This is called the Küstner polygon.

Ratio of $\frac{\text{vel. of earth}}{\text{vel. of light}}$ is related to Solar parallax & dist of Sun

3 factors – any 2 give 3^rd.

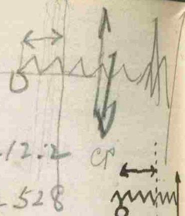
Present value K 20".47 ± ".02

giving Solar parallax 8".805

23/11/21

Vertical wheels

36	35.7	45	56.6
234	24.0	315	30.7



2.22.6 AW	4.11.5	2.12.2 CP
2 10.3	4.12.7	1.52.8
1 55.2	4.10.4	2 11.8
2. 10.0	3 50.9	2 17.6
1.99.5	1.99.05	1.98.6
Av. 2' 9.5"	2' 9.05"	2' 8.6"

Count no. of teeth from 0 to scale line betw 0 and arrow on head.

Periscope 337°

N.S. eyepiece on top

30 26.9	30. 22.0	E.W	6.1
30 19.0	30 23.2		6.3
30 22.9	30 22.6		Av. 6.2

Av. 30 22.75

Θ Aquilla.
mag 3.4
telescope to pole

Lat Camb 53° 13' ∴ Colat 36° 47'

337 50 − 217.7 = 120°

R. A 20h 7m
Dec S 1° 2'

Point telescope 121° 2'

26/11/21.

Determination of Star Magnitudes.

Statistical Study of stars — no. to 10 mag is 3 times no to 9th mag.

If equal distribution & no absorption it can be shown. factor 3.98 — log. 3.98 = 0.60
2.512 — 0.4

The cutting down of factor to 3 shows a dying away (except in galactic systems like Milky way.)

·1m is lowest easily obtained

·01m " recently attempted.

1855 Catalogue B.D. Bonner Durchmusterung
not accurate now.
1910 catalogues can generally be relied on.

Differential Magnitudes & Absolute Mag.

D. mags not so difficult. If you have 2 known stars say 8.52m & 9.2 then if unknown lies between interpolate between for it.

Harvard Standard Sequence
(since revised — Mt Wilson values now taken.
North Pole
(awkward at Camb. because Shepshanks cant get to N. Pole.

Sequence Sirius -1.5 to star of 15^m.

Visual & Photographic Magnitudes.

Visual depends on eyesight & ∴ not exact but closer than might be expected.

Photographic is rendered difficult by diff colours of stars & ∴ diff wavelengths.

Visual. Suppose 2 stars of diff colours judged of equal brightness - by limiting absorption reduce the apparent magnitude & find them no longer of equal brightness. (i.e. increase intensity of a red & blue light & brightness does not increase the same in each

This is the Purkinje phenomenon.

Usual method - photometer - Standard star & unknown star in same field. Use a Nichol prism to polarise light then put a doubly reflecting Iceland Spar in front & get extraord & ord image & the positions depending on rays polarized in 2 dir. $\perp^r$.

L = luminosity

Intensities are $\cos^2 i$ & $\sin^2 i$

$L_1 \cos^2 i$ $L_1 \sin i$

$L_2 \cos^2 i$ and $L_2 \sin^2 i$

Adjust to get $L_1 \cos^2 i = L_2 \sin^2 i$

$$\therefore \frac{L_1}{L_2} = \tan^2 i$$

A C Pickering (Harvard) made $1\frac{1}{2}$ million determination by this method.

For variable stars - practised eye comparisons are relied on.

Photographic: 20^{min} exposures on one star gives equal effect as $\cdot 2^m$ .. .. 2^{nd}.

then $10 L_1 = L_2$

$m_1 = m_2 + 2.5^m$ sign always neg.

This is found not quite right

Given by. $L\,t^{(0.7 \text{ or } 0.8)}$

i.e. 5 times exposure req^d then just over 4 times less bright.

Latest & Most Accurate Method

Put course grating or grid of wires of equal thickness & equal spacing over object glass

This acts like a grating giving primary undisturbed image of star & 2 1st order images; 2, 2nd; 2, 3rd etc.

——•—— from dimensions of grid calculate positions of these images

See Physical Optics Text for formulae.

Now take unknown star & compare magnitudes directly

known ....•...

unknown ..•..—

Schraffier-Kasette by Scherwartzschild

Instead of comparing total amt of light concentrated at point draw it out + measure brightness per unit area. (blackening method)

broadening by extrafocus. not quite uniformly distributed.

Colour Index = Photographic - visual

for blue it is -0.5 about

to +1.9^m

photographic magnitude smaller means star is brighter.

Draper Series

O hot stars

B - Helium Stars (Orion Stars Rigel

A - white star (Sirius (Balmer Series of H.)

F - metallic lines appearing

G - Sun

K

M - red stars Betelgeus spectrum of bands of chem compounds coming in. Titanium Oxide appears.

R
I
N

Above B still more intense stars i.e. hotter

Side line from K to R + N.

N has carbon compounds.

Russell Sur Stevens Ann XCI

former theory that this gave ordinary star history from hot to cool.

Recent theory - Star starts at comparatively cool star M + goes up scale to distance depending on its mass + then goes down again.

It is now possible to differentiate betw those going up + those going down i.e. G_{up} + G_{down} can be separated

Temps. M 3000°

G 6000°

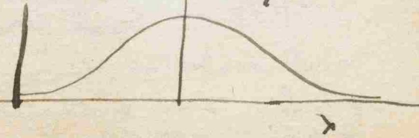
A 11000°

B possibly 20000° not known.

If star radiated as a black body, Wien's law of displacement towards violet takes place + i. colour index + spectral type are measures of one another

Stars do not radiate absolutely as black body but nearly

Wien's Law + Plancks Curve



$\lambda \theta = K$.

Spectral Types are thus classified to mag. 16. especially the Globular clusters

gt majority are at same mean distance

& ∴ apparent mags are measure of real mags. & ∴ classify them in Drapers series.

Stars much nearer not so easy to classify.

<u>Absolute Magnitude</u>

Convention for brightness of star indept. of distance - is chosen as

10 parsecs or 0".1 parallax.

On this arbitrary scale

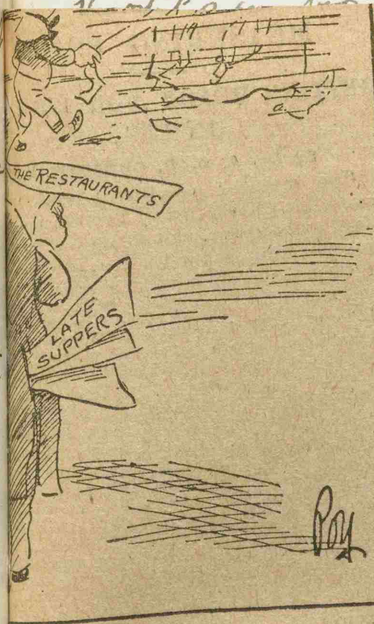
Sun wd. be $5^m.0$

i.e. at that dist. it wd appear a 5^{th} mag.

-26^m apparent mag of Sun.

At Observatory. 29/11/21. When Eros was fairly near Earth about 1903 Observations were made at Cambridge, Greenwich & Toulouse & 4 or 5 other places in France & Spain & several in Sweden & Denmark & in US. etc etc. These results taken over 5 yrs. were reduced & compared by Keith at Cambridge who deduced Solar parallax, & as bye products the moon's mass & complete data regarding orbit of Eros. For this he recd. French prize.

[illegible] its ellipticity, inclination [illegible] deduced from 3 [illegible] of its dec. & R.A.



[Y]ou believe it? He still imagines he [is] his nurse!"

[La]te suppers indicate that John Citizen is [th]at Dora no longer rules.)

through his interpreter. "I know it has the best Government for the people. But the smoke! It goes up and it comes down and it stays, making the air so thick! It is not the atmosphere we are used to."

Returning to the Near East, the Baha[i] found himself in Palestine early in the war and had a narrow escape from death at the hands of the Turks when they were evacuating the country. Such was his undeniably salutary influence in the Holy Land that, at Lord Allenby's suggestion, he [was knighted last year].

magnitude

<u>Selenium</u>

[illegible] of stars - resistance [illegible] to local temp. changes.

Light on film of [illegible] emitted & measured in [illegible]

changes outside of cell [illegible] very pot. - 150 volts [illegible] overcome resistance [illegible]

[illegible] measure change [illegible] movement of fibre [illegible] two. + & - plate.

[illegible] - absolute is very [illegible] invariably with that [illegible]

Spectral Types are thus classified to
mag. 16. especially the Globular Clusters
gt majority - are at same mean distance
& ∴ apparent ...
real mags. + ...
Drapers series ...
Stars much ...
Classify

Absolute Ma...
Convention for ...
of distance
10 parsecs
On this arb...
Sun wd ...
i.e. at that ...
5th mag.

At Observatory.
29/11/21
were made at ...
& Toulouse + 4 or 5 ...
& Spain + 2 ...

MEASURING SPACE.

GETTING READY FOR BIG TEST IN 1931.

By Dr. A. C. D. CROMMELIN.

The number of tiny planets, whose orbits lie (in the main) between those of Mars and Jupiter, mounts up year by year, and now approaches 1,000. The majority of them have no interest save to the professional astronomer, but it happens that three of very special interest have been under observation in recent months. The first of these is Eros, which was discovered by Dr. Witt, of Berlin, in 1898.

It was soon found that the orbit of this planet approaches that of the earth within 14 million miles, or much nearer than any other planet. This makes it peculiarly adapted for determining the scale of the solar system, and in particular the distance of the sun, by means of simultaneous observations at distant observatories. It will not make one of its closest approaches till February 1931; preparations are already being made for utilising that important occasion to the utmost.

The much less close approach (some 29 million miles) at the beginning of 1901 has already been used for this purpose, but the coming event should give a much more reliable determination.

There is a second method of finding the sun's distance from Eros: the earth disturbs its elliptical motion very largely, and it is possible to deduce the earth's mass compared with that of the sun, and hence to deduce the distance of the latter.

Calculations of this character have just been carried out by Dr. Witt and Mr. Noteboom. They find for the sun's distance by this method 92,897,000 miles, while the other method gives 92,822,000. Both figures are subject to revision after the observations of 1931, but it will be seen that

Continued in Next Column.

Continued from Previous Column.

the zone of uncertainty is already fairly small.

During the past 60 years the accepted estimate of the sun's distance has been changed from more than 95,000,000 miles to less than 93,000,000 miles. It is a great triumph to have got the uncertainty as small as it is, but everyone hopes that 1931 will make it smaller still.

The second interesting planet is Alinda, discovered by Prof. Wolf, of Heidelberg, in January 1918. This planet approaches the earth's orbit nearly as closely as Eros does, but it has a much more eccentric orbit, so that it requires four years to complete its path, whereas Eros takes only 1¾ years. It is such a tiny body (some three miles in diameter) that it is utterly invisible save when within a few millions of miles of the earth, and it is a matter of considerable congratulation that it has now been re-observed after completing a revolution round the sun, so that the size and shape of its orbit are placed beyond doubt.

The third body was found in 1920 by Dr. Baade, of Bergedorf Observatory, Hamburg, and is found to have an orbit so extremely elongated that it extends practically from the path of Mars to that of Saturn, taking 13 2-3 years to complete the circuit. From this orbit one would be inclined to call it a comet, but there is absolutely no fuzziness in its appearance, so that the name comet (which means "hairy") is unsuitable. We may conjecture that it is a minor planet which at some epoch passed very close to Jupiter and had its orbit entirely changed by his powerful attraction.

& in US. etc etc. These results taken over 5 yrs.
were reduced & compared by Keith at Cambridge
who deduced Solar Parallax & as bye products the
moon's mass + complete data regarding orbit of
Eros. For this he recd French prize.

The orbit of any body, its ellipticity, inclination
etc etc may be completely deduced from 3
consecutive observations of its dec. & R.A.

3/12/21.

... magnitude
Selenium
... of star - resistance
to local temp. changes.
Light on film of
emitted & measured in
changes outside of cell
to neg plate. 150 volts
overcome resistance
liberate electrons
... measure change
movement of fibre
... + & - plate.
- absolute is very
... smoothly with that

Spectral Types are thus classified to
mag. 16. especially the Globular clusters.
Gt majority are at same mean distance
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& Toulouse + 4 or 5 ...
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MEASURING SPACE.

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...EVED COUPLE: "Woul... is in tow ...
(Deserted late trains and unpatronised ... not yet awake to the fact

to the right quarter to secure my arres... and conviction on two criminal charges o... taking noxious drugs with the intention o... procuring a miscarriage, and, after prema... ture birth, with concealment of the same... Last Tuesday morning 'they' again de... manded the payment of the £600, tellin... me that 'they' had the evidence of the em... ployment of the nurse who attended me i... my illness.

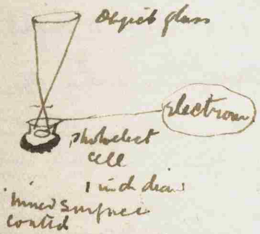
"Eventually 'they' told me 'they ... knew I could pay them £50 at once and b... ... morning next collect the £150 fro...

& in US. etc etc. These results taken over 5 yrs. were reduced & compared by Keith at Cambridge who deduced Solar parallax & as bye products the moon's mass + complete data regarding orbit of Eros. For this he rec.d French prize.

The orbit of any body, its ellipticity, inclination etc etc may be completely deduced from 3 consecutive observations of its dec. & R.A.

3/12/21.

Another Method of Measuring magnitude
Photoelectric. Old Method Selenium
Small coil exposed to light of star – resistance changes. Difficulty due to local temp. changes.
New Method. Light on film of alkali metal – electrons emitted & measured in electrometer.



Charge outside of cell to neg pot. 150 volts to overcome resistance to liberate electrons
String electrometer to measure charge by movement of fibre betw. a + & – plate.

Method is differential – absolute is very difficult – compare luminosity with that of a standard star
accurate to 1 in 300.

Difficulty due to keeping electrons vertical & still moving with tel. so that capacity of lead &c does not alter.

Hence value of Sheepshanks tel at Camb.

Apparatus for affixment to fixed observation end is now on order.

→ Magnitude is propl to log of luminosity.

In photographic measurements

mag. $m = a + b\sqrt{d}$ d = diam.

Probable error same for 1st mag as for 8th for example – as obvious from fact that by longer exposure the faint star gives as determinable a diam as the bright.

6th mag is $\frac{1}{100}$ 1st mag.

Photo-el-cell is best adapted to bright stars. chiefly due to Guthnick. Berlin Obs.

Brightness of Sun

Abbot found an irregular variation superimposed on regular 11 yr. period of sun spots variation. This irreg. variation was of order of 10% & was checked at Algiers & Mt Wilson hence there seems to be this 10% change in heat output. But the photo el. method disproves this. Cause of changes Eddington does not know, but probably due to atmospheric changes.

e.g. eruption of Alaska volcano made world wide abs. of solar heat below average for a year.

If there be a change of 10% in solar due to atmos. conditions then also it should be same for a planet.

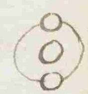
This was done & they were almost constant to ·01%. Corrections were made for unequal brightness of surface of Mars & for variation of rings of Saturn.

Variable Stars

Hopeful field for professional astronomers

3 classes. (1) Eclipsing variables (Algol)
(2) Cepheid " (δ Cephei)
(3) Long period (Mira Ceti)

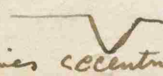
① Simple idea – 2 change in light one may be dark & one light &c regular eclipse of one by other



usually luminosities are quite diff & light curve is

if eclipse be total

if not total



This curve gives eccentricity of orbit &c but not scale of orbit.

This gives information re densities of stars.
Dynamically in gravitational units
density is $\rho = T^{-2}$
Divide between the 2 stars.

$M_1 + M_2 = \dfrac{a^3}{p^2}$ a = semi maj. axis of orbit in astron units.

Kepler's Law. p = period

Ratio of radii from light curve. call it equal $\therefore 2M = \dfrac{a^3}{p^2}$

$\frac{4}{3}\pi\rho = \dfrac{M}{r^3} = \dfrac{1}{2}\dfrac{a^3}{r^3}\dfrac{1}{p^2}$

$= \dfrac{1}{2}\left(\dfrac{a}{r}\right)^3 \dfrac{1}{p^2}$.

Hence density of star.

This has been worked out for 80 of the Algol class.

Stars of type G (like Sun) about 1

others are about .0001

This is incontravertable evidence of density less than atmosphere though spectrum analyses is similar.

Since 1925 note 4. Algol variable of very long period is ε Aurigae 27 yrs -

4 Period very short & absolutely regular one almost black wd give complete extinction distance apart very little.

Algol - period 69 hours approx with minima 69 mins

(3) Long period (Mira Ceti)

150^a. - 600
300 - 400

not absolutely regular periods.
variations 20 - 30 days from average & maximum brightness not always attained.

This is a single star (probably giant star) with an explosive "disease" at a stage of its growth.

Variation in light up to 5 mag.

Physical change - spectrum alters

(2) Cepheid.

1 or 2 mag. change.

Period short. 3 - 7 days up to 30 or 40

large no $\frac{1}{2}$ day period

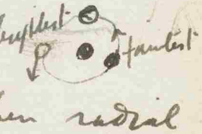
called Cluster variable.

Absolutely regular periods

Physical change - spectrum alters

Diff betw. (1) & (2)

If (2) is orbital motion

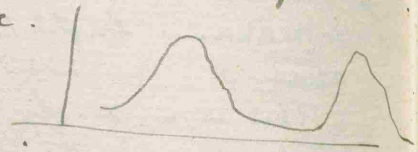


min. light must occur when radial light has min. value.

This is true for (1) but not for 2.

In (2) it is brightest when approaching & faintest when receding $\therefore$ not an eclipse one theory is that it is moving in a resisting medium hence one side bright & other dark on receding. This should give a retardation of period & this is calculated & detected for δ Cephei

Light curve.



Another difficulty in orbital theory
Calc. size of star & find it greater than orbit this O.K. as for ex Earth round moon 3000 mile rad.
but find what size orbit for one is neces. for period observed & find one is just grazing other thus "not room for binary theory" hence binary hypothesis very improbable.

Other theory Pulsating star alternately swelling & contracting.

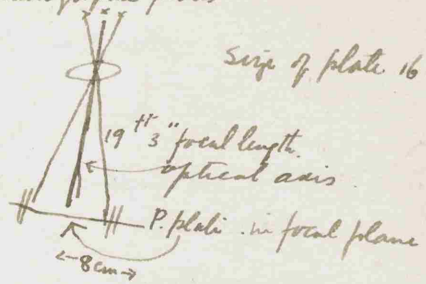
Betelgeuse? Cephei variable?

At Obs. 17/1/22.

Treatment of photographic plates.

Sheepshanks.

Size of plate 16 cm²



$\alpha = \dfrac{8}{19.5 \times 12 \times \frac{5}{2}}$ in circular measure.

α (in ' arc) $= \dfrac{8 \times 60 \times 57}{19.3 \times 30} = 47'.25'$

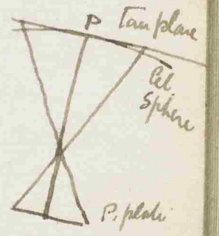
16 cm gives 94'.5' arc.

Hence field is 48' on either side of central star

Plate has cross lines numbered up to 32 hence get coordinates of any star on plate.

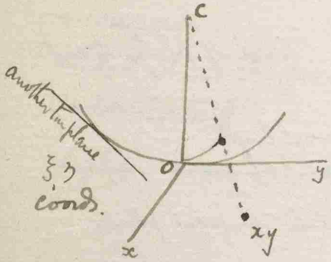
Photographic plate is similar to projection on tangential plane.

A gt ⊙ projects into a str line on tan plane



angle betw 2 gt ⊙ ≠ angle betw the projected str. lines unless the gt ⊙ pass thro. P where tan plane touches.

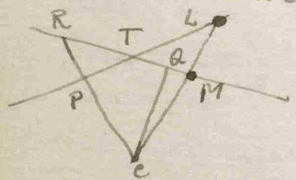
Invert diagram for simplicity



Show that $\xi = \dfrac{Ax+By+C}{Kx+Ly+M}$

$\eta = \dfrac{A'x+B'y+C'}{Kx+Ly+M}$

If same star be on both coords are related as above.



Partic case. of image of star in plane CPQ & on both plates. M on one, L on other.

PL = x
QM = ξ

Let rad. cel. sphere = 1.

$\xi = \tan QCM$. express in terms of x.

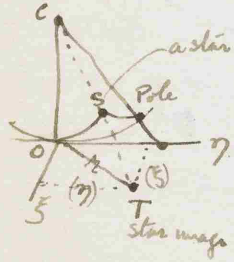
$= \tan(RCM - RCQ)$

$= \dfrac{\tan RCL - \tan RCQ}{1 + \tan \quad \tan}$

$= \dfrac{x - c}{1 + cx}$ where $c = \tan RCQ$
$= RQ$
= coord of centre of one plate on other.
= a constant for that plate.

Standard Coords. Positions on plate are affected by tilt of plate by aberration, etc. etc.

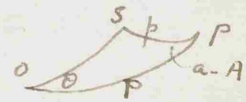
The ξη coords assuming η axis is in meridian
ξ ⊥ to meridian



To get standard coords in terms of dec + rt. asc.

A = rt. asc.
N.P.D (instead of dec) = P } O
a
p } for star

O = centre of projection (ideal pt)
OP = P
SP = p
$\angle SPO = a - A$
$\angle SOP = \angle \eta OT = \theta$ by proj^n of pt C thro O.
OT = r



$\xi = r \sin\theta$
$\eta = r \cos\theta$.
$r = \tan OCT = \tan OS$
$\xi = \sin\theta \tan OS$
$\eta = \cos\theta \tan OS$

(1) $\cos OS = \cos p \cos P + \sin p \sin P \cos \overline{a-A}$

(2) $\sin p \sin \overline{a-A} = \sin\theta \sin OS$

(3) $\sin OS \cos\theta = \cos p \sin P - \sin p \cos P \cos \overline{a-A}$.

from 1, 2, 3

$\eta = \dfrac{\cos\theta \sin OS}{\cos OS} = \dfrac{\cos p \sin P - \sin p \cos P \cos \overline{a-A}}{\cos p \cos P + \sin p \sin P \cos \overline{a-A}}$

$= \dfrac{\sin P - \cos P \tan p \cos \overline{a-A}}{\cos P + \sin P \tan p \cos \overline{a-A}}$

Let $\tan q = \tan p \cos \overline{a-A}$

Then $\eta = \tan(P - q)$ (in circular measure

$$\xi = \frac{\sin Q \sin OS}{\cos p \cos q \ldots} = \frac{\sin p \sin \overline{a-A}}{\cos p[\cos P + \sin P \tan q]}$$

$$= \frac{\sin q \tan \overline{a-A}}{\cos(P-q)}$$

To find meaning of q.

ΔSRP.

$RP = x$

SR ⊥r OP

then $PR = q$.

$\cos x \cos \overline{AA} = \sin x \cot P - \sin \overline{a-A} \cot 90°$

$\therefore x = RP$.

For reverse transformation

$$\tan \overline{a-A} \sin q = \frac{\sin \overline{a-A} \sin q}{\cos \overline{a-A}}$$

$= \sin \overline{a-A} \tan p \cos q$.

$\xi = -$

Elim. q.

then $$\tan p \sin \overline{a-A} = \frac{\xi \sec P}{1 + \eta \tan P}$$

$$\tan p . \cos \overline{a-A} = \frac{\tan P - \eta}{1 + \eta \tan P}$$

It is possible to show.

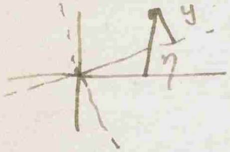
$$p = P - \eta + \frac{\eta^3}{2} + \frac{1}{2} \xi^2 \cot (P - \tan^{-1} \eta)$$

These formulae come into all Parallax work.

Astrographic Telescopes for standard work

13 ft focal length $1'$ arc = 1 mm on plate.

Thus by measuring plates & calculating standard coords correcting for orientation, refraction, temp changes, aberration (earth 30 km/sec.). In 1896 various observatories similarly equipped began to tabulate the heavens.



$\xi = x \cos\alpha - y \sin\alpha$ $+ c$

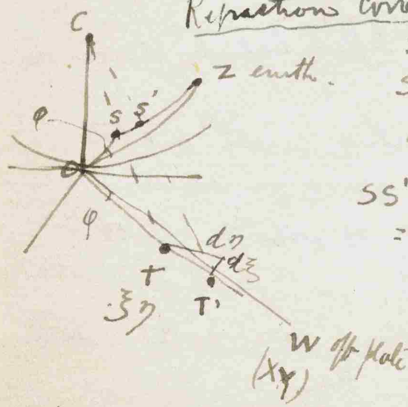
$\eta = x \sin\alpha + y \cos\alpha$ $+ f$

for orientation.

for change of origin add constants.

6 or 7 stars give sufficient measurable data for accurately determining the constants by method of least sqs.

Refraction Correction



S' is actually seen

S is real posn without atmosphere

SS' by laws of ~~diffraction~~ refraction

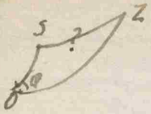
$= B \tan SZ$

$B = 58''$ arc generally

$TT' = SS'$ to first order terms.

$$d\xi = TT' \frac{Y - \xi}{\sqrt{\overline{X-\xi}^2 + \overline{Y-\eta}^2}}$$

$= B (X - \xi) \tan SZ / r$



$$\cos SZ = \cos OS \cos OZ + \sin OZ \sin OS \cos\varphi.$$

$$= \frac{1}{\sqrt{1+\xi^2+\eta^2}} \cdot \frac{1}{\sqrt{1+X^2+Y^2}} + \frac{OW}{CW} \cdot \frac{OT}{CT} \cos\varphi$$

$$TW^2 = OT^2 + OW^2 - 2\,OT\,OW \cos\varphi.$$

$$(X-\xi)^2 + (Y-\eta)^2 = \xi^2+\eta^2+X^2+Y^2 - 2\,OT$$

$$\cos SZ = \frac{1+X\xi+Y\eta}{\sqrt{}\ \sqrt{}} = \cos\psi\left(\text{———}\right)$$

where $\psi = OZ$.

Eventually

$$\tan SZ = \tan\psi\left[1 - \operatorname{cosec}^2\psi\,(X\xi + Y\eta)\right]$$

$$d\xi = \beta\left[X - \xi(1+X^2) - \eta XY\right]$$

$$\delta\xi = -\beta\{\xi(1+X^2) + \eta XY\}$$

$$\delta\eta = -\beta\left[\xi XY + \eta(1+Y^2)\right]$$

These are the differential displacements.

To get XY coords of zenith on plate.

colat $= C =$ polar dist. of Z.

$S =$ sid time $=$ RA of zenith

$\therefore$ have C & S for A & P.

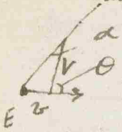
then $\tan\varphi' = \tan C \cos(S-A)$

$$Y = \tan(P-\varphi')$$

$$X = \frac{\tan(S-A)\sin\varphi'}{\cos(P-\varphi')}$$

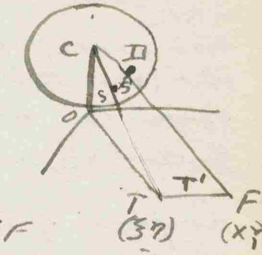
31/1/22.

Correction for Aberration similar to that for Refraction



$$\alpha = \frac{v}{V}\sin\theta$$

$C\Pi$ is v-dirn.



Displaced dirn is nearer plane of earth's way.

SD of O projects into $TT'F$

TT' is dx. measure $dx\ dy$ of it.

SD is θ.

T if no aberrn

T' with aberrn

$$\cos\theta = \frac{1 + \xi X + \eta Y}{\sqrt{1+X^2+Y^2}\sqrt{1+\xi^2+\eta^2}}$$

$$= \cos OD\,\{1 + \xi X + \eta Y\}$$

$OD = \varphi$

$$\sin\theta = \sin\varphi\left[1 - \cot^2\varphi\,(\xi X + \eta Y)\right]$$

$SS' = TT'$ almost as OC, TC are never more than $\frac{3}{4}^\circ$ apart under photo. plate condns.

Finally get $d\xi = Y_0 \dfrac{X}{\sqrt{1+X^2+Y^2}} - Y_0 \dfrac{\xi}{1+\sqrt{X^2+Y^2}}$

Hence Star at centre of plate has aber. effect of term 1. & differential effect is 2nd term.

Similarly for $d\eta$.

Scale error is a linear correction also

$\therefore$ Total of all corrections is a linear function.

$\therefore$ Say Standard coords $= x, y$ given by

$$x = \xi + a\xi + b\eta + c$$

$$y = \eta + d\xi + e\eta + f.$$

The $abcdef$ are very small quantities

Old system of working out each correction for each star now abandoned & the generalized correction only is taken.
Find abcdef from 3 or more known stars on plate (or on each $\frac{1}{4}$ of the plate – Smarts way) & from their Ra & Dec. get the coords & the constants abcdef.

Comparison of Plates at Several Years Interval

x y coords referred to dotted axes on both plates & compare for possible Proper Motion
find ξ by $\xi_1 = x + ax + by + c$
& η " $\eta_1 = y + dx + ey + f$.
This gives standard coords for old plate
Similarly for new plate $\xi_2 = x + a'x + b'y + c'$
$\eta_2 = y + d'x + e'y + f'$.
Proper Motion is given by

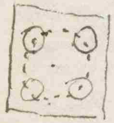
$$\xi_2 = \xi_1 + \mu_\alpha$$
$$\eta_2 = \eta_1 + \mu_\delta$$

then $\mu_\alpha = a''x + b''y + c''$
& $\mu_\delta = d''x + e''y + f''$.

Modern plates are exposed upside down to insure accurate superimposition.
In practice give one plate a small known displacement $\delta x, \delta y$, & then any star whose images have not this $\delta x, \delta y$ between them have Proper Motion.
$\Delta x = ax + by + c + p_\alpha$ if no correction abc go out

leaving p_α only. Similarly a p_δ.
Probable error in P. Motions .6" per century.
Old Transit ⊙ method very slow.
1 centuries work done in 5 yrs with the Sheepshanks –
Similarly for Parallax (Formula worked out by Sir Rbt Ball –

7/2/22



select say 10 stars (faint so that their P.M will be negligible more or less on circum. of a ⊙.

$ax + by + c = \Delta x$
⋮
$aX + bY + 5c = \Delta X$

Smart took 3 groups of 5 – from 3 sets solve for a.b.c. add

a = 3.63
b = 2.62
c = 555

X = 12.6, Y = 9.2 } are coords.
ΔX, ΔY } is displacement of 1920 plate from 1900 plate.

If ΔX is greater than average ΔX the excess is P.M.
Let $ax + by + c = \Delta x - \mu\alpha$
or $-\mu\alpha = ax + by + c - \Delta x$
Star No. 3. Δx was 613
$-\mu_\alpha$ = 46 + 24 + 555 – 613 = 12 scale units.
This is 1" a century with prob. error $\frac{1}{2}$" century

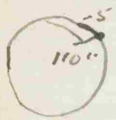
Star No 7. X = 21 + Y = 9.1 Δx = 707. –49 or nearly 5" century

Star 34
η Cassiopaea. X = 17.0 ΔX = 1782
Y = 23.3 ΔY 453
$-\mu_\alpha$ = 1104 μ_δ = 529

Interval betw plates 17.6 yrs.
Unit of scale = 0".0176 arc.

$\therefore$ per century $\mu_\alpha = \frac{1104 \times 176 \times 10^{-4}}{176 \quad 10^{-3}}$

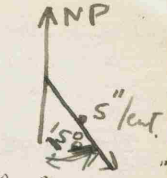
= 110.4 sec. per century = 2 min. approx.



μ_δ = 52".9 /century
i.e. 52" nearer pole
& R. Ascen. changes. 220 or 210" of arc per century approx
i.e. 14 sec. time earlier per century

Boss's Tables of P.M. from transit ⊙ obs. extending over a century — considered correct to 0".6 per century.
2 days plate measuring + calculating confirmed Boss's result absolutely.
In comparing R.A's + decs. of diff. years change in posn of Equator due to precession must be allowed for.
Constant of precession is derived from obs. of transit of bright stars .i.e. ∴ P.M. of any partic star is relative to mean motion of bright stars.
These bright stars are in 2 drifts + a stationary group.
Hence advantage of Photographic Method.
Mr Smart took a plate of Pleiades last summer
They are a moving cluster but some have a P.M. of 5 or 6 + some

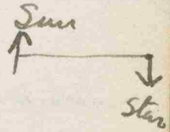
1 or 2" /century. Group motion
This gives means of telling which are actually belonging to the cluster

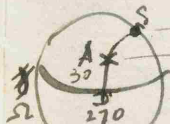


P.M. mov 5.3"
Photo 2.3"

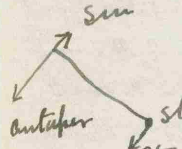
Occasionally a faint star has a great P.M. (18"/cent) + is fairly near)

Parallactic Motion. Movement of Sun in Space.
If star head on no change.
" " ⊥r to Suns Way
Stars posn alters



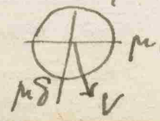


Solar apex. Opp dirn or Antapex



Star displacement along great ⊙ SA

R.A's of stars are neg.
η Cass P.M. 1100
other stars about 40 i.e. $\frac{1}{25}$ PM of η Cass.
If this is all due to Solar plx. then other stars is 25 times further away than η Cass.
V is Stream Vector
Plotting μ_α, + μ_δ for a groups – if they



group themselves along one dirn it shows a stream.

Δx

$a = 3.63$
$b = 2.62$
$c = 555$

Δy

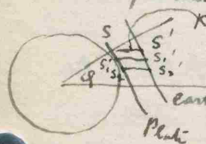
$a_1 = -0.57$
$b_1 = +4.56$
$c = -20.$

Star No 48 . $X = 27.0$ Δx 783
$Y = 28.8$ Δy 100

$$-\mu_\alpha = ax + by + c - \Delta$$
$$= 98.01 + 75.456 + 535 - 783$$
$$728.466$$
$$-783$$
$$-54.534 \text{ or } -55$$

$\mu_\delta = -4$. use 5-figure multiplication Tables -

Notes on previous lecture by A.S. Eddington copied from C. Payne

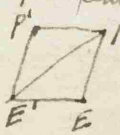


Effect of Earth's way on plate - size altered + plate tilted.
Latter is a 2nd order change
Distance is 1st order effect.

$\varphi = \angle$ between plate centre + apex of ⊕'s way
thus distance corresponds to $1 + K\cos\varphi$
+ size on plate to $\frac{1}{1+K\cos\varphi}$. Field is shifted.
Correction for tilt of plate is negligible.

$\xi - x = ax + by + c$. $\eta - y = dx + ey + f$

Transformation formulae are obtainable.
Linear interpolation for refraction + aberration
All this holds for nova + single stars - but not for planets or comets since we get E'P



+ we want EP or E'P'
necessary to date back the obsn. so as to get E'P'

E'P' is now published under heading "Astrographic place".
Generally use at least 10 ref. stars on a plate.
Plates are 2° sq. and there are 40,000 sq. deg.
∴ 100 000 ref. stars wd. be reqd. This is a large demand -
(1). Proposed new plates standard 10° square.
(2). about 6000 intermediate stars, others referred differentially.
All these have themselves a P.M. Thus historical stars must be worked in

Clock Stars

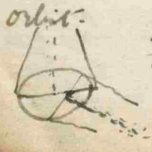
There is a periodic error partly due to seasonal effect + diurnal clock variations
Erroneous P.M's introduce a "wave" as time goes on. Observe clock stars for 12 hours + take mean - this gives true error approx.
Daylight observations involve errors -

Measurement of Parallax of Stars

Obsy. 11/2/22.

1. Sheepshanks - Camb. Method

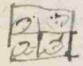
Hincks + Russel 1905-6. Réseau mesh system of standard lines are printed onto photo. plate. + the coords. of parallax star are thus obtained relative to same imaginary axes.
Major parallax when Earth at opp. ends of the orbital axis which is ⊥ to direction of star
This is not necessarily the major axis of orbit.



The times when earth is at extremities of the reqd. axis may not be favourable for photo. then take obs. at other time + calculate the major parallax from the lesser values

2. Greenwich Method. They have a very fine ruling machine giving the axes relative to which the coords. are taken. There are 7 observers working on their observations

3. Oxford. They select a dense part of sky & take 10 consecutive plates of it twice a year giving about 150 star images. By reso line method they measure up the positions. but it is an awfully slow process as they do everything by Least Sqs. instead of by Smart's short cut of evaluating constants from 3 or 4 groups symmetrically placed as far as possible.



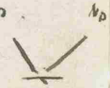
Oxford has just published result of obsns taken about 10 yrs ago.

Parallax + P.M. may be of same order. So that a year or two of P.M. may double or obliterate parallax - hence need for accurate knowledge of P.M.

When taking photos thro. glass (i.e. plate reversed) 3 times the exposure is reqd.

R.S.E. 18/2/22 Caelostats.

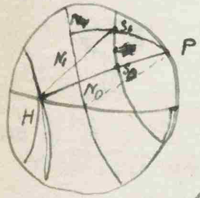
1. Siderostat - Coudé like Sheepshanks fixed with axis towards N. Pole. Limited to certain parts of sky.



2. Caelostat proper. Polar axis in plane of mirror

normal to mirror is always on eq.
Star in posns $S_1 S_2 S_3$ + its light must be brought to a pt ~~~~ H. cutting cel. eq. at $N_1 N_2 N_3$ where N_2 is midpt of HS_2

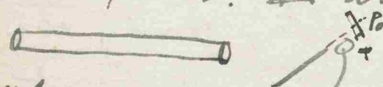
∴ drive Caelostat at rate that normal follows posn of N. i.e. $\frac{1}{2}$ rate of star i.e. once in 48 hrs.



Setting in R.A. is done in mirror. Setting in dec. is done by setting telescope

This is specially used for temporary erections in special eclipses etc. Not very good for spectroscopic work.

Lay tel. horizontally on piers or packing cases. + drive mirror by a clock



N.B. In S. hemisphere direction of rotation is altered.

Not necessarily exactly horizontal - a slight downward tilt is more convenient

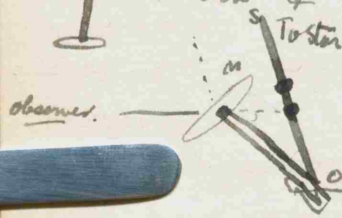
Necessity of setting plane of mirror in polar axis. Mount a small theodolite on axis as T. + thus see if star is in line. Azimuth ~~error~~ error in mounting mirror is thus detected also altitude.

Object on meridian gives former.
Star at 6-hour angle gives altitude.

3. Heliostat. gets over difficulty of (2) of having to shift everything for change of declination

This gives double motion to mirror.

mounted on a pillar with rotation motion & tilt —



Incident ray ∥l to fixed dirn of star. & ray emerges horizontal.

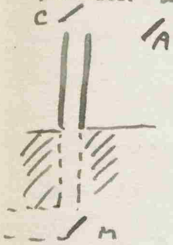
Adjust SO to star & sheaf action at O alters dirn of mirror M

Drive like an equatorial 24 hrs.

Delicate easy sliding jointing at O hence less easy to set up than 2 but more useful when fixed up well.

One disadvantage is that field is rotating which is not case with caelostat. ∴ latter better for photography but for spectroscopy this is not so essential as very short exposure only is reqd & ∴ heliostat is preferable for this.

4. Tower Telescope. Vertical beam instead of horizontal. (this obviates errors due to vertical air currents etc.)



Put caelostat mirror on top of tower C & another adjustable mirror at A to ensure 2nd degree of freedom.

Mt. Wilson is like this ~~with~~

Another mirror M directing rays into a constant temp chamber is being ~~set~~ up at Potsdam — Result not known. Eddington is doubtful of its success —

Historical.

Bradley — Greenwich — first observations with any precision. (Just before him Halley —) about 1760.

Auwers re-calculated Bradley's values for Proper Motions & published about 2000 stars. 1890?

Groombridge's Catalogue (4000) up to 8th mag. & up to 15° Pole — an amateur at Blackheath —

Astron. Gelschft. Catalogue. 1880. down

to 10th magnitude - covering almost all N. hemis.

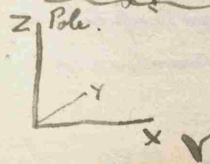
Boss Prelim. General Catalogue - best to date. 6000 stars well scattered & with reliable accuracy -

Up to 1905 Solar Motion was the aim of these observations - i.e. Sun rel. to stars or stars rel. to Sun. There is an obvious tendency in one direction noticeable in any group of 10 or 12. (N.B.)

Sir Wm Herschell - 7 stars known with fair accuracy gave him the first determination of Solar Motion.

Halley - was first to detect P.M of a star i.e. Arcturus. He took Lat. + Long. (latter may be due to precession but not former) & found both had moved -

Eqn for Solar Motion



If star has a vel. XYZ rel. to us it becomes form eqns

$$-X \sin\alpha + Y \cos\alpha \left[+\delta m + \delta m \sin\alpha \tan\delta = \mu_\alpha\right.$$
$$-X \cos\alpha \sin\delta - Y \sin\alpha \sin\delta + Z \cos\delta = \mu_\delta$$

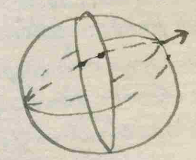
2 eqns for each star & solve by least sqs. or group stars in small region & take mean values instead of for each star.

Constant of Precession comes in & reqs. an unknown. [ ] to be determined also from obsns since mom. of inertia of earth not known well enough.

$$\left[X \cos\alpha \cos\delta + Y \sin\alpha \cos\delta + Z \sin\delta = V\right]$$
Alternative method of Airey.

Near stars come in with very great weight. exclude any evidently exceptional stars i.e. of 400" instead of 4".

Kobold 1st attempt.
find pole of that small arc of P.M. & so for all - Thus get plane of greatest flattening & ⊥ to this the dirn of Solar Motion



Airey → 270° +34
Kobold → 270° -20

Astrophysical Journal - Vol 48. p 205
Monthly Not Vol 77 p. 16 596
79 p. 2
Cepheids " Vol. 81 no 4
Russel on Stars Nat. vol 93 p. 227
252
281

Presidential Address of Prof. A S Eddington MA., M.Sc., FRS.
to Brit. Assoc. on Cardiff 1920
The Internal Constitution of the Stars.

Diffuse gaseous stars are called "giants" & are the most powerful light-givers - 3/4 of naked eye stars including Aldebaran, Canopus, Arcturus, Antares. The older dense stars are dwarfs.
Star life begins effectively as a low-temp. giant of M-type, red star; contracts with rise of temp, to a maximum pt. depending on its mass; then as it further contracts it becomes cooler & is a dwarf from O downwards. Its luminosity is much less on downward path - yellow stars. "Bloated giant has far larger surface than compact dwarf & gives correspondingly greater light". Also definite spectral differences exist distinguishing the age. Giant star has material + ethereal heat energy former ∝ temp, latter ∝ temp.^{4} & in these stars the two forms are about balanced due to very low density & high temp. "It is as though the stars had been measured out, their sizes determined, with a view to this balance of power.... a deep significance in the evolution of the cosmos into separate stars".
Thus the problem is not how the energy is brought to the surface, but rather how it is held back. It is radiative equilibrium, not convection. and the outflow of ethereal energy is a wind (ether waves have mass & momentum) with a mechanical effect opposing gravitation - called radiation-pressure.
Linking observational data (masses, densities & total radiation) with theory we can obtain the approx. value of the average molecular wt. and the opacity or permeability to radiant energy.
Wave length from stars is like soft X-rays 3-30 Angstroms
Since in star atoms wd. be highly ionized the average wt., in limit of all electrons outside nucleus being free, is about 2 as lower limit for upper limit 200 or ∞. These give limits of mass absorption coeff. 10 to 130. Other data point to 35 C.G.S. units or av. mol. wt. 3.5
This is of same order as found in lab. for hard X-rays & explains ability of star to store energy. But the opacity seems to be the same for all stars, hence the absorption coeff must be approaching a limiting value - a saturation effect, never coming into play in comparatively weak intensities of lab. experiments; but present in stars. Barkla suggested scattering would account for opacity but on all available data the opacity is definitely higher than scattering could produce.
Opacity being constant, the total radiation of a giant star should be a function of its mass only indept. of its temp or density. This total radn. is measured by the luminosity & is found observationally to remain constant throughout the giant life
Russell having based his giant & dwarf hypothesis partly on fact that luminosity is nearly the same for all giant stars of every spectral type. Thus ranges of mass are found to be in a ratio 3:1

for practically all the giant stars. Extending the reasoning to dwarf stars & considering the deviations from gas laws finding the reqd constants by the case of our Sun, it has been shown that probably a star must have ½ mass of Sun to reach the temp. of the lowest spectral type M and 2½ mass of Sun to reach the hot type B.
[O B A F G K M.] The theory also gives the
dwarf ← → giant
probable diff. in brightness of the giant M & dwarf M & this agrees with obsn very closely. Though impossible to check this theory — It is also impossible thus far to disprove it by all the traps set for it. "If it lies it has shown some discretion in lying without being found out.

"The greatest need of stellar astron today is a means of measuring the apparent angular diam of stars" - at present a theoretical calculation from surface brightness. Hopeful trials at Mt Wilson range finder arrangement. Probable order will be as calculated Betelgeuse ·051" Antares ·043" etc. Aldebaran ·022" Arcturus ·020" Pollux ·013" Sirius ·007"

The proportion of the weight of a star supported by radiation-pressure depends solely on the stars total mass & molecular wt., not on density or opacity.

Av. mol. wt. 3,	mass ½ Sun, proportion supported by radn pres. is	·044
" " " "	5 × Sun " " " "	·457
" " 5	½ Sun " " " "	·182
" " "	5 × Sun " " "	·645

Thus between limits ½ Sun to 5 × Sun the radn press. passes from insignificance to preponderance and probably 90% giant stars fall within these limits. This points to the probability that a globe of larger mass would become unstable with a small rotation or perturbation & is break up & continue to divide until masses within the limits of stability were reached. Thus the prediction that the material of the stellar universe will aggregate primarily into masses chiefly lying between 10^{33} and 10^{34} gms & this is the magnitude of the masses of stars observed astronomically. Narrowing the theoretic limits still further: take mol. wt. 3.5 assume most crit. condn is when ⅓ grav. is counterbalanced (by analogy with rotating spheroids when centrifugal force opposes gravitn & creates instability) then critical mass is just twice mass of Sun & stellar masses may be expected to cluster round this value.

Source of heat from Sun & Stars — Contraction theory of Helmholtz insufficient. Kelvin showed it dated our Sun to 20 million yrs, an age now "treated with no more respect than Archbishop Ussher's" since Geo. Darwin, Rayleigh & others showed several hundred million was nearer the truth. Applying this old theory to stars it points to formation of the giants only 80,000 yrs ago. "The telescope reveals to us objects not only remote in distance but remote in time. We can turn it on a globular cluster & behold what was happening 20,000, 50000 or even 200,000 yrs ago & as Shapley says the verdict appears to be "no change". It is thus difficult to resist the impression that the evolution of the stellar universe proceeds at a slow, majestic pace with respect to which these periods of time are insignificant." The contraction hypothesis is further insufficient to explain the data from the Cepheid pulsation variability

as an alternative consider the liberation in the star of sub-atomic energy. Experiments of Rutherford & Aston point of H being the basis of other elements with electrons binding the H atoms together. Thus He is 4 H bound by 2e though mass of He, 4, is not 4 × mass of H (1.008) but less by 1 in 120 approx.
"Mass cannot be annihilated & this deficit must represent the mass of electrical energy set free in the transmutation."
"If 5% of a stars mass consists initially of H atoms which are gradually being combined to form more complex elements the total heat liberated will more than suffice for our demands & we need look no further for the source of a stars energy"
"Sir E Rutherford has recently been breaking down the atoms of oxygen & nitrogen, driving out an isotope of He and what is possible in the Cavendish Lab. may not be too difficult in the Sun." Stars are probably crucibles in which the lighter elements which abound in nebulae are compounded into the more complex & varied elements needed for a world of life.
"The radioactive elements must have been formed at no very distant date; their synthesis, unlike the generation of helium from H, is endothermic. If combinations requiring the addition of energy can occur in the stars, combinations which liberate energy ought not to be impossible.

"In ancient days two aviators procured to themselves wings. Daedalus flew safely thro middle air across the sea & was duly honoured on his landing. Young Icarus soared upwards towards the Sun till the wax melted which bound his wings & his flight ended in fiasco. There is something to be said for Icarus – he brought to light a constructional defect in the flying machines of his day. Cautious Daedalus will apply his theories where he feels most confident they will safely go, & their hidden weaknesses cannot thus be brought to light. Icarus will strain his theories to the breaking point till the weak joints gape. But if he is not yet destined to reach the Sun & solve for all time the riddle of its constitution, yet he may hope to learn from his journey some hints to build a better machine"

23/2/22.

Special Lecture at Newnham Sci. Soc.
by Prof. A.S. Eddington on Relativity:

From the 19th century we inherited our ideas of space & time being independent & absolute. With the refinements of measurement it became possible to detect a small interval of time which would elapse between the theoretical time of return of a beam of light sent transverse to the earth's way and one sent in the dirn. of the earths way, but no delay was found to exist by Michelson & Morley & other experimenters following in similar tests. i.e. the earth's motion through space has no effect on a beam of light transmitted through space. (and is not detectable *①) Lorenz put forward his Contraction theory. i.e. as the basis of matter is electromagnetic it is assumed that a "wind" of ether striking matter end on compresses it into less space than it would occupy if sideways on. This was not entirely satisfactory or sufficient to meet the other troubles arising from consideration of Time.

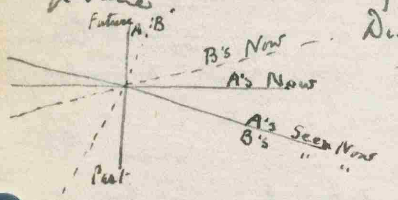


Diagram represents the mathematical construction of the "Now" lines of 2 observers A & B who have an instant in common & ∴ a line of events "seen now" in common. B is on a star moving with a different velocity from A through space so the Now lines cannot coincide.

Consider the measurement of dist. between two points. If stationary & ruler is stationary then one length, but if moving like a ray of light & ruler is fixed to earth it is a different set of circumstances & the length is not relatively the same.

*③ In measuring things on the Earth we are all agreed but if measuring points in the universe our measurements relative to Earth wd. not be the same as those of an observer on a star making his measurements relative to his star. Thus we must break away from our geocentric conception of the Universe which we erroneously thought we had laid aside at the time of Copernicus.

Newton stood on the earth with the molecular pressure bombarding him upwards & saw an apple fall & stated the law of force with which he viewed its descent. Einstein has taken the point of view of the apple & stated the law of force with which it would view Newton apparently being projected up towards it with the Earth shoving him from below. For consider a man fallen from an aeroplane with an apple in his hand on which he lets go his grasp. It does not fall below him but remains right there at his hand. It is in no field of

force relative to him. Thus Einsteins restatement of the law of force is given as relative to some set of axes of reference and as both space + time are relative it is necessarily expressed mathematically in four dimensions. It includes all Newton's laws of momentum, energy, gravitation etc in one very complex, but symmetrically beautiful set of equations of partial differentials.
All efforts of deduce the vel. of the earth through the aether from the Maxwell Electromagnetic equations fails by the elimination of the unknown quantity leaving the relation 0 = 0 to proclaim that there is no use in asking a mathematical equation a question which in the nature of things it cannot answer.

Thus the relativity theory takes us out of Euclidean geometry into a world of non-euclidean geometry which, being 3-dimensional beings we cannot visualize. Einstein does not consider the ether no longer essential but subject to "puckers" or curved in a fifth dimension - it is this curvature which appears to us as a gravitational field of force. Lengths adjust themselves automatically with the optical properties so that no change is measurable no matter how delicate the apparatus for it is all subject to the same modifications. Expressing Einsteins law in words it becomes. The radius of curvature of the universe is a constant both in time + space. or conversely: a given length or rod of measurement referred to a given frame of reference is always a constant fraction of the curvature of space.

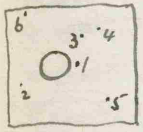
The confirmations of Einsteins hypothesis are the observed deflection of a ray of

light in passing the Sun; the correct predict[ion]
of the movement of Mercury's perihelion
&, though still rather in doubt, the crowding
of the Fraunhofer lines toward the Red
end of the Spectrum; also the extreme
beauty of the mathematics.

Geometry is essentially an experimental
science - Euclidean works accurately
enough on the earth & has therefore
been adopted but that does not
prove it applicable to the Universe.
A very true analogy is the relativity
in value of money exchanges
A £1 is a £1 in England but
due to various causes its value
changes when comparisons with
other countries are made.

Astronomical observations on the Effect of
Gravitation on Light. Observatory 20/2/22.

Possibilities (1) no displacement
(2) Newtonian deflection 0".87 on corpuscularity of light
(3) Einstein deflection 1".75 × $\frac{2}{r}$.
Two Expeditions: Sobral & Principe



Weight of any other observations almost
negligible.
Superimposition of images if possible
with perfect conditions would be the
most satisfactory.

Star ① was lost in corona.
Assume for (5) - anticipated deflection 0".65
1" = 60 scale div.
∴ 0".017 = 1 scale div of measuring machine
With perfectly black images we cannot hope for
less error than 6% or 7%. With less good
images up to 20%. Beyond (6) the prob.
error is of same order as effect to be
measured.

Instruments used

A. Greenwich Exped^n to Sobral.
I Greenwich astrographic object glass with a
coelostat arrangement used.
Trouble originates in action of ☉ on coelostat
mirror - This spoiled the work.
II Dublin lens - used with an 8 inch coelostat
Took 19 plates in Greenwich telescope
" 8 " " Dublin "
Of latter 4 gave a large scale photograph
Stayed 2 mos. to get a set of check plates of
same fields of stars.
One was thro glass the rest direct. Former

was treated as standard plate against which
the eclipse plates + check plates were measured.
4" images from pt. of view of measurement were
nearly perfect.
Δx, Δy were measured + if there were no
displacement $\Delta x = ax + by + c$ rigorously
allowing for Einstein term
$\Delta x = ax + by + c + \alpha E_x$ where E_x is
displacement at 50' from centre of ~~Sun~~ ☉
50' being mean for stars measured.
Also $\Delta y = dx + ey + f + \alpha E_y$.
Solve these eqns completely for all plates
Range of α for 7 plates. 0.073 to 0.148.
mean 0.120
Using same formula for check plates
α is −0.015 to +0.042.
mean 0.015
This indicates images or measurement bad
Thus α = 0.105 ?
From Y values 0.111 to 0.319
Checkplates. 0.007 − 0.060 mean 0.129
Combining x with y, α with δ get
α = ·100 in micrometer = 0".625 at 50'
Reduced to limb of Sun = 1".98
Probable error 4-in results. 6%
No systematic error of any account.

R.A.

	Obs.	Calc.
11	−0".19	−32
5	−29	−31
4	−11	−10
3	−20	−12
6	+10	+4
10	−8	+9
2	+95	+85

Dec.

4	+83	+74
5	+100	+87

II Astrographic Telescope.
Images diffused + out of focus. effect of ☉'s heat
on mirror Images not symmetrical.
gives 0".93 on limb.
Prob. error. 20% — Rejected.

Principe Expedn. Oxford Astrographic 13" lens
Plates K L M .... Z taken but only
X + W showed stars. Measured by
superimposition with Oxford check plates
Results turned out extremely well on above
basis.
1".61 ± 0".30 Confirming Einstein's Theory.
Displacement is thus radial

Mercury Perihelion data from this Theory
gives value 29" instead of 43" per
century —

Clusters 28/2/22.

Ord. method of getting distance by taking
plate at 6 mos interval is no good due to
extreme distances.
Globular clusters. like our stellar system?

Universe  " all glob. clusters are in one half
2. also none near Milky Way —
3. All coming towards Milky Way
at 500 km/sec Average.
Whereas spirals are moving from Milky Way
at 1200 or so km/sec Av.

1/ Take angular diams & assume all have same no. of stars & density: & ∴ get relative distances away –

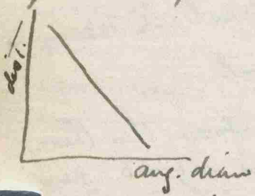
2, take mean mag. of say 30 brightest stars

cluster 1 say 15.5 then say ② is

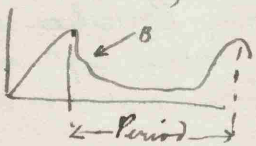
" 2 " 16.5 so many times

further off than ① – this again assumes kindred nature & absolute size.

Comparing the results of these 2 methods an agreement wd. tend to confirm truth of assumptions.



3 Cepheid Variables (5 days) (like Pole Star)



3 The bump B is a definite characteristic of the curve & cannot be explained by eclipse theory. Jeans $A\cos^n(t-T) + B f(t-T)$

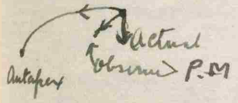
where f is same for all cepheids but A + B are determinable for each. An explosion in the star at intervals.

or Pulsation theory – tested by movement of lines in spectrum.

[β Lyra Eclipsing variable – displacement of lines leads to idea of a 3rd body –]

Knowing the app. mag. & parallax then the abs. mag. is deducible – this has been done for many cepheids.

All stars wd. appear to move towards Solar Antapex if no Proper Motion of its own.



Average for all stars

$\upsilon = 0''.002$ 1"

$S = 0''.018$ toward antapex

Abs. mag. is size of star were at unit distan...

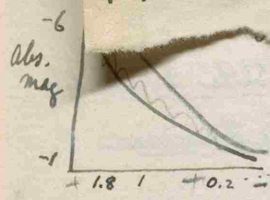
our S...

to nas...

πάντων μέτρον ἄνθρωπος

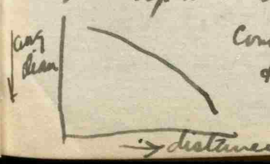
ῥητά καὶ ἄρρητα

Demosthenes



find its app. mag by looking at it or taking a plate and its period by obsn for a month and read off the abs. mag. from chart & thus get its distance.

In the Clusters there are Cepheid Variables. Consider Cluster ω Centauri etc. Often up to 100 variables in one cluster.



Construct curve for say 20 clusters & apply to cluster "21" & compare with other determinations ① & ②

343

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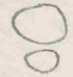
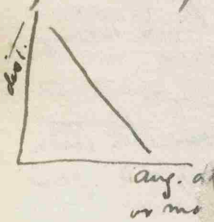
2/ Take mean mag. of say 30 brightest stars cluster ① say 15.5

" ② " 16.5



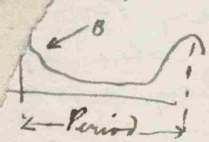
Then say ② is so many times further off than ① – this again assumes kindred nature + absolute size.

Comparing the results of these 2 methods an agreement ... to confirm truth of assumptions.



... variables (5 days)

... stars)



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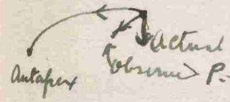
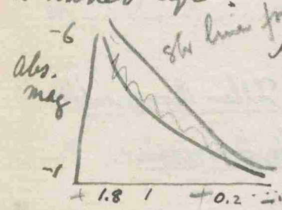


Average for all stars
ν = 0".002 1"
S = 0".018 toward antapex

Abs. mag. is size of star were at unit distance i.e. 0.1" arc parallax – then our Sun wd. be 5 mag. or just vis. to naked eye

See Spencer Jones Gen. Astr. p. 343



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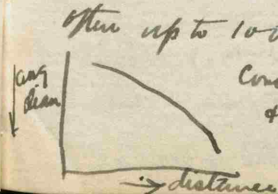
In the Clusters there are Cepheid Variables

Consider Cluster ω Centauri etc.

Often up to 100 variables in one cluster



Construct curve for say 20 clusters + apply to cluster "21" + compare with other determinations ① + ②

nearest globular cluster is about 6000 parsecs distant. Distant ones about 70,000 parsecs

<u>1 parsec = 3 light years</u>.

Are we a globular cluster or the main part of the Universe?

Motion of clusters towards M. Way.

& open clusters (like Pleides in M. Way

There are about 83 glob. clusters no more discovered since Herschel's day.

The extreme distance of these clusters is upheld by there being no detectable P.M.

<u>Notes from Eddington's Stellar Movements</u>
<u>& the Structure of the Universe.</u>

27/3/22

<u>Ch. 1.</u> Principal data are.

1. <u>Apparent posn</u> in sky.
2. <u>Magnitude</u> or <u>apparent brightness</u> measured on logarithmic scale. 6th mag fixed traditionally each mag above is indicative of 2.512 times more light + 5 mags. corresponds to $100 = (2.512)^5$ times the brightness

$$\log_{10} \frac{L_1}{L_2} = -0.4(m_1 - m_2)$$ where

L_1 L_2 are intensities of light from 2 stars of mag. m_1 m_2

Classification is photometric or visual & photographic.

The intrinsic <u>brightness</u> or <u>abs. luminosity</u> is calculated from the app. brightness if the distance be known i.e. the parallax ω''.

If L be the abs. luminosity

$$\log_{10} L = 0.2 - 0.4\,m - 2\log_{10}\omega$$

Take luminosity of Sun as unit

[Sun's magnitude in stellar units wd. be $-26.^m1$ [see 9 pages further on]

$M_{abs} = m + 5 + 5\log\pi$

3. <u>Type of spectrum</u>: Draper series (Harvard)

O bright bands on faint continuous background helium & hydrogen just showing. Very hot

B (Orion type, Rigel &c.) He predominates

A (Sirian type) Intense Balmer H series

F. (Calcium type) Ca lines intense H less intense

G (Solar type) numerous metallic lines appear

K Metallic lines quite outweigh H

M (red stars. Betelgeus &c) Titanium oxide flutings become marked.

N (red) very akin to M but shows in addition flutings of C compounds

Sir Norman Lockyer says there are stars of ascending & descending temp in each class

King & Schwartzschild have classified according to Colour Index

4 <u>Parallax</u> - angle subtended by one astron. unit (i.e. radius of earth's orbit) at distance of star. This periodic displacement is superimposed on the

uniform proper-motion of the star, but readily
distinguishable from it. Only certain to 0".01
Unit of stellar distances is 1 parsec
or dist of a star having a parallax of 1".0 arc
This is equal to 206 000 astron units or about
3 light-years.

5- Proper-motion - apparant angular motions
Greatest known is C.Z. 5^h 243 9^{th} mag Kapteyn at Cape
Obs? 870" per century which would carry it
over an arc equal to Orions belt in 1000 yrs.
Probable error 1".0 per century is satisfactory.

$$\text{Linear speed} = \frac{\text{annual proper motion}}{\text{parallax}} \times 4.74 \text{ km/sec}$$

6. Radial Velocity - vel. in line of sight - measured
by spectroscope on Doppler Principle of
displacement of lines towards red or violet
according as vel. is from or towards earth.

Mass & density - This is deduced for certain
stars which are binaries (1) Visual binaries
(2) spectroscopic binaries (3) eclipsing variables
It is estimated that probably $\frac{1}{3}$ of the
stars are really binary.

Ch. 2. Lens- or bun-shaped system of stars
about the galactic plane around & on which
is milky way.

galactic plane

Sun just off centre
Globular clusters are mostly right outside our
system. The galactic plane is the median plane
of our system.

Stars of each spectral type seem fairly well distributed
throughout the bun though the early types may cluster
nearer plane. 3 classes of motion are
detectable thoroughly intermingling in bun
Stream I moving towards R.A. 94° Dec +12.
" II " " 274 " -12
" 0 apparently stationary.
Stream I moves faster than Stream II, 34 and 19 km/sec & has
more stars in it. in ratio of 3:2.
Relative to Sun whose vel. is 20 km/sec.
towards R.A. 270° Dec. 35° Stream I moves
towards R.A. 91 Dec -15° & Stream II. RA 288 Dec -64.
Stream 0 contains most of the Orion B type & some
of all types.

There is a regular progression in the size of the
linear motions from the youngest to the oldest
stars. Average vel. 6.5 km/sec for Type B.
" " 17 " " " M.

Eddington favours hypothesis that in primitive state
the star forming material was scattered much
as the stars are now - densely along galactic
plane & more thinly at distances from it
Where it was rare small stars developed rapidly
& have acquired large velocities in falling in
towards plane. Where it was rich large stars
which evolved slowly were formed & not having
fallen far in have low velocities.

No stream motion appears in Types B_{0-9}
Max.m " " " A
Less & less " " " F, G, K M N
due to more & more haphazard motion of these stars.

due to distance at wh. formed & ∴ less parallelism to galactic plane in their motion.

Beautiful star clusters - some surrounding us, some apparently outside our Sun but all in one hemisphere & in part of Sky of Sagittarius & Ophiuchus near a brilliant patch of Milky Way "undoubtedly the most extraordinary region in the Sky"

"It is the "white" spiral nebulae which it is thought are outside our stellar System & possibly themselves stellar systems coequal with our own.

Ch. 3. Table of 19 nearest stars provides data fairly representative of stellar system in some cases i.e. Large no. of binary stars 8/19 & very different in other cases.
Note that Arcturus is 150-350 × as bright as the Sun Antares 180 × , Rigel + Canopus 2000 × . But Sun is probably not far below average because in all catalogues the brightest naturally predominate unduly connection between speed & luminosity is indicated.

	Luminosity	Mean transverse speed
9 Brightest stars	48. - 0.25	29 Km/sec
10 Faintest stars	0.1 - 0.004	68 " "
19 nearest stars to Sun		

Ch. 5 Solar Motion. absolute motion is indeterminate as no fixed frame of reference.
Convention considers the flock of stars as a whole to be at rest i.e. centroid of stellar system fixed i.e. centre of mass or rather of mean position.

Position of Solar Apex RA. 270°.5 Dec. +34°.3 N?
from Boss 6188 stars prop. motion
W.W. Campbell 1193 " radial vel. 268°.5 +25°.3
Speed of Solar Motion 19.5 Km/sec. (±.6)

Evidence that the Dec. increases with diminishing mag. for later spectral types but Boss' value seems best average. 38 $\rho\ \delta\ \psi/2$
Annual motion of Sun = 4 times radius of Earth's orbit hence value as a base line for parallax observations

Abs. Mag. + Space Velocity. W.S. Adams &c Astrophys Jr. 1921.

Solar Apex. 270°.9 A.R. +29°.2 Dec.
V = 21.48 Km/sec.

The formula is derived $\pi = K \dfrac{\mu}{T} 10^{0.2(M-m)-1}$
where π = parallax $K = 4.737$ Km/sec.
μ = geom mean of p. m.
T " " " tangential motion not corrected for Solar motion.

Assume solution of relation between M & v to be
$\log \bar{v} = a + bM + cM^2$

Results. All types

Space vel.	$\log \bar{v}$	$= +1.408 + 0.0632M - 0.00240M^2$
Tang. "	$\log \bar{T}$	$= +1.256 + 0.0870M - .00303M^2$
Av. Radial "	$\log \theta$	$= +1.219 + 0.0321M + 0.00020M^2$

The variation of vel. with spectral type among giant stars is fully confirmed both by space velocities & radial motions. Among the dwarf stars there seems a doubtful relation. If any exist it is a reversal of effect found for giants .. i.e F + G stars more rapid than K. M stars.

1350 stars F. G. K. M considered show increase in Av. space-vel for decrease of 1 mag. in brightness varies with type but is of order of 3 Km/sec.

Eqns. $x = V\cos\alpha\cos\delta - \frac{K}{\pi}(\mu_\alpha \sin\alpha\cos\delta + \mu_\delta\cos\alpha\sin\delta)$
$y = V\sin\alpha\cos\delta + \frac{K}{\pi}(\mu_\alpha\cos\alpha\cos\delta - \mu_\delta\sin\alpha\sin\delta)$
$z = V\sin\delta + \frac{K}{\pi}\mu_\delta\cos\delta$

where $x\ y\ z$ are vel. components in equatorial system in km/sec. V radial vel. rel. to Sun α, δ are rt. as & dec. $\mu_\alpha\ \mu_\delta$ corresponding prop. motions, π parallax & $k = 4.737$ km/sec

If $x_0\ y_0\ z_0$ are the vel. comp. of Sun referred to centroid of a large no. of stars

$x_1\ y_1\ z_1$ of star relative to this centroid are $x_1 = x + x_0$ $y_1 =$ $z_1 =$

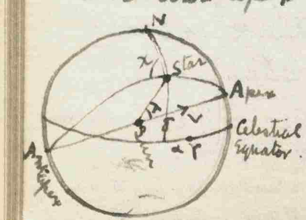
and space vel. of star corrected for solar motion is $v = \sqrt{x_1^2 + y_1^2 + z_1^2}$

1/5/22.

1/5/22 Theoretical considerations in regard to Statistical investigation of relation between abs. mag and linear vel. ⊥r & ∥l to line joining star & Solar Antapex for giant stars ($M < 3.4$) of Spectral types $G_8 - K_2$.

Stars selected from "Contributions from Mt. Wilson Obsy" No. 199, 1921 Parallaxes of 1646 stars by W.S. Adams. Proper Motions μ, μ' in R.A. & dec taken from Boss Prelim. Catalogue

North Pole - Star - Solar Antapex angle χ taken from Groningen Publications 9-16 1902 also Star-Apex distance λ. Apex (1810) 273° R.A. +29.5° Dec. N



Linear vel. towards & ⊥r to antapex would have to be multiplied by 4.74 to be expressed in km/sec.

The formulae are :-

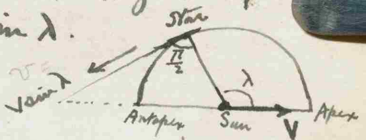
$$\parallel^l \text{vel} = 15\frac{\mu}{\pi}\cos\delta\sin\chi + \frac{\mu'}{\pi}\cos\chi = v_1'$$

$$\perp^r \text{vel} = 15\frac{\mu}{\pi}\cos\delta\cos\chi + \frac{\mu'}{\pi}\sin\chi = v_2$$

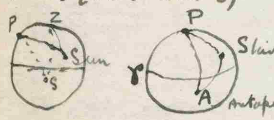
since Boss gives μ in sec of time along equator and μ' in " of arc toward pole.

The ∥l vel is here relative to Sun whose motion V towards Apex must be resolved onto Star-Antapex direction and added or subtracted from v_1' to give v_1

$$v_1 = v_1' \pm V\sin\lambda.$$



If χ is not obtainable in the Groningen catalogue it may possibly be found in Percy Davis's Star Azimuth Tables (60°N to 60°S) Rules for finding χ thus are:-



∠ZPS = H.A PA = 90 - Dec Antapex

SP = 90 - Dec Sun PS = 90 - Dec Star

PZ = 90 - Lat. APS = RA(Star) - RA(Antapex)

Lat (in tables) corresp to Dec. Star

Dec of Sun (") " " Dec Antapex

H.A. (") " " R.A.(Star) - R.A.(Antapex)

I If Star be N dec use Contrary names (since antapex is S dec)
If Star be S use Same names

II Star N. If ∠APS is + ($0^h < APS < 12^h$)

$\chi \equiv 360 -$ Tabulated Azimuth (∠PSA)

If ∠APS is − ($12 < H < 24$)

Subtract R.A. Star from 24 & add this neg angle to R.A. Antapex

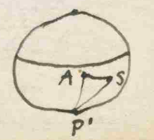
& $\chi \equiv$ Tabulated Azimuth

III Star S. If ∠APS + ($0 < APS < 12^h$)

$\chi = 180 +$ Tab. Az.

If ($12 < APS < 24^h$)

$\chi \equiv 180° -$ Tab. Az.



Solar Physics Observatory & Laboratory of Astrophysical Department. Visited May 8th

All the instruments removed from South Kensington in 1913. where Huggins, Sir Norman Lockyer & Rowland worked. The telescopes used by Huggins are mounted ||l to one another & have spectroscopes & photographic appliances attached. One is mere curiosity - the other in occasional use with a 15" glass. These with a brass inscription plate were presented to Camb. Obsy. in 1908 by the Roy. Soc.

The Spectroheliograph is used daily - if possible - for taking the solar disc & prominences by monochromatic light - i.e. λ for the K line of calcium. The slit is 4 in. long. 2 inches being the solar image diameter. The reflector is due N. of the heliograph instrument & has R.A. & dec. setting & clockwork regulator adjustments & is so fixed that as the Sun moves the reflected ray always goes due S. into the other bldg thro' a lens which directs it onto the slit & along to the prisms & back ||l to the photographic plate slit the whole being on a sliding base so as to move ~~the~~ slit across the disc image in from 30" to 2 or 10 min. according to exposure. For prominences a black wood disc (size according to season of year) is put in front of 1st slit to cut off all but prominences. They have a remarkable series of plates taken during the period of the May 1919 Eclipse showing development & breaking away of a prominence. They hope to get an adjustment for Hydrogen lines as well. The necessity for sticking to one λ is because the rays from top & bottom of slit hit the prism differently & ∴ have different deviation depending on the λ & this is corrected for by a curvature in the 2nd slit. Ca, H, & Fe spectroheliographs are the only ones taken at the various Obsys elsewhere & very few of the latter two.

The Newall dome has the 25 inch aperture refracting telescope for spectrographic observations for Radial velocities, &c. presented 1890 by R.S. Newall F.R.S. from his private obsy. 1870. Across this dome passes a beam of reflected sunlight through an aperture to the adjacent room containing a double coelostat - mirrors 16", object glass aperture 12" focal length 60 ft., & a spectrograph of the Littrow form.

A large 36" reflector was being set up in 1913 & is not yet in working order.

The standard spectrographic Roland grating framework is in another room with a temporary prism replacing the Roland grating wh. was taken to Odessa in 1914 for Eclipse observations & is there still unfortunately.

There are framed various interesting spectrum photographs showing displacement of lines between E & W limbs of sun, spectrum of various meteorites (Ag. from mint 99.9 % pure supplied to Sir N. Lockyer for comparison in arc spectrum)

Tables for Great ⊙ Sailing . J. T. Towson F.R.G.S. 1861.

Contains a linear index chart for determining lat & long. of vertex - i.e. furthest S. or N. pt. of the gt circle joining the ship to th' reqd destination. Then tables give first course. This can be utilized for my problem of North Pole - Star - Antapex angle X. where star is ship's position, antapex is destination. Use as before Antapex (1810) 93° R.A. −29°.5 Dec.

From Mt Wilson Obsy Catalogue of 1646 stars by Adams - his prob. error for Abs. Mags. is given as $\pm 0.3^m$; for parallaxes as $\pm 0''.4$

Relation between cross vel., distance and abs. mag.
Let suffix t = true & o = observed.

Ratio $$\frac{v_t}{v_0} = \frac{d_t}{d_0} = \sqrt{\frac{L_t}{L_0}} = \sqrt{10^{-0.4(m_t - m_0)}}$$

$$= e^{-0.2 \log_{10}(m_t - m_0)}$$

If B represent the app. mag. of two stars of different distances $B = \frac{L_1}{d_1^2} = \frac{L_2}{d_2^2}$

$$\therefore \sqrt{\frac{L_1}{L_2}} = \frac{d_1}{d_2}.$$

Velocity.
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... divided into five
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...errors of abs. mag
... Prof. Eddington

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Royal Astronomical Society

CENTENARY CELEBRATION

CONVERSAZIONE

Monday, May 29, 1922

EXHIBITS

Photographs and Transparencies	Dr. STEAVENSON
Aids to Calculation, Ancient and Modern	Mr. F. ROBBINS
Historical Instruments	Mr. D. BAXANDALL
Ancient Books	Mr. W. A. PARR
The Probability Curve	Prof. PLUMMER
The Newton Relics	*lent by the* ROYAL SOCIETY
Portrait of Galileo	*lent by* Mr. G. H. GABB

During the evening there will be an

EXHIBITION OF LANTERN SLIDES

in the Lecture Room, with explanatory remarks

P.M.		
9.0	—Sunspots, 1820-1920	Mr. MAUNDER
9.20	—Mars and Jupiter, 1820-1920	Rev. T. E. R. PHILIPS
9.40	—The Comets of the Century	Dr. CROMMELIN
10.0	—Some Fellows of the Society	Mr. HOLLIS
10.20	—Instruments of the Period	Mr. HEPBURN

Velocity.
mentioned, 287
$G_8 - K_2$ + giants
& their 1st vel.
Then rearranged
divided into five
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Prof. Eddington

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$+ dM$
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the mean mag.

Absolute Mag. & ~~Cross~~ Velocity.

Of the 1646 stars previously mentioned, 287 proved to be of spectral types $G_8 - K_2$ + giants $M < 3.4$. These were listed & their $\perp^r$ vel. worked out as explained. Then rearranged according to abs. mag. & divided into five groups of as nearly equal weight as possible in order to correct for accidental errors of abs. mag & hence obtain a general formula of the form $v = a + bM$.

Effect of accidental errors of abs. mag. — as worked out by Prof. Eddington

M denotes true abs. mag.
μ .. observed .. ..
v .. true cross-vel.
V .. observed " "

Then $V = v\, e^{-q(\mu - M)}$ 1.

where $e^{5q} = 10$ so that $q = 0.4605$

The frequency of an error in determination of abs. mag. $\mu - M$ will be propl. to

$$\frac{h}{\sqrt{\pi}} e^{-h^2(\mu - M)^2} d\mu. \quad 2.$$

For a homogeneous group such as K giants the distribn of true abs. mags conforms sufficiently nearly to the error law. We may set
frequency of abs. mag M to M + dM

$$= \frac{k}{\sqrt{\pi}} e^{-k^2 M^2} dM \quad 3.$$

(Measure all mags. from the mean mag. of the group as zero)

Then the frequency of μ to $\mu + d\mu$ will be

$$\left.\begin{aligned} &= \frac{K'}{\sqrt{\pi}} e^{-K'^2 \mu^2} d\mu \\ &\text{where } \frac{1}{K'^2} = \frac{1}{K^2} + \frac{1}{h^2} \end{aligned}\right\} \quad 4.$$

K' is determined from the statistics of the group by finding the ~~stars~~ mean deviation of the frequency curve of magnitudes.

$\eta = \dfrac{\Sigma(fx)}{\Sigma f}$ where x is deviation from mean and $\Sigma f = N = 287$.

and $K' = \dfrac{.564}{\eta}$ (see Brunt's Combination of Observations).

h is found since the prob. error of the mags is given by Adams as 0.3^m

and. $h = \dfrac{.477}{\text{prob. error}}$. (see Brunt)

Hence K is found –

We shall have that the sum of the v's for all stars between $M + dM$ and $\mu + d\mu$ is given by

$$\frac{h}{\sqrt{\pi}} e^{-h^2(\mu - M)^2} d\mu \cdot \frac{K}{\sqrt{\pi}} e^{-K^2 M^2} dM \cdot v e^{-q(\mu-M)}$$

and the sum for all stars between μ and $\mu + d\mu$ is obtained by integrating for all values of M

Thus $\Sigma v = d\mu \dfrac{hK}{\pi} \displaystyle\int_{-\infty}^{+\infty} e^{-h^2(\mu-M)^2 - K^2 M^2 - q(\mu - M)} \cdot v\, dM$

$$\left.\begin{aligned} &= d\mu \cdot \frac{hk}{\pi} e^{\frac{1}{4}q^2 - K^2(h^2\mu^2 - q\mu)/(h^2+K^2)} \int_{-\infty}^{\infty} * \\ &* \cancel{e^{-h^2(\mu-M)^2 - K^2M^2 - q(\mu-M)} \cdot v\, dM} \\ &* e^{-(h^2+K^2)\left(M - \frac{h^2\mu + \frac{1}{2}q}{h^2+K^2}\right)^2} \cdot v\, dM. \end{aligned}\right\} \quad 5.$$

-unction of the obs.

$M + cM^2 + \text{——}$

to take

$- \dfrac{h^2\mu + \frac{1}{2}q}{h^2+K^2})$ l

$\mu^2 + q\mu)/(h^2+K^2 \cdot ($ *

$) \dfrac{\sqrt{\pi}}{\sqrt{h^2+K^2}}$

-ibuting to this sum

$\dfrac{h^2K^2}{h^2+K^2} \mu^2$

$\left(a + \dfrac{\frac{1}{2}qb}{h^2+K^2} + \dfrac{h^2}{h^2+K^2} b\mu\right)$

-tion between

-r a number of homogeneous groups)

$e^{-(\frac{1}{4}q^2 - K^2 q\mu)/(h^2+K^2)}$

be a linear function

$+ b'\mu$

-eviations & hence

-n (b') and

-oo)

Then the frequency of μ

$$= \frac{k'}{\sqrt{\pi}} e^{-k'^2}$$

where $\frac{1}{k'^2} = \frac{1}{k^2}$

k' is determined f

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of the frequency curv

$\eta = \frac{\Sigma(fx)}{\Sigma f}$ where

and

and $k' = \frac{.564}{\eta}$

h is found sin

the mag is given

and. $h = \frac{.4}{}$

Hence

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ν's for all sta

$\mu + d\mu$ is given

$\frac{h}{\sqrt{\pi}} e^{-h^2(\mu-M)^2}$

and the sum f

μ and $\mu + d\mu$

for all values of

Thus $\Sigma \nu = d\mu$

$= d\mu . \frac{hk}{\pi} e^{\frac{1}{4}q^2 - k}$

~~$* e^{-h^2(\mu-M)}$~~

$* e^{-(h^2+k^2)(M}$

OBSERVATORY,
CAMBRIDGE.

1922 May 4

Dear Miss Douglas

The enclosed will show how I propose to eliminate the effect of accidental error in the determinations of absolute magnitude in the final working up of your cross-velocities.

I will not guarantee the algebra is quite right; it must be gone through again. But you will see that it is quite practicable to allow for the accidental error. And it makes the investigation rather interesting as the effect is presumably considerable and to a large extent vitiates Adams's own treatment of the data.

Yours sincerely

A. S. Eddington

nction of the abs.

$M + cM^3 + —$

to take

$- \frac{h^2\mu + \frac{1}{2}q}{h^2+k^2})$

$\mu^2 + q\mu)/(h^2+k^2) . (*$

$) \frac{\sqrt{\pi}}{\sqrt{h^2+k^2}}$

ributing to this sum

$\frac{h^2k^2}{k^2} \mu^2$

$(a + \frac{\frac{1}{2}q}{h^2+k^2} +$

ation between

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homogeneous groups)

$e^{-(\frac{1}{4}q^2 - k^2 q\mu)/(h^2+k^2)}$

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$+ b' \mu$

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$= \frac{K'}{\sqrt{\pi}} e^{-K}$

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$\eta = \frac{\Sigma(fx)}{\Sigma f}$ wher

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and $K' = \frac{.564}{\eta}$

h is found s

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and. $h = \frac{.4}{\mu}$

Hence K

ν's for all st

$\mu + d\mu$ is give

$\frac{h}{\sqrt{\pi}} e^{-h^2(\mu-M)}$

and the sum

μ and $\mu + d\mu$

for all values o

Thus $\Sigma \nu = d$

$= d\mu . \frac{hk}{\pi} e^{\frac{1}{4}q^2 -}$

~~$* e^{-h^2(\mu-M)}$~~

$* e^{-(h^2+k^2)(M}$

If the velocity v is a function of the abs. mag. M, say $v = a + bM + cM^2 + \text{—}$

this can be evaluated -

It is probably sufficient to take

$$v = a + bM$$

$$= a + \frac{h^2\mu + \frac{1}{2}q}{h^2+k^2} + b\left(M - \frac{h^2\mu + \frac{1}{2}q}{h^2+k^2}\right)$$

Then (5) becomes.

$$\Sigma v = d\mu \frac{hk}{\pi} e^{\frac{1}{4}q^2 - k^2(h^2\mu^2 + q\mu)/(h^2+k^2)} . (*$$

$$* \left(a + \frac{b(h^2\mu + \frac{1}{2}q)}{h^2+k^2}\right) \frac{\sqrt{\pi}}{\sqrt{h^2+k^2}}$$

The number of stars contributing to this sum is eq (4)

$$= \frac{d\mu}{\sqrt{\pi}} \frac{hk}{\sqrt{h^2+k^2}} e^{-\frac{h^2k^2}{h^2+k^2}\mu^2}$$

Hence by division.

$$\bar{v} = e^{(\frac{1}{4}q^2 - k^2 q\mu)/(h^2+k^2)} \left(a + \frac{\frac{1}{2}qb}{h^2+k^2} + \frac{h^2}{h^2+k^2} b\mu\right)$$

giving the observed relation between velocity and abs. mag.

Having evaluated $\bar{v}$ for a number of values of μ (the five homogeneous groups) then evaluate $v' = \bar{v}\, e^{-(\frac{1}{4}q^2 - k^2 q\mu)/(h^2+k^2)}$

This should turn out to be a linear function of μ namely $v' = a' + b'\mu$

(Work out the standard deviation & hence the regression equation (b') ~~and~~ ~~transform origin to zero~~)

Then $b' = \frac{h^2}{h^2+k^2}\, b$

$$a' = a + \frac{\frac{1}{2} q b}{h^2+k^2}$$

from which a, b can be calculated and the origin transformed to zero.
Multiply throughout by 4.74 and the eq^n becomes. $v^{km/sec} = a^{km/sec} + b^{km/sec} M$

If in each of the five groups the n stars of exceptionally high cross vel. are omitted (take $n = 3$ and 6) giving eq^ns

$v_1 = a_1 + b_1 M$
$v_2 = a_2 + b_2 M$
$v_3 = a_3 + b_3 M$

a comparison of these is made by considering the effect of imposing an upper limit on v.

Find X such that

$$\int_0^X e^{-x^2} dx = \frac{N-n}{N}\int_0^\infty e^{-x^2} dx$$

true arith mean $= \int_0^\infty x e^{-x^2} dx \div \int_0^\infty e^{-x^2} dx$

adopted mean $= \int_0^X x e^{-x^2} dx \div \int_0^X e^{-x^2} dx$

$$\therefore \frac{\text{true mean}}{\text{adopted mean}} = \frac{\int_0^\infty x e^{-x^2} dx}{\int_0^X x e^{-x^2} dx} \cdot \frac{N-n}{N}$$

$$= \frac{1}{1-e^{-X^2}} \cdot \frac{N-n}{N}.$$

then $b' = \frac{h^2}{h^2+k^2}\, b$

OBSERVATORY,
CAMBRIDGE.

1922 Dec 11

Dear Miss Douglas

I enclose a draft of the pap. Please look through it to see that there is no misunderstanding.

I find that Adams gives $\pm .40$ as the probable error of his parallaxes, and I think it would be desirable if you would recalculate the results with this value. It will be interesting to see how large a difference this makes.

If you can let me have the results of this calculation I will then go through the paper again.

Yours sincerely

A S Eddington

Will you put a Christian name at the head of the paper — to prevent yourself being addressed as Mr.!

then $b' = \frac{k^2}{l^2+k^2} b$

OBSERVATORY,
CAMBRIDGE.

1922 Dec 18

Dear Miss Douglas

I have rewritten most of the paper and made some alterations. You may be interested in § 3 which is new; it would I think have been a rather easier way if it had been thought of earlier.

I cut out the discussion of the results (ratio of masses, etc.) because it is clear that so much depends on the adopted value of the probable error $\pm 0^{m}.3$; it was rather an accident that the agreement was so good at the first shot.

There are a few slight changes in your figures - it is rather difficult to explain why they were necessary

(1) The formula had to be $a + b(m - 1{\cdot}01)$ not $a + b(m - 1{\cdot}06)$ in order to agree with the theory

(2) In your solution for $\pm 0^{m}.4$ the <u>a</u> was not quite right.

(3) I disagree with your probable error for b which was too low I am sure. I have recalculated it.

If you agree with it as it now stands, will you send the paper direct to the Assistant Secretary, Royal Astron. Soc. Burlington House, London W. 1.

I expect you had better keep the sheets of calculations.

If anything is not clear or you would like to talk about it, come round one afternoon this week. I shall be away for a bit after Christmas.

The meeting is on the second Friday in January. I hope you will be able to come. Tea at 4·30.

Yours sincerely

A S Eddington

1923 Jan 25

Dear Miss Douglas

Here is the proof. You might read it through to see if you can pick up any more errors. If it is all straightforward return it (within 3 or 4 days) to Assistant Secretary R.A.S. Burlington House, W. 1. (The M.S. need not be returned). If there is anything doubtful send it to me.

I shall order 50 reprints of which you can have any number up to 25. If that suits you please fill in 50 on the slip. If you want more increase the number proportionately (I dont think they cost more than about 10/- ~~for~~ per 50). I am afraid I am always very negligent about distributing reprints

You will see I have added a paragraph at the end and that brings me to the question of using the other component. I think it is well-worth trying. Divide into 5 groups as before and take the mean longitudinal motion paying attention to sign; – also for each group take the mean of the foreshortening factors (~~cos~~ sin star apex). ~~If you work out just as before you will get the~~ Divide one by the other to get the $\bar{v}$ supposed to be the parallactic motion. (I think that is better than correcting each separately for the foreshortening factor.) Then work out as before the corresponding $v = a + bm$. b should be zero, and if it is not, that means that we have not chosen the right value of h. Of course a should be 20 km. per sec., but that only checks the systematic accuracy of Adams's spectroscopic magnitudes. It will probably be easier to use the method of § 3 using $\log_e v = a' + b'm$; then since $b = 0$

$$b' = \frac{-q\,k^2}{h^2 + k^2} = -q\,\frac{k'^2}{h^2}$$

is the equation determining the proper value of h.

OBSERVATORY,
CAMBRIDGE.

If this proves satisfactory it will be desirable to go on to examine another group.

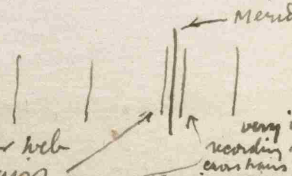
I am afraid there is no very satisfactory way of rejecting excessive motions (in the parallactic motions). If done at all it had better be done sparingly.

Yours sincerely
A S Eddington

...t to the Assistant
...x. Burlington House,
...er keep the sheets of
...ear or you would like
...nd one afternoon this
...a bit after Christmas.
... second Friday in
...l be able to come.
...A S Eddington

april 6th/23

Visited Royal Observatory Greenwich
with introduction from Mr Smart to Mr John Jackson.
Large dome with two telescopes erected 1896 by Sir Christie. Astr. Roy. 1881-1910
(1) 26" refractor - photographic work a.m. + p.m. for parallax data for Catalogue of Stars 64° N. to Pole - now doing 9th mag. stars with 11.0 stars as comparison stars.
(2) 30" reflector fitted with spectroscopic adjustment.
Room where parallax plates are measured & results calculated + recorded. Fine old man in charge then explained theory + methods. He does not measure by contact from R & then from L + take mean but makes one shot for centre - (unlike Camb. + Paris method)
Nice circular office of Astron. Roy. Sir Frank Dyson. Very friendly - had just been reading the Paper in M. N. R.A.S. by A.S.E. + A.V.D. & was very interested in result & in the way it contradicted Adams result + that it was "just like Eddington" to have seen the fallacy in latters treatment.
(3) Large tel. mounted like the Northumberland + erected by Sir Geo. Airey - Astr Roy till 80 yrs old. Now used for photographing visual binaries 1000 yr periods + 200 ? astron units dist. between
(4) Transit Circle
on zero parallel.



Time is found daily + checked with that found at Paris + Moscow + broadcasted daily from then. At 1 p.m. large ball at top of mast falls to roof of dome.

Chronometer room. with hundreds of instruments being tested for Admiralty - No. of sec. error recorded daily. Saw first Chronometer made by Harrison about 1720 - big affair in glass case 2' x 2' x 1½' Also first small one 1754 by order of Brit Gov.
(5) Cookson's floating telescope - see p. 29. for determining zenith & hence changes in latitude. Cookson was nephew of old Newall. On walls of this small dome are old telescopes of historic interest notably that with which Bradley studied ~~the~~ aberration of light
Octagon Room. The original building put up when Chas II founded the Observatory. Now the council room with portraits on walls + fine view across the river + over London
Two groups of domed bldgs. surrounded by a wall situated on top of a 150' wooded steep hill giving appearance of a fortification.

april 12th/23

Visited Paris Observatory

Sent in card & request to M. le Directeur. M. Baillon Baillaud
an old white-haired gentleman - very courteous.
rather deaf & feeble - knows A.S.E. & Dyson & Plaskett.
Saw Mme Chandon who operates the new Transit
Circle - a beautiful 4' instrument so mounted
that it can be raised & reversed on its axes to
eliminate any error - an electrical micrometer
face adjustment as at Greenwich. Obsns are
made daily with it & the old one like that
at Camb & Greenwich, & with a smaller old
type one. The new instrument was designed
by M. Baillaud and nephew Bruty son of the
late Prof. of Physics at Sorbonne. 4ft focal length
M. Baillaud, Jr. head of photographic dept.
showed me the visual & photographic equatorial
by which chart of heavens was made &
now obsns of 1885-90 are being used
for proper motion determinations. Very
fine measuring microscope (N.B. He always
approaches image from R & from L & takes
mean) Also saw machine for ascertaining
app. mag. by comparison of image with a
graded series of images down to 12th mag.

Saw large telescope about 60 yrs old 10 meters
long in lighthouse tower up 160 ft. used for
visual work on planets & binary stars

The Obsy was founded by Louis XIV &
has beautiful grounds laid out as at
Versailles with converging avenues of
plane trees. Statue at entrance to
Le Verrier with bas reliefs of solar system
& of Paris Obsy.

Sir Robt Ball 1886. "The Story of the Heavens", of boundless
interest and of exquisite beauty, leads to the contemplation
of grand phenomena in nature and great achievements
of human genius."

Uranus - discovered by Sir Wm. Herschel 1781.
ex-Hanoverian soldier - musician & organist at Bath
made first high magnificat.n telescopes & set himself
to examine all the bright stars & found a disc.
Neptune - its position, orbit, mass & period calculated
by J. C. Adams at Cantab & reported to Astr. Roy. Sir
Geo Airy in 1845. After 9 mos. Airy asked Challis to use
the Northumberland telescope at Camb. to chart that
portion of sky in order to find planet. Meanwhile
Le Verrier of Paris Obsy. had made similar computations
with same results & asked Berlin Obsy. to locate it
as they already had one chart of that region -
Before Challis had compared his two sets of measurements
showing the planetary motion, Dr Galle at Berlin
found & announced the discovery.
Other major planets: known from antiquity.
Tides. Attraction by moon (or sun) draws up
water on near side & draws away solid
earth ~~from~~ far side - hence bulge of water on
both sides - Source of energy is Earths
rotation - result of expenditure of energy is
retardation of earths rotation i.e. elongation
of day, and the increase of distance of
moon - Tendency is to make day = month
i.e. time of rotation of earth = period of rev. of moon.
Lunar tides once very powerful before solidification
of moon account for its day ~~or~~ period of
rotation being now equal to its period of revolution
When moon left earth the ratio of $\frac{\text{earths day}}{\text{month}}$ was unity &
each was 3 or 4 hours - As moon retired ratio became
less to a limiting value of $\frac{1}{29}$ then increased to its
present value of $\frac{1}{27.3}$ & with moon still retreating is
approaching unity again with day = month = 57 x present day.

Harvard College Observatory May 1924.

Director: Dr Harlow Shapley.
Photographic Dept. Prof. King

Draper Telescope. Historic. 15" lens. 2½ ft. Mertz & Moller. Munich
Gift stimulated by gt. comet of 1843. Tel. erected 1847
first photograph of a star 1850 taken with this telescope.
Director of Oby was Dr Bond & with it he discovered
a satellite of Saturn and the inner ring. Attempt to
measure size of satellite by a pin hole bore in a plate
just size of satellite & afterwards exposed to disc of planet
thus getting relative sizes of the pinhole to Saturn. Thus
he obtained estimate of 10 mile diameter.
In 1891 the 8" Draper Tel. was erected. Experiments on
best way of preventing settling of dew. Old method was
by applying an alcohol flame. King uses a cap with
low c.p. carbon bulbs round it giving just enough warmth
to prevent dew from condensing.
another tel. has the largest objective prism in America
" " mounted too delicately on 8" base is upset
by ordinary wind & too unsteady for photo work
" " on a tremendously strong horseshoe mounting
with a 16° Metcalf lens. Metcalf is a minister who
focal length
guid leaves. sometimes out in the woods in camp
this has an arrangement for using an air pump
to produce a partial vacuum behind the p. plate
so as to curve it in such a way that the spectral
images near edges are in focus as well as at
centre of plate. 2 Objective prisms of glass, 15°,
are in front of object glass. 60'–90' exposures
for ordinary plates.

A fixed end, caelostat type behind & over library with the
adjustable mirror controlled from above used every fine
night by Mr Twinkle for visual & photographic
observations on variables.

3 Telescopes are now at Arequipa in Peru at the
Harvard Obs. 9 there. 8000 ft altitude. meteorological
station at 10,000. on east side of the Andes. Most of
the objective prism spectrographs for Draper Memorial
catalogue taken there. 250 000 classified by Miss Cannon

Photographic laboratory: Clocks regulating each tel
with adjustable weights & scale of weights for correcting
for refraction lag near horizon etc.

Table for lunar observations.

Dec. | hour angle → | increasing to 6h | decreasing to 12h

Photo plates are tested by standard exposures to a
standard lamp thro apertures □ □
King has noted that contrasts are greater if the
plates are left long undeveloped.

see separate special notes on conversations with
Dr Shapley & notes on problem of ray filter
and of stellar magnitudes and types & motions.

Wellesley College Observatory for teaching Astronomy.
Equipped for teaching astronomy to 250 girls May 26th, 1924.
Time determinations. 2 transit instruments & radio
& chronometer. 1 good equatorial for photo work
& clock regulated driving motor. 2 small observing
telescopes, photographic room, library & large room
for doing problems on plaster of paris celestial globes.

Saw 1st chronometer made by Harrison 1720 circa. big affair in a glass case 2 × 2 × 1/2 feet - also first small one 1754 - for the Br. Gov -

(5) Cooksons floating telescope for determining Zenith & hence changes of latitude. Sent from Camb. - Cookson was nephew of Old Newall -

(6) Octagon room - original bldg. when Chas II founded the obsy - now now a council room with portraits all around -

L'HOTELLERIE

CARCASSONNE

Le ....

On walls are
old telescopes
notably the one
with which Bradley
studied the aberration
of light

Shown around by
Mr John Jackson

ASCENSEUR
Chauffage Central

TOILETTE A EAU CHAUDE
DANS TOUTES LES CHAMBRES

CHAMBRES AVEC BAIN PRIVÉ

TÉLEPH. 4-34

now used for photographing visual binaries. (1000 yr period 400 astron units dist between.

(4) Transit Circle

O meridian

(aligned by Pole Star)

Colimator

Spectr. well

time found daily

also at Paris & Moscow

& broadcasted to Greenwich

at 1 p.m. large ball descends on flagstaff.

Chronometer room - for testing admiralty instruments

no of seconds error recorded daily

Greenwich Observatory Apr. 1923

1

Dome with 2 Telescopes erected 1896 by Sir Wm. Christie Astr. Roy. 1881-1910

1) 26 in refractor - photo work am & pm for parallax work on catalogue of stars 64° N to Pole. Now doing 9th mag. stars with

(2) 30 in reflector fitted with spectroscopic adjustment.

Office of Astr. Roy. & room for measuring & calculating parallax results.

3) Large tel. mounted like the Northumberland erected by Sir Geo. Airy (Astr. Roy.) till 50 yrs old

Merton

Phil Trans
Vol. 222 p. 369 – 400 ..
A 603 . 27. 4. 22.

Sobral 1919 Eclipse

7 plates
7 stars on each

Dyson Edd & Davidson.

Silberstein . p 408

Weyl pp v, vi, vii

Majority 33rd – 34th

Stone	Light	Material
32	.0016	.9984
33	.106	.894
34	.570	.430
35	.850	.150
36	.951	.049

Lightest star just below 33rd.
Heaviest just above 35th
Majority 33rd—34th

"SOMETHING BEYOND OUR DREAMS."

ASTRONOMER'S HOPE.

MYSTERY OF MARS.

EVE OF STRANGE DISCOVERIES.

In his address to the Royal Astronomical Society yesterday at Burlington House, Piccadilly, W., on the occasion of the hundredth anniversary of its foundation, Professor Eddington, president of the society, dealt with the progress of astronomy in the past century, and alluded to the feeling "that they were on the verge of something greater than their dreams could shape"—discoveries which, it may be, have yet to come.

He mentioned as a definite fact, which not the most sceptical would deny, the existence "of what is presumably some kind of vegetation on Mars," that planet of mystery.

CINDERELLA OF SCIENCE.

The record of astronomical progress during the last 100 years, he said, was one of continuous advance, followed by periods of exhaustion.

He was told that there was a period shortly before X-rays and electrons came to the fore when the physicist had given up anticipating any radical advance. He thought the big discoveries were already garnered. The feeling, so present with them to-day, that they were on the verge of something greater than their dreams could shape, had not yet disturbed their placid progress.

"The centre of most rapid progress has shifted from time to time," continued Professor Eddington, "and the various branches of astronomy have had their ups and downs. I suppose that in recent years the department of planetary astronomy has been in the depression of a wave. At least it seems to be so in comparison with the more sensational progress in our knowledge of the sun and stars. But the depression has by no means reached stagnation.

"On the theoretical side we have Taylor's important investigation of tidal friction in the Irish Sea, which, true to its name, is responsible for a considerable proportion of the suction and dissipation of energy on this planet. And Jean's researches have given us new ideas of the origin of the planets which attend the sun and of the singular (perhaps even unique) character of this system. If the department of planetary astronomy is now the Cinderella of our science she yet has dreams that her prince is waiting for her

MARS VEGETATION.

CHANGING WITH THE SEASON.

"It is startling to-day to read a passage from Huxley's essays: 'Until human life is longer and the duties of the present press less heavily, I do not think wise men will occupy themselves with Jovian or Martian natural history.'

"Martian and—I almost fear to mention it—lunar natural history are no doubt thorny subjects, but, notwithstanding Huxley's censure, probably the most sceptical among us would admit that the observation of seasonal changes of what is presumably some kind of vegetation on Mars is a recognised astronomical pursuit."

In reviewing the general advance of astronomy during the century they could not but be struck by the tendency to leave the little system ruled by the sun and to penetrate deeper into the vast world outside.

"We celebrate to-day our centenary," concluded Professor Eddington, "and by a simple subtraction sum—subtracting 100 years from May 30, 1922, you arrive at the date of our foundation, January 12, 1820, according to the most accredited calculations—which I trust no mathematician here will venture to criticise. The great event caused a ripple in the ether which has spread out ever since in widening circles. To-day that ripple embraces about 10,000 of the fixed stars. The re-in 1846, for Adams's memorandum was left at the Royal Observatory in October 1845. We have no time to-day to dwell on this remarkable episode in our history (it was to this society that Airy gave his account of the circumstance), but we may perhaps glance at the portrait of Challis, who sent Adams to Airy and then seems to have washed his hands of the affair, and of Mr. Graham, who made the observations which included Neptune, and from which it would have probably been identified but for the fatal cup of tea hospitably offered by Mrs. Challis."

Professor Turner explained that by the time the tea was finished the heavens had become obscured by clouds.

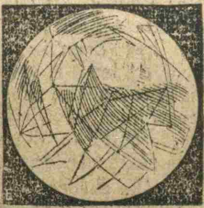
THE KING AND SCIENCE.

A message from the King was read at the meeting thanking the society for a loyal message they had sent. During the last hundred years the civilised world has witnessed a wonderful advancement in astronomical discovery, and the King is proud to know that the Royal Astronomical Society, both here and in the far corners of the Empire, has given its full measure of support towards the success of these valuable achievements. "You can rest assured that the King watches with interest and admiration the patient, diligent, and unobtrusive manner in which the Fellows of the Society conduct their unremitting research, in the hope that they may, by piercing the hidden mysteries of the skies, add step by step to the store of scientific knowledge, and thus contribute so much that is essential to the progress of mankind on land and sea."

LIFE ON MARS?

RIDDLE OF THE RED PLANET.

Can life possibly exist on the planet Mars? In two years' time United States astronomers are to make special observations at the Mount Wilson Observatory, California, to discover what amounts of water and oxygen—these two necessities of human life—there are upon the planet. Dr. Charles E. St. John, of the Mount Wilson Observatory, one of the several famous foreign astronomers who are now in London in connection with the centenary celebrations of the Royal Astronomical Society, told a *Daily Mail* reporter yesterday afternoon that he was doubtful whether there was any intelligent life on the planet.



The Mars "Canals."

"Our knowledge of the appearance of Mars is very vague," he said. "Mr. Barnard, who has the best eye of any American astronomer, can only say that he sees some kind of forms on the planet.

"Only one part of the visible surface can be examined closely at one time, and the drawings of the so-called 'canal' systems are built up from piecemeal observation. The planet is seen in a continual shiver, as it were. It won't keep still for you.

Draper 15' Historic

Mertz & Moller

Munich

1843 Comet caused the gift of this

1847 erected

1850 first photo of a star.

15' lens

22 ft

Bond at that time

Director of obsy.

Sat at Sat

inner ring

tried to get size of sat by pin hole here in

8" Draper 1891

Dew

Old methods alcohol flame hot sandbags.

New idea of a cap with C. bulbs.

24" refl has the largest objective prism in Am.

12' on 8 in. refl too delicate

wind shakes it.

Horseshoe Tel Very strong mounting.

suction at back of plates.

Metcalf 16'

Minister who makes lenses.

two 15" prisms over object glass.

from over

a metal disc covering saturn

+ giving estimate of rel. size of

planet to pinhole.

10 miles drawn was his result.

Caelostata type.

fixed eye piece end.

adjustable mirror – used by Mr Twinkele

for vis + photo word on Variables.

Arequipa + Jamaica.

'moon markings not moisture

probably effects of light of surface

of moon.

Kings Photo Lab. Clock weighting for

lag or reverse due to refraction.

Table → hour angle

dec ↓

→ increasing to 6h

← decreasing to 12h

increasing ↓

Plates tested □ □ by

exp. to Standard Lamp

contrasts are greater if

left long undeveloped.

The heavens declare the
glory of God & the
firmament sheweth His
h

that it seems hardly
worth while writing
We took C. M. to Capt.
applejack last night &
there was great fun blasting

· PHYSICS · NOTES ·

MISCELLANEOUS

&

ELECTRON THEORY OF CHEMISTRY
by J. J. T.

&

ASTRONOMY
by A. H. S. Gillson.

A. W. Douglas.

Atomic Collisions.

Sir E. Rutherford. 22/10/22.

Terrestrial collisions

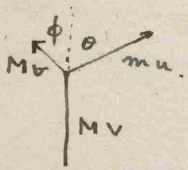
1 Steel balls: time of contact $\frac{1}{5000}$" in which time the elastic constant is often exceeded & permanent strain takes place.

Kinetic Thy. H mols $v. = 1.84$ Km/sec.
probably no interpenetration of mols on collision.
Vel $\propto \sqrt{T}$ abs. $2.73\ 10^6$ °C in Sun $v = 184$ Km/sec.
Veloc. of fall onto Earth — energy of mass $= G\frac{m}{a}$
$v = 11.5$ Km/sec.
" " fall into Sun $v = 600$ Km/sec.
i.e. 2 stars falling into one another produces heating effect of: 1 gm falling into Sun $\frac{1}{2}mv^2$
$= 43\ 10^7$ gr cal per gm.
Spec. heat gives Temp 43 million °C.
This wd. give a vel. to an atom of order of 1000 Km/sec

Vel. of neg. electron in P.D. 1 volt
$\frac{1}{2}mv^2 = Ve$ $v^2 = 2\frac{Ve}{m}$ $\frac{e}{m} = 1.77\ 10^7$ Emu.
1 volt $= 10^8$ "
$\therefore v = 5.95\ 10^2$ Km/sec.
$v = 5950$ Km/sec for 10 000 volts.
Hence vel. of mol in K. Thy. = fall thro .036 volts.

Vels in Vacuum tube 1000 - 10000 volts.
hot wire 1 volt —
+ rays 10000 - 50000 volts.
Radio active rays α 8 million volts.
β $\frac{1}{2}$ " to 3 million volts

Mathematics of Collisions.



Momentum is conserved
Energy " "

Vertical $MV = Mv\cos\phi + mu\cos\theta$ (1
$0 = Mv\sin\phi - mu\sin\theta$ (2
$MV^2 = Mv^2 + mu^2$ (3

Hence $u = \frac{2VM}{M+m}\cos\theta$ (4

$\tan\phi = \frac{m\sin 2\theta}{M - m\cos 2\theta}$ (5

$v = \frac{V}{M+m}\left\{M\cos\phi \pm \sqrt{m^2 - M^2\sin^2\phi}\right\}$ (6

Case 1. one α hitting another α.
$M = m$ $u = V\cos\theta$
$\tan\phi = \frac{2\sin\theta\cos\theta}{1-\cos 2\theta} = \cot\theta = \tan\frac{\pi}{2} - \theta$.
$\therefore \theta + \phi = \frac{\pi}{2}$.

Case 2. $\frac{m}{M}$ small eg. α hits electron.
$u = 2V\cos\theta$ approx
for head on collision $u = 2V$ – ie it communicates

Since its vel
$\tan\phi = \frac{m}{M}\sin 2\theta$.
$\sin 2\theta =$ or < 1 $\therefore \tan\phi < \frac{m}{M}$.
$\therefore \phi < \frac{m}{M}$
$\therefore$ α ptcl only turned thro $\phi < \frac{1}{1800\times 4} < \frac{1}{7000}$

Case 3. $\frac{m}{M}$ large. α hits gold atom or another.
$\alpha = \frac{2V}{M+m}\cos\theta$.

Case 4. Try α mass 4 + C mass 12. Comparable masses.
α turned thro $\frac{\pi}{2}$
$\frac{v}{V} = \frac{\sqrt{44-16}}{16} = .707$.
Range $\propto$ vel^3. $\therefore$ Range after recoil $= .707^3 \times$ initial range.
$\frac{u}{V} = \frac{8}{16}\cos\theta$ $= .35$ cm.
$\tan\phi = \infty$
$M - m\cos 2\theta = 0$ $\therefore \cos\theta = \sqrt{\frac{2}{3}}$
$\therefore \frac{u}{V} = \frac{1}{2}\sqrt{\frac{2}{3}}$

Case 3. α + Au 200
$\frac{v}{V} = \frac{49}{50}$ ie vel. only reduced 2%.
+ Au is sent off at very nearly 45°

Case 5. α 4 H 1
$u = \frac{2V\times 4}{5}\cos\theta = \frac{8}{5}\cos\theta$.
Head on col. $\theta = 0$ $u = \frac{8}{5}V$.

Case 6. Consider max^m angle of defl^n of α hitting H.

$\sqrt{m^2 - M^2 \sin^2\phi}$ must be +.

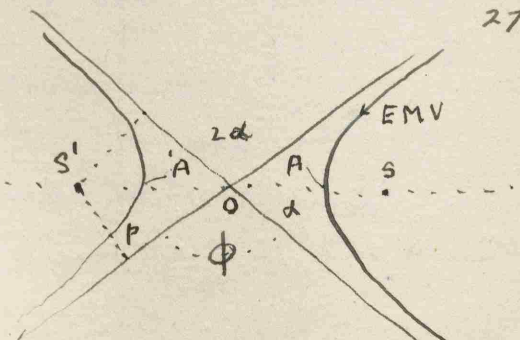
$\therefore m^2 > M^2 \sin^2\phi$

$\sin^2\phi < \frac{1}{4} \quad \therefore \phi < 14^\circ\, 30'$

Possibility of interpenetration of atomic systems
2 bodies can occupy same space at same time as it were.

Hence an α can get into the empty space surrounding the nucleus & is hardly disturbed by the electrons in that space but comes under the powerful influence of the nuclear forces.

27/10/22.



On assumption of no energy loss, photos should give data to deduce masses – i.e. given M θ φ find m.

Consider orbits.

1. Head on col.

$\frac{1}{2} m u^2 = NeE/b$. $\quad b = \frac{2NeE}{Mu^2}$

b is closest approach to nucleus.

2. Hyperbolic deflections. see diagram above.

$SA = SO + OA = SO(1 + \frac{OA}{OS}) = p\,\mathrm{cosec}\,(1 + \frac{1}{e})$

$= p\,\mathrm{cosec}\,\alpha(1 - \sec\alpha) = p \cot \alpha/2$ = apsidal distance

1. Conservation of momentum.
$pV = v \cdot SA = \text{const}.$

2. Conserv^n of energy.

$\frac{1}{2} MV^2 = \frac{1}{2} Mv^2 + \frac{NeE}{S'A}$.

$v^2 = V^2(1 - \frac{b}{SA})$

$= V^2\, p^2/S'A$

$\therefore p = S'A(S'A - b)$

ϕ = ∠ of defl^n $= \pi - 2\alpha$

$\alpha = \frac{\pi}{2} - \frac{\phi}{2}$. $\quad \cot\frac{\phi}{2} = \frac{2p}{b}$.

Closest approach b for 170°
$\frac{4}{3}$ b .. 90°

— sheet of gold area a thickness t
n atoms per cc
No of atoms $a t n$.
area exposed to α ptcl $= \pi p^2 A n t \;/ A$

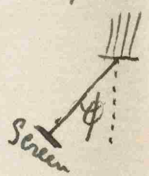
Chance of hitting within radius p of any atom is $\pi p^2 n t$.

$p+dp$, p chance of hitting within p & $p+dp$ is $2\pi p n t$.

Probability that α is deflected thro angle $> \phi$ is $\frac{\pi}{4} n t b^2 \cot^2 \frac{\phi}{2}$.

Hence ϕ_1 & ϕ interval gives prob.
$\frac{\pi}{4} n t b^2 \left(\cot^2 \frac{\phi_1}{2} - \cot^2 \frac{\phi_2}{2}\right)$

In practice by scintillation get no deflected through ϕ.



No scattered thro ϕ per unit area $= \dfrac{n t b^2 \operatorname{cosec}^4 \phi/2}{16 r^2}$

$\because \dfrac{4 n^2 e^2 E^2}{M^2 V^4} \;::\; (\text{nuclear charge})^2 \;::\; \dfrac{1}{r^4}$
$:: \operatorname{cosec}^4 \phi/2$

Note 1. considering masses M & m substitute for $\frac{1}{M}$ $\;\frac{1}{M} + \frac{1}{m}$

Note 2. α + He Relative to either the other is describing a hyperbola – but not actually in space. see C. G. Darwin.

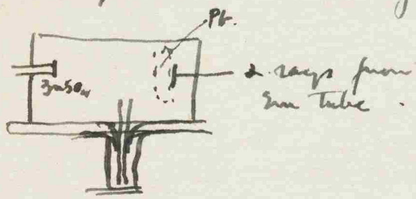
General problem

Closest dist. $= p \cot \frac{\alpha}{2}$
$= \dfrac{n e E}{V^2}\left(\frac{1}{m} + \frac{1}{M}\right)(1 + \sec\theta)$

$p = \dfrac{n e E}{V^2}\left(\frac{1}{m} + \frac{1}{M}\right)\tan\theta$ general relation.

In case of H – the closest distance is never actually less than 5 × (theoretical value) due to motion away of H as α approaches. i.e. "H. is not pinned to one spot".

First expts Marsden & Geiger.



Mic or source can revolve but dist is constant

Count first at say 150° when Em was strong as it decayed (4 dys ½ value) count at lesser angles till in 1 month counts directly opposite.

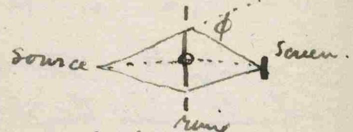
Within limits of expt. accuracy these expts showed that theory based on inverse sq. law did hold for heavy atoms (Ag, Au) over a certain range.

Closest appr. of α (7 cm. from Ra C) to a lead atom head on col. $b = 4.5\ 10^{-12}$ cm.
nuclear charge 82 e
defln 180
" 1° $30 b = 1.2\ 10^{-10}$.
If vel. is ½ then $4 \times 30 b = 4.8\ 10^{-10}$

Range of inv. sq. is within K ring of electrons but not quite to nucleus.

R. thinks M + G.'s results too good & that within 10^{-12} the inv. sq. law should begin to break down

Chadwick 1920.



compared scattered beam with direct beam cut down to $\frac{1}{1000}$ by a rotating wheel. Counting up to 200 per min

	At. no.	Chadwick
Cu.	29	29.3
Ag.	47	46.3
Pt	78	77.4

angle $\phi = 30°$ was used.

Hence inverse sq law holds very accurately.

$n^4 \propto \frac{1}{u^4}$ shown by Maxwell.

If law of force $\frac{1}{2^p}$

no. scattered $\propto \left(\frac{1}{V^2}\right)^{\frac{2}{p-1}}$

If $p = 2$ no. $\propto \frac{1}{V^4}$

$p = 3$ $\propto \frac{1}{2^2}$

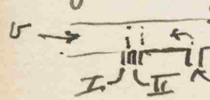
$p = 4$ $\propto \frac{1}{2^4}$

Isotopes - Dr Aston. 4/11/22.

Latest work is identification of isotopes of tin, barium & selenium. Some of these overlap in their position in the At. No. table & are termed Isobars. Thus an isobaric isotope has same no. of protons & electrons in its nucleus as the isotope of some other element & hence the same At. wt. & nuclear charge - but evidently a different configuration in space - Probably also there is a difference in At. Wt. always accompanying diff. in packing of nucleus but not gt. enough to be detected with present experimental accuracy of 1 or 2 in 1000. All isobars found so far have At. ~~weights~~ even nos.

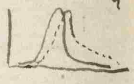
Tin shows a definite departure from the whole-no. rule supposed to hold previously & certainly holding from He to Sn. This change is 4 or 5 in 1000 & is interpreted as signifying a distinctly new mode of nuclear packing. The five isotopes of tin have a relative whole no. sequence within themselves.

G. Hertz has tried to separate isotopes of gases by diffusion - Lord Rayleigh method.



different rates of diffusion give different distances to which the 2 types of mols will penetrate against the influx of some other gas at vel. v. Then draw off isotopes I & II. This

Aston considers mechanically impossible as coeffs of diffusion are so nearly alike & the Maxwellian distribn curve wd. give maxima points so nearly overlapping that no separation is pos.



Dillon Clark + Hinchy (Dublin) claim to have separated isotopes of lead by repeated chemical reaction PbCl — Pb — PbCl < Pb, PbR ; — PbR — PbCl < Pb, PbR.
giving figures 185.83
·76
and 185.67
·62

This is not considered possible axiomatically by R. & Aston.

Replying to Sir Jos. Larmor Aston said Mme Curie has shown the % of Chlorine isotopes to be same in sample from Plutonic rock & from Sea floor & he considered there is no process in Nature to change the % or separate out the mixture i.e. that they were formed in that proportion & always will be so except when separated in a lab. i.e. the probability of formation of one type or another is a definite variable factor.

Larmor expressed his regret at breakdown of the whole-no law & the hope that it would prove to be an experimental error. Aston denied the latter possibility.

<u>Electron Capture Theory</u> Prof. A.S. Eddington
Cavendish Society 29/11/22

For giant stars.
Lanes Theory. Mass $\bar{\rho}$ av dens –
Homer Lane. radiative equilib.
m = mol. wt. often less ~~than atomic wt.~~ as the electrons are split broken off Assume m = 3 approx for the degree of ionization probable –
Then temp. & dens. & temp gradient at every pt on the star

Coeff of opacity – K.
H = flow of heat /cm²
Eqn joining K H & temp grad.
for giant stars (ρ < for air possibly)
K = 20 to 40 – cf. K for X rays.

Typical case T = 5600000°
ρ = ·068
m = 2.83
K = 23.

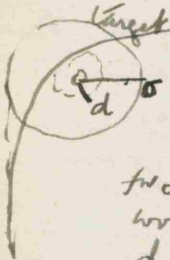
Stefans law gives amt of energy abs./sec/gm.
= 4.33 10^{22} ergs = 2.40 10^{33} quanta.
= no of electrons emitted from atom per sec.
= no " " captured per sec.
if radiative equilib. exists.

No of indept. ptcl/gm = no of electron/gm almost
i.e. no/gm = $\frac{2.0\ 10^{33}}{1.2\ 10}$
av. speed 1.47 10^4 cm/sec = 600 volts

Consider no of targets which might act as capturers.

For iron radius of target =

i.e. nucleus 10^{-12} Target is region round nucleus & 1st electron right is far out.



Target electron takes hyperbolic path.
& so radius of true target
is $d = 8 \times 10^{-13}$ cm.
& this must be radius of nucleus.

for Impact is inelastic or else electron would not be captured

$$d = \frac{H^2 a n}{2\eta\pi R c v} \quad \frac{A}{2N} \quad \frac{(mkT)^{1/2}}{\rho}.$$

Constants

At wt
A

Formula determined astronomically.

for giant stars. $T^3 \propto \rho$

$$mkT^{\frac{1}{2}} = \text{const.}$$

hence it seemed probable K = const.

for T great m is less & so changes counter act one another in the expression.

Account is taken of inc. of mass with vel. by putting for d $d + \frac{1}{2}Nb$.

Rel-ty predicts that mass of whole system remains constant probably hence above correction not essential.
~~Certainly~~ probably a value between d & $d + \frac{1}{2}Nb$ wd be the correct value to take.

Larmor's formula for radiant energy does not apply in case of stars.

E. Assumed that an electron going with this vel. wd not knock out another el — Is this justified.

Energy is ~~a~~symmetrical in ether (quantum) but also symmetrical in matter —

Sir E.R. Capture & emission of electrons from & ptcl.

Agreement of main figures better than expected when one works at STP. & the other in a giant star.

Apsidal distances differ by 100
Is it a question of nucleus at all.
E.R. thinks not. but that capture is a matter of ring orbits

A.S.E. Point is to find border line between capture & non capture.
Idea of capture at a definite ring (K ring for e.g. does not appeal — "sticky ring" not an impediment to E.R.'s fast moving electrons.

Milne. Continuous spectra in corona & nebulae probably due to capture of electrons.

A.S.E. All ~ 10^{-10} sec. absorption & emission
σ depends on nature of element etc —
d is constant.

Mr R. Fowler
18/1/23.

Quantum Thy of Spectra

Frequencies depend on atom or mol.
Intensities depend on method of excitation etc.
+ is the statistical side of the Quantum theory.

Historically - on statistical side study of Q. thy. began to explain discreps betw thy + obsn
Intensity of radn betw λ and $\lambda + d\lambda$

$$E_\lambda d\lambda = \frac{\frac{hc}{kT\lambda}}{e^{hc/kT\lambda}-1} 8\pi kT \frac{d\lambda}{\lambda^4}$$

$p = c/\lambda$ $\quad$ p = frequency

$$E_p dp = \frac{hp/kT}{e^{hp/kT}-1} \frac{8\pi kT}{c^3} p^2 dp$$

$$= \frac{8\pi h}{c^3} \frac{p^3 dp}{e^{hp/kT}-1} \quad \text{Experiment.}$$

Rayleigh Jeans Law gives $E_p dp = \frac{8\pi kT}{c^3} p^2 dp = 8\pi kT \frac{d\lambda}{\lambda^4}$ Theory

These agree for high T or low p.
otherwise they differ.

$k = 1.372 \; 10^{-16}$
$h = 6.55 \; 10^{-27}$ Planck const erg sec

Plancks explanation assumes.
jumps of energy equal to hp.
in emission or absorption
Theory is now watertight. though at pts not quite proved theoretically.

1. Problems of spe. heats of gases + solids were solved by this theory -
2. Problems of photoelectric effects came under this theory.

Light on a metal surface.
freq. p_0 if $p > p_0$ electrons are given off
" $p < p_0$ nothing happens.
the maxm vel. of emission of electrons depends only on p
$\frac{1}{2} m v^2_{max} = h(p - p_0)$
Einstein showed that if Q. thy holds the above eqn is merely law of conservn of energy -
No classical theory of radiation gives this observed fact of maxm vel. of emissn being dependent only on p.

This formula used by Mr Ellis for determining frequencies of γ rays with complete consistency.

[Threshold value p_0 in some cases of u-v light can be done only by Millikan
Experimental difficulties are so great]

Jeans says photoel. effects are best proof that Plancks quanta have a physical reality.

Conversely. Wave theory explains diffraction etc & on q. thy no explan.n is possible

No generalized thy. embodying the two has been yet forthcoming.
But C G Darwin has an idea that faith complete rels. in conserv.n of energy as a statistical truth (like 2nd law Thermodynamics)

Grst success of q. thy is its applic.n to thy of spectra.

(1) Empirical Rules of Spectra & notation
(2) Bohrs orig. theory of H spectrum.
(3) Development by Sommerfeld etc. to explain Stark & Zeeman effects
(4) Bohrs correspondence princ. (most important idea of all)
(5) Intensities of Spectral lines.
(6) X ray spectra.
(7) Band Spectra
(8) Bohr's Thy of atomic Structure.

(1) Spectra of 2 classes. Emission Spectra. & absorption "
Latter is a part of the former - & only a part. this is one gt. snag to classical theory.
another 2 classes Optical Spectra. X-ray "
has a theoretical meaning. Latter is indep.t of chem & physical properties. former is characteristic ~~also~~ of atoms in free state.
X ray Spectra are indep.t of method of excitation
Optical " may be of more than one kind depending on whether 1 electron has already been lost.
(Si. has 4 distinct optical spectra)
i.e. former depends on outside of atom valency electrons wh. give the chem props.
& latter on inside of atom.
Stimulation by impacts of electrons or by absorption of radiation (resonance spectra)

References. Sommerfeld. Atombau & Spektrallinien (easy German)
Foote & Mohler (non math.)
Bohr Theory of Spectra & Atomic Constn.
Fowler Report on Series in Line Spectra.

The whole theory is in essence due more to Bohr than Sommerfeld.

Optical emission Spectra. Line and Band
latter are really lines but so arranged that under
moderate dispersion appear like bands.
line spectra assigned to atoms.
band " " " mols.

Lines fall into series of a general type.
Iron spectrum not yet sorted out into series
Neon also very complicated system is now
nearly sorted into series by Paschen.

great range req^d Shortest line recognized
is 500 Å
Å = 10^{-8} cm. in p^t of 1S - 2P for He.
greatest for optical spectra 100000 Å

→ Simplest of all spectra is H atom
Balmer Series
in Å units:
6562.793
4861.327
4340.466
4101.738

3978.075 3889.052 3835.381 3797.900

ratios are 1 $\frac{20}{27}$ $\frac{125}{189}$ $\frac{5}{8}$
$\frac{44}{81}$ $\frac{16}{27}$ $\frac{45}{77}$ $\frac{500}{846}$

or $\frac{9}{5}$ $\frac{16}{12}$ $\frac{25}{21}$ $\frac{36}{32}$ $\frac{49}{45}$ $\frac{64}{60}$ $\frac{81}{77}$ $\frac{100}{96}$

i.e. $\frac{m^2}{m^2-4}$ where $m = 3$ for 1^st
4 " 2^nd
etc.

i.e. reciprocal wave lengths exhibited by
$\frac{1}{\text{wave length}}$ called wave numbers.

Electron Theory of Chemistry

Sir J. J. Thomson.
Cavendish Lab.
Jan 19^th 1923.

Atom - not billiard ball exterior
but holes - empty space with electrons
at pts. holding frontier.
machine gun defence strong as steel
This idea applied to chem leads naturally to Mendel table
& idea of valency - coord numbers, active atoms or mols
& partial valency theory.

Chemist Dalton lead to at. thy of matter becoming an
accepted hypothesis of science. Physicists benefited most
chem. combin^n is surface of discontinuity betw chem
& physics - the great barrier.
no mechanical theory of chem until we know
something of the machines involved - i.e. a
picture of atom & mol.

Structure of Atom
1897 discovery of electron $\frac{1}{1700}$ wt of H. atom
given out by every type of matter & always
the same ∴ the el. a part of the atom of
every element.
Counting no. of el. in atoms of each element
no. of el = at. no of the element = approx $\frac{\text{at. wt}}{2}$

In H only 1 el. ∴ look for another source of its
mass. also electrons always neg. ∴ look for
a pos. since the atoms are normally neutral.
& there must be a + charge equal to at. no.
By positive ray method find charge & mass
on these other constituents of atoms.
The carrier of the + charge depends on the atom
& has a mass just equal to mass of atom.

Hence practically all the mass is due to the pos. charge
& At. wts are all multiples of mass of H.
Hence assume H atom as the unit of mass.
Thus if there were At. Wt. no. of H units the
+ charge wd be twice the - neg charge
∴ balance this excess by mixing up half as many
electrons with the At. Wt. no. of H units +
Hence the pos. part has a structure of its own
& on it depends radio activity
The outer structure of electrons governs
the chemical properties

Detection of electrified particles very delicate,
of unelectrified particles very crude.
∴ if there are unelectrified constituents also in
the atom they have & may for a long time
escape our detection
But el. & pos. part accounts for mass
& takes us a long way with phys + chem props.

Question of arrangement & configuration.
Equilibrium under mutual repulsion & the attraction
of the + charge.
Law of force - If inv. sq. law held for all
distances this atom could not exist.
Earnshaws Theorem - no equilibrium for inv.
sq. law bodies - i.e. orbits
& an individual orbit for each electron.
Saturns rings - all in same orbit - but they
attract ∴ stability
Electrons repel ∴ every electron must
describe a different orbit
Very complicated for heavy atoms

A theory is a tool not a creed.
Choose a less cumberous tool & assume
law of force $\frac{Ee}{r^2}\left(1-\frac{c}{r}\right)$
It is arbitrary
but considering dimensions of atoms it seems
reasonable to amend old law

Build universe of (1) mass
(2) measure of elect. charge.
You cannot build 3 units out of 2.
& a full theory of matter demands
measure of length mass + time
∴ study of electrostatics not sufficient.
Hence study currents & electromagnetic
phenom & get a 3rd unit the vel.
of light.
Natural unit of length is diam of electron
i.e. 10^{-13} cm i.e. $\frac{1}{100000}$ part of size
we attribute to atoms.
This not useful. 10^{-8} comes in often & there
must be some law which involves a unit
comparable to atomic dimensions.
In the above eqn at c force changes from
+ to -.
Old classical theory provides no unit of
this order.
Quantum theory might be made to do so.

c may vary from one atom to another.
Positions of equilibrium
(2) repulsion just sends the el. beyond the dead point
of c. (3) around eq. (4) tetrahedron.
5 (6) cube (7) 5 and 2 and end of axis

8. a skew ~~square~~ cube 4 up + 4 below twisted round thro 45^0. (Oxygen)

Crisis at this point the electron are getting nearer + nearer + ∴ greater & gtr repulsion & danger of explosion since + nucleus is finite & limited

Thus 9 cannot be in stable equilib of they be distributed at equal distances from nucleus.

a + nucleus of 9 can hold 8 in inner layer but 1 is 5 or 6 times further out in beginning of a new layer.

22/1/23.

2^{nd} shell can only hold 8 el. so 17^{th} el. has to go out to form a 3^{rd} shell.

Lithium - At. no. 3. ∴ 3 electrons are available but of these 2 are bound closely (like a He) + only one is free.

Thus - Li	Be	Bo	C	N	O	Fl	Ne
1	2	3	4	5	6	7	8
Na	Mg	Al	Si	P	S	Cl	Argon

Repetition of no. in outer layer + hence of any properties depending on this no.

Hence periodic law like Mendeleefs.

Valency depends on no. in outer layer

Thus if we take away an el. from an atom we should alter its valency.

Thus a + O atom has 5 el. in its outer layer like neutral N + should have same valency.

Consider NH_3 Can we get O^+H_3. Put O + H in a discharge tube + by pos. ray method find mol. wt. of the compounds present. We find one of 19 corresponding to OH_3

Some atoms can have 2 or more up to 7 for Hg. knocked out.

Thus O^{++} is like C in its valency CH_4 hence there should be $(OH_4)^{++}$ + it is found by pos. rays.

Inert gases - 8 el - i.e. complete ring.

Take out 1 - valency like Fl. 7

FlH ∴ also $(NeH)^+$ at. wt. 21.

Ne^{++} has 6 el like O + forms $(NeH_2)^{++}$ but cannot be proved bec. there is an isotope of Ne with same at. wt.

N^+ is like C ∴ NH_4 should exist but its at. wt. is 18 like water + so cannot be proved present.

$(Na)^+$ acts inertly like Ne + their spectra should show similarities same arrangement though diff. lines due to diff nucleus.

Zeeman has compared K^+ with Argon + finds striking similarities.

Now consider the addition of an el. or of several el. to atoms not already complete with 8 in layer.

Thus Fl^- behaves like Ne.

N^- " " O.

C^- " " N.

These are more difficult to prove experimentally; spectrum method must be used.

Valency cannot alter unless no of el. in outer layer is altered.

With high nuclear charges it is poss. that an el. may be trapped inside & then or several el

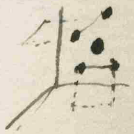
two or more consecutive elements wd. have same no. in outer layer & hence similar props & same valence.

This is the case of Fe Ni Co group
Rodium
Rare earth " "
Pt - group "

i.e. the additional el. have been trapped by an inner layer.

Alotropic modifications of atoms. One arrangement wh. is most stable i.e. P. Energy minimum. but there may be other ways involving small increase of P.E. overbalanced by some other factor.

i.e. for 8 el. the twisted cube has least P.E. but put 4 at corners of a tetrahedron + 4 more at corners of another similar but larger tetrahedron. This is stable though less so than twisted cube.

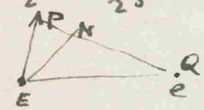


Thus if there be 2 forms with very little diff of PE the tendancy to alter from one to the other is not great - but the arrangements of el furthest out may be very different & hence different chem. props.

N 5 el. 2 poss. forms. 5 in one layer or 4 el near centre & one outside.
Rayleigh's work on "active N."
In pos. ray tube there is always found much N^+ i.e. 4 el in tetrahedron.
When it becomes neutral the newly arrived el probably takes up a posn of equilib. far outside giving this alotropic form of N.

26/1/23. Size of atom. depend on pos attrn of centre & neg. repulsion mutually. equilibrium position is termed the radius of the atom.
Consider central charge E. el. charge e.

$\frac{Ee}{r^2} - \frac{C}{r^3}$ Assume this law of force.



force on P. due to el.

$$\frac{e^2}{PQ^2}\cos EPQ.$$

$\therefore$ for equilibrium. $\frac{Ee}{r^2} - \frac{C}{r^3} = \sum \frac{e^2 \cos EPQ}{PQ^2}$

drop $\perp^r$ from E on PP.
$PQ = 2EP \sin PEN.$
$\cos EPQ = \sin PEN.$
} Insert these $\rightarrow$ $= \sum \frac{e^2}{4r^2} \frac{1}{\sin PEN}$
$= \sum \frac{e^2}{4r^2} \frac{1}{\sin \frac{1}{2} PEQ}.$

$$\therefore \frac{Ee}{r^2} - \frac{C}{r^3} = \frac{e^2}{4r^2} \sum \frac{1}{\sin \frac{1}{2} PEQ} = \frac{e^2}{4r^2} S_n$$

$$\therefore \frac{E}{r} = Ee - \frac{S_n e^2}{4} \quad \text{or} \quad \frac{C}{e^2 r} = \frac{E}{e} - \frac{S_n}{r}$$

$$\text{or } r = \frac{\frac{C}{e^2}}{\frac{E}{e} - \frac{S_n}{4}}$$

giving radius for atom with any no. of electrons

C varies for diff elements.

23/1/23. Quantum Theory. Contd.

Series $N'\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$ $n = 3, 4, \ldots$

$\frac{c}{\lambda}$ c not well known λ very well known

use $\nu = \frac{1}{\lambda}$ $\nu = N\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$

N is Rydberg's constant.

Curtis value $N = 109678.3$

Other series: $N\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$ Lyman series.

$N\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$ Paschen ..

Spark spectrum now attributed to an ionized atom.

arc " " the ordinary spectrum.

Arc spectrum given by $N\left(A - \frac{1}{(m+\mu)^2}\right)$

N = Rydberg No.

A & μ constants for element.

Spark spectrum given by $4N\left(A - \frac{1}{(m+\mu)^2}\right)$

X ray spectra (K lines) unlike optical spectra repeat themselves with Mosely progression from element to element.

Combination Principle of R ?

Empirical generalization. wave no of any line in spectrum of any element can be expressed as the difference of 2 terms. ∴ the diff of frequencies between any two lines may be expected to be the freq. of another line

Given wave nos. ν_1 ν_2 then look for $\nu_3 = \nu_1 - \nu_2$

Some exceptions but not yet known why.

Series $\nu = N\left(A - \frac{1}{(m-\mu)^2}\right)$ gives a series converging towards maximum ν at NA.

If $\nu' = N\left(A' - \frac{1}{(m'+\mu')^2}\right)$ limit NA'.

terms of series depend on $\frac{N}{(m+\mu)^2}$

4 sets of terms

(1) Sharp Terms. $mS = \frac{N}{(m+s)^2}$ (1S.

(2) Principal $mP = \frac{N}{(m+p)^2}$ (2p

(3) Diffuse $md = \frac{N}{(m+d)^2}$ (3d

(4) Fundamental (misnomer) call them by their finder Bergman $mb = \frac{N}{(m+b)^2}$ (4b

Main sets of lines.

(1) Principal Series. 1S − mp
1S is the limit.

(2) Sharp Series. 2p − mS
2p is limit. ~~2p − md~~

(3) Diffuse 2p − md

(4) Bergman 3d − mb

The normal absorption spectrum is usually the Princ. series with 1S as limit.
but in exceptional cases it may be 2) or (3)
i.e. Al. group.

1911 Rutherford's nuclear Thy. of At. structure.
1913 Bohr's Thy & Plancks Thy.

$e = 4.774 \; 10^{-10}$ ESU
$= 1.591 \; 10^{-20}$ EMU
$\frac{e}{m} = 1.769 \; 10^{7}$
$m = 0.90 \; 10^{-27}$ gms.
mass H atom $1.662 \; 10^{-24}$ gms.

no of mols in 1 gm. mol $= \frac{1.008}{1.662} 10^{24}$
$= 6.06 \cdot 10^{23}$
$h = 6.53 \; 10^{-27}$

Bohrs explanation of H atom – the simplest case.
one electron must give the Balmer Lyman & Paschen series.
No explanation can be got from classical mechanics & electrodynamics.
It is a very definite problem as λ's are known to 1 in 1,000,000 accuracy –

Bohrs assumption: (1) energy emitted is hp if frequency of radiation be p. If E_1 be energy before radiation & E_2 after
Then $p = \frac{1}{h}(E_1 - E_2)$

(2) certain stationary states where el. can exist for some indefinite time without radiating. Change from one stationary state to another is accompanied by emission (or abs.) of energy according to eqn. (1)
Thus Ritzes combn. princ. is hereby incorporated.

(3) apparently ad hoc assumption.

$$\left.\begin{aligned} \nu &= N\left(\frac{1}{m_2^2} - \frac{1}{m_1^2}\right) \\ p &= eN\left(\quad " \quad \right) \end{aligned}\right\} \text{ then } E_m = \frac{-chN}{m^2}$$

Consider el. revolving in circular orbit of rad. a. not radiating – otherwise obeying mechanical laws.

25/1/23 ω = freq. of revol.

$\therefore$ vel of el – $= 2\pi a\omega$.

$\frac{1}{2}m(2\pi)^2 a^2\omega^2 - \frac{e^2}{a} = -W$ = energy eqn.

$\frac{e^2}{a^2} = m(2\pi)^2\omega^2 a$ $\left[= \frac{mv^2}{a} \right]$ = radial accel.

$\therefore W = \frac{1}{2}\frac{e^2}{a}$

$2a = \frac{e^2}{W}$

$\omega^2 = \frac{e^2}{4\pi^2 ma^3} = \frac{2W^3}{\pi^2 me^4}$

Make this fit stationary states

Put $W = \frac{chN}{m^2}$ for stationary state.

Insert this value

$2a_m = \frac{e^2 n^2}{chN}$ and $\omega_n^2 = \frac{2c^3h^3N^3}{\pi^2 me^4 n^6}$

Fundamental idea here introduced.
Plancks radn. th. agrees with classical th. for large quantum numbers $\therefore$ this must also agree.
On classical th. for n large gives.

(1) $\frac{\omega_n}{\omega_{n+1}} \to 1$ as $n \to \infty$

(2) p_n should be identified by the frequency of el. in its orbit.

Hence these two should hold.

$\therefore p_n$ same as ω_n for n large.

$p_n = Nc\left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right) = \frac{2Nc}{n^3}$ in limit from quantum theory.

$\therefore$ put this $= \omega_n$

$\therefore$ In limit $4N^2c^2 = \frac{2c^3h^3N^3}{\pi^2 me^4}$

$\therefore N = \frac{2\pi^2 me^4}{ch^3}$ Rydberg no.

$= 2\pi^2 \frac{0.899 \; 10^{-27} \; (4.774)^4 \; 10^{40}}{3 \; 10^{10} \; (6.55)^3 \; 10^{-81}}$

$= 1.093 \; 10^5$

Observed value. $1.096783 \; 10^5$.
This is the 1st experimental check & is very hopeful

Michelson had suggested that angular momentum might be simple multiples of h.
Bohr tested this

$\omega^2 = 2\pi,$ $= \frac{2\pi me^4 n^4}{4c^1} \times$ ——————

$\omega = n\frac{h}{2\pi}$?

Hence (3) becomes Angular momentum

in each stationary state is $n\frac{h}{2\pi}$.

normal state of H atom is when W is greatest

numerical values. $2a$ = diam of atom
$= 10^{-8}$ order.

Consider nucleus with el.-ch. Ze + 1 el.

W_n = neg. energy $= \frac{chNZ^2}{n^2}$

$\nu = Z^2 N\left(\frac{1}{n^2} - \frac{1}{m^2}\right) = N\left(\frac{1}{\left(\frac{n}{2}\right)^2} - \frac{1}{\left(\frac{m}{2}\right)^2}\right)$

Series wrongly attributed to H.

ζ-puppis series (Pickering) is

$N\left(\frac{1}{4} - \frac{1}{(m+\frac{1}{2})^2}\right)$

found in helium by Fowler.

Other series $N\left(\frac{1}{(\frac{3}{2})^2} - \frac{1}{n^2}\right)$

$N\left(\frac{1}{(\frac{3}{2})^2} - \frac{1}{(n+\frac{1}{2})^2}\right)$.

Bohr said $Z = 2$ in above series

∴ they are due to He ionized

This is now accepted & was the 2nd great confirmation of Bohrs assumptions.

Bohrs theory then suggested that N wd differ slightly with diff. elements of finite mass.

This was found.

Consider. M, Ze. nucleus. m e.

Consider c.g. of system fixed.

r_2 r_1 M c.g. electron $(mr_1^2 + Mr_2^2)\dot\theta = H$. const.

$\frac{1}{2}m(\dot r_1^2 + r_1^2\dot\theta^2)$ = KE. of el.

$r_1\ r_2$ radii vectores changing.

$+ \frac{1}{2}M(\dot r_2^2 + r_2^2\dot\theta^2)$ KE of nuc.

$= \frac{Ze^2}{r_1+r_2} + E_{(const.)}$

Put $r = r_1 + r_2$

$r_1 = \frac{M}{M+m}r \qquad r_2 = \frac{m}{M+m}r$.

Put $\frac{1}{\mu} = \frac{1}{m} + \frac{1}{M}$

$$\boxed{\begin{aligned} \therefore\ & \mu r^2\dot\theta = H \\ & \tfrac{1}{2}\mu(\dot r^2 + r^2\dot\theta^2) = \frac{Ze^2}{r} + E. \end{aligned}}$$

i.e. General case – for elliptic orbits etc. gives formula as though nucleus were fixed.

Return to circular orbits: $N = \frac{2\pi^2\mu e^4}{ch^3}$
formula & generalize it

$\therefore N_{(for\ H)} = \frac{2\pi^2 m M_H e^4}{(m+M_H)ch^3}$.

$N_{He} = \frac{2\pi^2 m M_{He} e^4 (4)}{(\quad)}$ ← Z^2

$\therefore \frac{N_{He}}{N_H} = \frac{4M_{He}(M_H+m)}{M_H(M_{He}+m)}$

$= 4(1.000407)$ not 4 exactly

Observed values Curtis $N_H = 109678.3$ Ratio 1.000406
$\frac{1}{4}N_{He} = 109722.9$

This suggested Moseley's work on inside electrons. K electron acts like the H electron & a similar formula should hold.

Try to connect Bohr's 2nd assumption with Planck's oscillator.

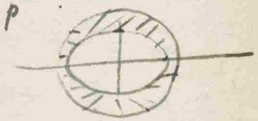
S.H. motion of an el. along a line $\xleftarrow{x}$

Freq. ν. $\left.\begin{matrix}KE\\PE\end{matrix}\right\}$ $\frac{1}{2}m\dot{x}^2 + \frac{1}{2}\,\overline{2\pi\nu}^2 x^2 = W$.

$x = A\cos(2\pi\nu t + \varepsilon)$

momentum $p = m\dot{x}$

$W = \frac{1}{2}m(2\pi\nu)^2 x^2 + \frac{1}{2m}p^2$.



p & x variables

gives an ellipse

axis of p : $\sqrt{2mW}$

" " x $\sqrt{\frac{2W}{m}}$

area $= \pi$(semi axes) $= W/\nu$

$W = nh\nu$ ∴ areas cut off between stationary states

area $= nh$.

Area of curve $= \int_G p\,dx = \oint p\,dx$ new notation.

∴ $\oint p\,dx = nh$.

J.A.T. contd.

for Li. only one free el. $\frac{E}{e} = 1$ & S_n vanishes

∴ $r_{Li} = \frac{C_{Li}}{E^2}$

for Be. 2 el. $\sin 90 = 1$ ∴ $S_n = 1$. $\frac{E}{e} = 2$.

$r_{Be} = \frac{C_{Be}}{2 - \frac{1}{4}}$

Element.	$\frac{E}{e}$	S_n	radius
Li	1	0	C_{Li}/E^2
Be	2	1	$\frac{4}{7}\,C_{Be}/E^2$
Bo	3	2.3	$\frac{4}{9.7}\,C_{Bo}/E^2$
C.	4	3.66	$\frac{4}{11.35}\,C_C/E^2$
N	5	5.2	$\frac{4}{14.8}\,C_N/E^2$
~~Fl~~ O	6	6.68	$\frac{4}{17.32}\,C_O/E^2$
Fl	7	8.08	$\frac{4}{19.92}\,C_{Fl}/E^2$
Ne	8	10.1	$\frac{4}{31.9}\,C_{Ne}/E^2$

Let $C = aN + b$ N = at. wt. a b constants. Substitute values.

Li	$7a + b$
Be	$5..a + .47b$
Bo	$4.52a + .412b$
C.	$3.88a + .322b$
N	$3.78a + .2702b$
O	$3.69a + .23b$
Fl	$3.7a + .21b$
Ne	$3.63a + .191b$

See diagram of at. vol. in festoons with at. wt

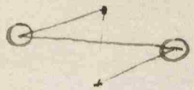
min. for at. vol. comes in middle of period. in most text books -

This list differs going down to minimum at end of period.

W.H.Bragg's method of determining radii 

Molecules –

1 el.



Mutual attraction & repulsion can balance for certain dimensions
chemists say there is a bond
physical interpretation in 2 el symmetrically placed –

2 el per atom

4 el at corners of a sq. ⊥r to plane of paper –
Chemists "double bond" not very stable

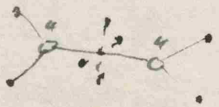
or

More stable but the end electrons will attract other mols making a close compact solid mass –

3 electron per atom

tendency to form a solid as before –

4 el



Carbon – Strong tendency to aggregate.

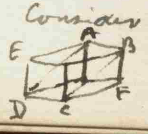
Other elements in this period are gases – why?
Return later to N

Consider O. 12 electrons in 3 sets of 4 el at corners of sqs. with 2 nuclei between – i.e. 2 cubes together with a nucleus at centre of each. i.e. 2 atoms of O gain stability by economy of el. as of walls in 2 semidetached houses – & are completed octets ∴ no tendency to further union ∴ gaseous.

Consider Fl. 14 el. 2 cubes with an edge in common – not a face. Thus 2 complete octets self-sufficient ∴ gaseous.

Consider Ne. each atom has 8 & is self contained & is a monatomic gas –

Consider N 2 atoms have 10 el. 2 octets impos.



Cube AC with diagonal planes ABCD having

2 N nuclei on either side of its middle pt. & 2 el. in plane ⊥r to line joining nuclei & ~~...~~ & 8 el. at corners of cube – Self contained ∴ gaseous –

Scattered light from a asymmetrical system of el. seen thro nickel is plane polarized ∴ an extinguishing the more asymmetry the less approach of the minimum pt. to extinction.
In N min. is lower than in O. i.e. O mol. less symmetrical i.e. ellipsoidal shape of mol. is more elongated than shape of N mol. wh. is more spherical.

29/1/23. Extension to Chem. Compounds.

Fl. 7 el ∴ add 1 H to get a stable compd.
O 6 " ∴ 2 H " " "
N 5 " ∴ 3 " " "
C 4 4 "

i.e. valence of Fl is 1, O 2, N 3 C 4 etc.
i.e. valency = 8 – no. of el. in outer ring.

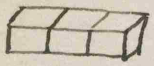
Consider O & Fl. each of the 6 el of O can bind one Fl atom – In this sense its valency is 6 i.e. making up octets for other atoms not for itself. here valency = no of el.
Thus 2 kinds of valency v_1 v_2
where $v_1 + v_2 = 8$.
N has valency 3 and 5.
Similarly N & P and Fl & Cl go together in Table & we expect to get compd. PCl_5.

CH_4 = H—C—H (H above and below C)   i.e. 8 el. around a C nucleus whose + charge is 4.
this wd not be stable unless the H nuclei each exerted necess. attrn on its own el.

The alkali metals monovalent have not the same intense attractive force on their one excess el. & ∴ cannot replace H in the compd CH_4 as each atom could not pull sufficiently to make a stable system.

Chem. saturation means complete octet formation

Consider CO_2 no of el. to be considered is sum of + valencies = 4 + 2(6) = 16 and 3 nuclei. Can we form complete octets.



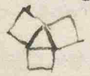
Yes. 3 with 2 pairs of common faces.

Consider skew cube having sq. faces & Δr faces. latter allow for a sharing of 3 el. if desirable.

Thus ClO_2 no of el = 7 + 2(6) = 19
3 skewed cubes with one common Δr face & one common edge. ∴ 5 saved 24 - 5 = 19.

SO_2 no of el. 18   3 cubes 1 face + 1 edge representing O = S – O

or   3 edges common ring molecule



S
O—O

Possible instability of these arrangements of octets

Thus a string of C's with cube faces in common is unstable. (organic chem. proves this 5 C's in string is maximum).

Similarly chain with edge of oxygen atoms in common is unstable.

a chain of CH_2's is stable. (70 in chain has been made)

It is pos. to +ly electrify a mol of complete octets whereas not pos. to negatively electrify. except in special cases.

e.g. O mol. is found neg. charged by making them have only an edge in common hence 2 more el. can be added.

Hence where there are "double bonds" it is pos. to change to single bond & negatively electrify.

2 Cl atoms with edge in common.
Replace contact along edge by contact at one corner ∴ 1 el. more can be attached. & mol. thus negatively charged.
Thus centres further away & ∴ less stable & probability of separation into 2 atoms one neutral & one neg. charged.

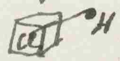
Fire an electron thro N or argon & it finds no home & retains its high vel.
If a fraction of a % of O be pres. the el. finds a place & loses its high vel. & charges the O atom or O mol. wh. it has divided into a "one bond" mol.

2/1/23.

Mechanism of Chem. Combⁿ.

Certain mols termed polar

Consider O mol. two cubes common face. the C.G. of + & - units coincide ∴ non polar & the resulting external field is very small.

Consider HCl.  centre of + el. not coinciding with centre of -

i.e. 2 charges equal & opp & not coincident

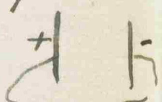
They act like a magnet. + ····· -

Hence external field gtr. than for O_2 & hence attraction (or repulsion) is gtr.

Vital distinction - polar type mols - & non polar mols

To detect measure spc inductive capacity of the substance in gaseous state

If polar mols. in an el. field they set themselves like small magnets.

This gives a great sp. ind cap. 

But in non polar mols (O_2)

& now the el. are shifted slightly also nuclei due to the El. field making the mols somewhat polar.

But kinetic thy of gases - i.e. high temp, high vel, more collisions ∴ less induction possible in these ~~non~~ polar mols at higher temps. while for non polars change is very small.

1st test. spec. ind. cap.

2nd abnormal decrease in above for polar mols

Sp. I.C of H_2O is very high 80.

Rule S.I.C. $\propto$ (refractive index)^2 . $K = \mu^2$

water wd. give 2 but is 40 times gtr.

Renecke found this true for gaseous H_2O also.

K being $= 40\,\mu^2$

for Oxygen $K = \mu^2$ and for most other gases

~~But~~ other exceptions is NH_3 Acetic & Formic Acids

Methyl & Ethyl Alcohol.

Chloroform $CHCl_3$.

These are all polar mols -

They dissociate salts

" have high surface tension

" negatively electrify gases passed bubbling thro them, liquid being pos.

gas bubbled thro non polar liquid eg. paraffin oil, no electrification

Why is water polar. If. 

But try skew cube. symmetrical

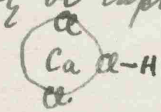
no symmetry possible from corners. + c.g.s coincident.

But If the 2 H's were one above & one below the square faces of skew cube symmetry only pos if they are at dist from nearest electron $> \frac{1}{2}$ diagonal of face & if this be > than attractive forces require too much energy is reqd. to maintain this symmetrical shape.

Thus theoretically a symmetrical non polar mol H_2O is possible though probably unstable — (never found experimentally.)

Consider $CaCl_2$. Saturated by every available el of the electro pos. atom gone to satisfy the needs of the el-neg. atoms.
∴ the Ca cannot attract another Cl
but if another Cl gets charged up elsewhere the Ca may be able to hold it in stable equil.
The former are the valency compounds
Latter is $\underset{++}{Ca}\,\underset{---}{Cl_3}$ negatively charged mol. existing as an ion or in a discharge tube.
Thus a polar mol may be captured

$Ca \cdot Cl_2 \cdot HCl$

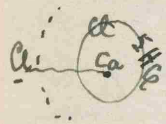


all held by attraction of Ca. Forces at work are identical to those in valency compounds. (Terms 2ndary valency or auxiliary valency are misleading.)

This addition of polar mols cannot go on indefinitely as there is a limit to the no. of systems wh. can be held in equilib by the +charged atom. Analogous to limit to el. in ring of atom.

This limit (not same as valency no.) is called the coordination number.

Consider Ca. Coord. no. is 6. Introduce 7 Cl atoms. break up into 2 rings as does atom with more than 8 el.



$CaCl_2 \cdot 5HCl$. One Cl^- goes to an outer ring & is easily detached & goes off as an ion.

i.e. the polar atoms are attracted to the $CaCl_2$ & force one Cl^- out.

Determination of Coord No.
If an octet (cubic) turns a corner to the + nucleus. it contributes 1 el. to inner layers. if an edge 2 if a side 4.
Thus if a corner there can be 8
" " edge 4
" " face 2
& therefore the coord. nos cannot be less than 2 or more than 8.
There are cases of 4 & 8 but majority are 6 i.e. 2 edges & 4 corners.

Thermal energy not sufficient to account for separation of atoms 5 volts reqd.
Room temp gives $\frac{1}{30}$ volt

5/2/23 Consider $Pt\,Cl_4$
This attracts polar molecules (espec. NH_3)
$Pt\,Cl_4$ by itself is not an electrolyte.
add NH_3. & get $Pt\,Cl_4(NH_3)_2$ not an electrolyte – Coord. no. not yet exceeded.
More NH_3 $Pt.Cl_4(NH_3)_3$
Pt has coord no. 6 here are 7 so one Cl is forced out.
& this salt is an electrolyte Cl —— $Pt(NH_3)_3Cl_3$

More NH_3 $Pt\,Cl_4(NH_3)_4$ 2 Cls driven out.
This salt is an electrolyte & still better conductor
Similarly $Pt\,Cl_4(NH_3)_4$ 3 Cl's out.
electrolyte & better condr than others.

Similarly add polar H_2O mol to HCl & the original compd is forced apart.

Chem combn presupposes an expenditure of energy to separate the atoms in the mol – making their Pot. energy lower

Analogy of water thro syphon – initial work to raise water to top of syphon then pot. energy tends to decrease & water flows of itself

Catalytic energy has this power. Expends energy to get the action started & is recuperated afterwards –

Baker & Dickson on influence of presence of H_2O.
Gases HCl + NH_3 ~~H_2SO_4~~ if water present combine immediately – if perfectly dry no combn.

also gases CO + O mix & send a spark thro & Explosion follows.
Dickson dried them & sent a spark thro & no combn followed.

Note that the atoms need not be free but so lightly bound that they easily rearrange themselves
Delicate method of detection of free atoms & in gt majority there is no conductivity in the gas ∴ no free atoms
One interesting exception Pot. Iodide
This vapour conducts elect easily $\frac{K}{I}\frac{+}{-}$
Dry it very thoroughly & there is no conduction – hence influence of the H_2O polar mol in producing ionization
Baker found pres. of water necessary also to the decomposition of Sal amoniac.

casual drying of a gas is ineffective to prevent its chemical activity due to K. Thy. of Gases data on frequency of collisions.

Suppose only enough H_2O vapour to give one to 1 in 10^4 HCl mols.

100,000,000 col. per sec. 1 in 10000 is of any use ∴ only $\frac{10^8}{10^4}$ are effective.

i.e. 10^4 per sec. mols are rendered capable of combn which is fast enough to seem instantaneous

All this action due to polar mols can be attained by artificially electrified systems such as an ionized gas due to X ray passage.

Eg. Take dry HCl + NH_3 & expose to X rays & find very little decrease of combn.

In normal gas STP. 2.8 10^{19} mols/cc.
Suppose water vap present 1 in 10^6
there would be 2.8 10^{13} mols water/cc
X rays produce 10^{10} ions/cc.
∴ 2.8 10^{23} are pres.
i.e. 3000 times more than 1st case.

Polar mols produce a concentration of the gas round themselves ∴ more collisions ∴ more combn on that account.

Action of minute drops of water is similar & keeps the HCl (say) held close with increased chance for chem. combn.

Certain substances have similar effect. metals - a surface or colloidal ptcls of a metal aid chem. reaction.

Thus vel. of chem reaction often varies with state of walls of the containing vessel.

Note Chem. Combn can in certain cases take place without aid of polar mols -

analogy between electrons in octets about nucleus & mols about mols.

eg. the chlorides mono chloride like atom with one el. di Chloride like 2 el.
tri " " 3 el.

Note Li. Bo etc. are solids
N. O Fl. " " gases by previous arguments. apply to these mol. combns.
Expect the Chlorides with few Cl's solid & those with high nos of Cl atoms gases.
This is actually so. All mono, di & tri

Chlorides are solids - But high ones with
5 or 6 Cl atoms are gases or very
volatile liquids.

Most atoms with few electrons are metallic
or at least good conductors of el.
The electrons carry the electricity.

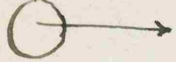
In mols. it is diff. not el. carriers
but it is an atom has the charge & cannot
move about.
∴ not metallic conduction but
liquify so atom will move & electrolysis
is the result.

By analogy mono. di & tri chlorides
wd be conductors when fused or
in liquid
& conversely for high Cl mols.
This is the case
Alkali salts are good conductors.
Others (e.g. $SnCl_4$) non conductors.
if 4 or more Cl's no conduction.

Similar considerations for H & less
sharply marked for other substances.

9/2/23. Surface of a solid or liquid is the seat of
el. forces since the mols are not coordinately
saturated
Air bubbled thro pure H_2O or other polar mol liquid
it comes out neg. electrified
but no effect thro other liquids like petroleum.

Bubble of air in H_2O under el. field moves.
Similarly particles of metal in suspension.
Majority move as though they were neg. elect.
& the water +.
Colloidal solⁿ of ferric hydroxyde acts oppositely
Helmholtz theory. two layers + & −



of motion in dirⁿ. of el field
+ inside − outside

Then particle was pushed & water pulled when
field was applied. Rel. motion is ∴ assumed
possible. $v = \dfrac{X\Phi}{4\eta}$ X = El. force
Φ = diff of potl.
Thus if v be measured η = viscosity of liquid.
the P.D. can be calculated
& if P.D is constant v is indepᵗ of the
size of the particle.

	P.D.	
Lycopodium	− ·035 volts.	moves as though neg.
Quartz.	− ·042	
Air	− ·056	
Arsenious Sulphide	− ·031	
Gold	− ·030	
Ag.	− ·033	
Bi	+ ·015	moves as tho pos.

note that the P.D. is fairly constant $\frac{1}{30}$ volt. This corresponds to energy of thermal agitation at ord temps.
Hence work reqd to separate one neg. charged ptcl from that layer is of order of energy available by thermal agitation.
Not a mere coincidence
It allows the layers to move relatively.

Hence partial ionization at surfaces is established.
Recall -
$PtCl_4$ add polar mols - [ ] $\begin{matrix} Cl \\ Cl \end{matrix}$
Pt acts like +ion + the 2 Cls are pushed out.

Ferric hydroxide in water + the hydroxyl OH goes into the water + the ferric part has the + charge.
In this case the colloidal ptcl has the ~~neg~~ pos charge + the water the negative.

$Pt\begin{matrix} OH\ H^+ \\ OH\ H_+ \end{matrix}$ Pt + water compound.
the Pt now has the neg charge + the H's are in the water + pos -
This is the most frequent type - water pos + particle neg.

Is vel. influenced by type of gas - i.e. O. or H or steam i.e. diff state of same substance

H_2O mols are polar + also a surface is polar since evidence that more neg ends "stick upwards"
Thus a P.D. might be established betw. a steam bubble + water / Thus quasi-chemical action may take place resulting in ozone for example - (always present in waterfalls.
Electrified
(Soap bubble) film suddenly bursts + contracts + every + cannot pick up a neg - i.e. 1 in 40,000 or so negatives escape -
actual measurement possible.
Introduce some free ions - they reduce the ptl. hence vel - is slowed up if not reversed by addition of salts in the water

To bring about chem. combn requires some initial energy to separate atoms
One way - Loss of p.e. by polar mols -
2nd " where no polar mols are present or seem to be reqd.
note except inert gases all atoms tend to combine but some mols are also chemically active.
Consider O mol. O_2 2 octets sharing a face i.e. 12 electrons. But if edge on there are 2 in common + 7 round each nucleus - This

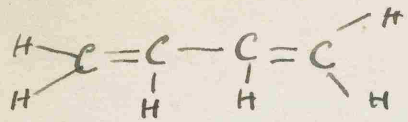
leaves the mol unsaturated & ∴ chemically active though less so than 2 isolated atoms.
Less work reqd to rearrange the el. in the atoms in the mol

12/2/23. In pos. ray work it is seen that the O mol can carry a – charge ?
i.e. in its chemically state with only an edge in common.
In a mol. usually there is no room for an extra el. ∴ rarely get a –ly charged mol.
Only in double-bond mols is it pos. –
Occasionally the H mol is formed –ly charged.
i.e. H mol + – – + becomes – + – +
& is chemically active theoretically though there is no chem. evidence of it –

Mixture of H & O at 600° temp explodes & chem. combn takes place
Dissociation of O mol into atoms only takes place at 1700
(Dissociation into atoms can be detected bec. increase in no. of independent systems means increase in pressure

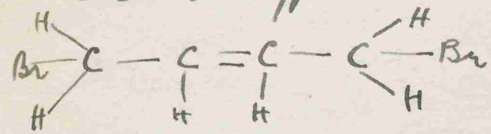
Similarly Cl_2 with edge in common if only a corner in common there is 1 el short or one octet of the mol is chemically active – Cl_2^- found in pos. ray plate.

Partial Valencies – Thiele's Theory –

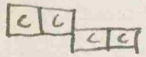


Single bond betw central C's & double to ends –

Attack this with Bromine & only the end C's are affected –

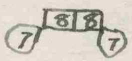


Consider original compd & distribn of electrons
= means side in common –
central bond – means edge in common –



when made chemically active the 2 end ones swing out to posn of common edge.
septets.

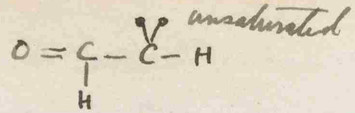
By symmetry & for maxm energy restored the 2 middle ones revert swing into face in common posn.



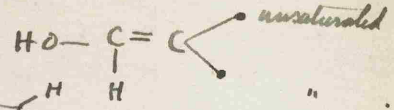
hence Br atoms ~~attack~~ to septets at ends.

Thieles Thy says a double bond is never perfectly saturated ∴ a residue of chem affinity.

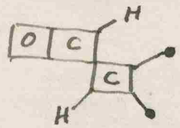
Consider



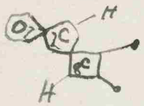
may become



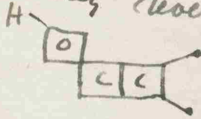
In first case



If this is made active

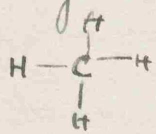


The O septet is now active & takes the H atom from the C octet & completes its octet leaving a septet C edge on to the other C septet & they revert to octets with common face.

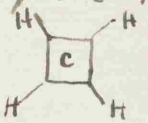


This transference of the H to the O only takes place when some of the atoms onto the C are electro negative.

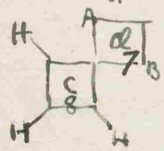
Marsh gas. CH_4



Substitute a Cl for one of the H.



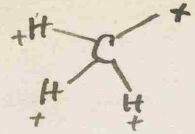
replace by Cl.



Cl + charge 6 + 6 more electrons.

the 4 el. at pts A, B cancel 4 pos at centre leaving +2 at centre and −2 at far end an electric doublet with the + nearest the Carbon & the − outside.

This gives



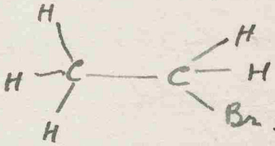
Electric doublet resulting from the Cl. This repels the + H's i.e. the 3 H's are now more easily detached

Hence substitution of an el. neg. atom in the compound increases the chances of detaching the remaining H atoms.

This also increases the concentration of neg. ions round the other H atoms of the mol or in a region of ions.

hence replacement of H is more easy.

Consider



This Br renders probability ghi for another H to be replaced on same side & not a symmetrical replacement.

$H_3C\,CH_2Br$ becomes $H_3C\,CHBr_2$

rather than $Br\,H_2C\,CH_2Br$.

Another rule regarding Oxidation

Further oxidation more likely on those molecules — Carbon compounds — already oxidized to some extent.

16/2/23. — not present —

19/2/23. Variable Valency.

Tungsten (e.g.) can form various Cl compds

Discontinuity in chem props in table after we pass Ca.

Li to Neon.
Na to Argon perfect regularity
in changes in props.

Atomic wts are a multiple of 4 or 4n + 3 - except. Berillium & Nitrogen.

Beyond these lighter elements this law does not hold.
For these lighter ones no Variable Valency is found.

From Ca upwards -

Sc. Ti. Va Cr. Mn Fe Ni Co. Cu Zn etc
no well marked sequence in valency
much variable valency amongst elements whose properties are closely similar
magnetic metals.
Exceptional props

Consider structure of an atom whose + charge at centre is great (say 16 or more) then it is poss. for more than 8 to be held in inner layers -

Hence addn of a new el. means an internal readjustment without change of outer ring & hence no great change in chem props - until inner layers are saturated under existing pos. charge & then further els. go to outer layer & progression of chem props recommences.

In one element several groupings might be almost equally stable with ±1 in outer layer & ∓1 in inner then valency wd. be v ±1 for that element

Cause of magnetic props uncertain
Various hypotheses to explain mag. forces.
(1) Ring electron
2) Relation betw. el. force & mag. force.
when law of force has inverse cube form.

Mag. forces depend on symmetry of distrib of electrons. if perfectly " - then no para mag. props. but if non symmetr

then is a resultant moment & the atom
behaves as if it were a magnet.
This unsymmetrical distribn is found
in this same group where nos in the
layers are changing ∴ probability
is that a mag. moment will exist.

Consider Fe as typical.
Ferric salts have mag. props depending only
on wt. of Fe in the compound
Ferrous salts - same is true but a
gm - of iron in ferrous salt will not
produce as much mag. effect as a gm
in the ferric salt.

Consider Ferrocyanide - latter overcomes
former & is on neg. side & compound
is diamagnetic
Ammonium salts of Co have former on
pos. side yet they are dia mag.

Hence mag. props may be said to
depend on method of combination.

Mag. props. depend not only on strength
of mag but on strength of field in
wh. mag is held. If latter forces are
very strong mag. force of atom
is not great i.e. no response by
the mag. to an externally applied
field

$K_4(Fe\,C_6N_6)$ has 6CN in inner layer
& the mag. force is very small & is
swamped by the diamag. force present in
every atom.

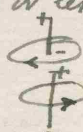
Consider O
strongly magnetic as an element but only
one compound, NO, is magnetic.

Explanation cannot be based on instability of
inner layer.  O mol.
Suppose the electrons to rotate (about long
axis of symmetry) in opp. dirns + − +
excess + ∴ magnetic.

But CO_2 octets + − + − Resultant
is zero.

Also C_2H_4 non mag. bec. arrangement
is such that the nuclei of H wd. have
to rotate too & not enough energy to
set this going. ∴ no rotation.

22/2/23. Diamagnetism

A prop. in all elements in more or less degree.
Consider 2 circuits. 
Induced cur. in 2nd is
opp. in dirn. to the cur. in primary circuit
Cur. is equiv. to a magnet thro centre of circuit
& ⊥r to its plane. Mag. field around

primary is diminished by presence of 2ndary.
This is like diamagnetism
A permanent mag. brought up near primary
in same sense increases field
This is like para magnetism.

If no secondary circuit but an electron
the field of the primary sets the electron
moving in a dirn similar to the cur.
in the circuit.

∴ a mag. field sets the el. in an atom
rotating in a dirn opp. tending to
make field a minimum
∴ diamagnetic.

---

vel. of el is $\frac{1}{2}\frac{He}{m}$
in a circle
Force is like that of a mag. at centre
with mag. moment is $\frac{1}{2} e v \omega$.
ω = ang. vel
$v = r\omega$.
mom. $\frac{1}{2} e r^2 \omega$.
" $\frac{1}{4} e^2 r^2 H/m$
for several atoms at diff radii
$\frac{1}{4}\frac{e^2}{m} H \Sigma r^2$
Mom per unit vol $= n(\ ") = kH$
K = const of mol. diamagnetism

$\therefore K = \frac{1}{4}\frac{e^2}{m}\Sigma r^2\, n$.

Div. by n to get value for each atom.

Thus knowing K from mag. obs.
calculate Σr^2.

Consider Cl. with 8 el. about it
∴ find dist of electrons from nucleus.
i.e. size of atom

Pascal has tabulated these
O is an exception in mag. props,
& comparison with Braggs value
shows same order in all cases & something
identical values of diameter.

	Diamagnetic Props.	Bragg's Crystal Method
H.	$1.05\ 10^{-8}$	$1.35\ 10^{-8}$
Cl.	$2.0\ 10^{-8}$	$2.1\ 10^{-8}$
Br.	2.4	2.38
I.	3.0	2.8

This method can be applied to find changes
in posn of el. during chem-reactions.

Light in chem-reactions.

2 types of chem-reactn (1) no change of
energy - i.e. no transference of el.
from one atom to another
2 HCl H loses its el - rearrangement
only -

This is the type embracing Oxidation & Reduction. In latter case El. neg. loses an el & el pos. atom gains one. & v. v. for oxidation.

Consider H & I

HI means H has lost its el. to the I this is oxidation (in strictest sense the O with 6 el gains 2 to form its octet) When HI dissociates the I loses one el & the H gains one — this is Reduction i.e. an el. falls into electro pos. atom.

When an el. falls into an atom light is emitted. i.e. this is the source of all light. This emission of light not necessarily within visible range or of intensity gt enough to effect eye.

Extreme case is chem action producing a flame

Phosphorescence of Sulphur

Light of glow worm due to oxidation no glow if no O. more glow if more O or O_2. This is the most remarkably efficient source of light known for it is all in vis. spectrum.

Animalculae in sea water light due to O in water.

fresh surfaces of metals like Na glow for a time — i.e. during oxidation.

The alkali metals are the most photo-electric i.e. electrons given off due to exposure to light. It is said that Rb is as sensitive to light as human eye. i.e. detect a candle at some hundred yards by el. emission.

If these metals be kept in dark absolutely for some weeks & then introduce a gas & some el. are given off. This is the emission of light due to chem action with O, Br, Cl etc gas. But its intensity is great at atomic distances though not gt. enough to affect eye.

The el. falls into the El. neg. element ∴ the light is due to the vibration of the el. neg. atom & on the posn it occupies as it might vibrate differently if free.

Thus. in H_2O the O is el. neg. & the spectrum obtained is that of O not of H.

H lines never got thus in oxidation not even in oxy hydrogen flame.

If there were Reduction — i.e. dissocn of H_2O into H + O the H lines are seen.

The frequency of the vibration by
Quantum law $\propto$ energy given off to get el. into atom
= work reqd. to remove el. from atom.
= ionizing potential.

One of great cases of reduction is in electrolytic cell – H given off at neg. pole. Cur. is carried by + charged H atoms. & an el. falls in & H normal atom is freed – no visible emission nor readily detected invisible radiation – why?
Because the H atom has some water mols attached & they being polar put their neg. ends nearest the +ion when an el. falls in at neg pole the neg. ends of polar water mols repel the el. & ∴ there is less energy to spare & it slowly makes its way in with nothing to spare.

Frequency of vibration $\propto$ this excess energy which is very small
∴ very little frequency, i.e. great wave length & far off in infra red.
∴ why expect any luminosity.
Q.E.D.

26/2/23. Solids.

Consider crystals – symmetrical distribn of atomic nuclei + electrons.
(1) The distribn must fill space.
Simplest case – cube or ~~||~~epipedon.
Next " " rhomododecahedron.
made up from a cube with ~~its~~ 8 cubic corners & 6 other corns out from each face.
14 corners – 4 faces meeting at 6 corns.
3 " " " 8 ".
Cubo octohedron – series of sq. & hexagonal got by planing off corners of faces former –

Case of 1 atom nucleus & 1 el – simplest cubic structure since each corner of any one cube is the meeting pt of 8 cubes –
i.e. consider each unit as having +1 at centre & $-\frac{1}{8}$ at each corner.
thus a neutral system.

The atoms give a lattice system in rectangular coords. The el. " " " " "
" ||le but shifted over midway – & equi spaced.

Assume atoms fixed (very heavy) & the el.

if displaced vibrate - in finite degree of freedom.

Fastest poss. vibration ~~motion~~ of el. is motion of whole el. lattice rel. to atom -

Atoms pull them back -

Calc. period of this vibration

Phil Mag. Apr. 1922.

Wave length of light 10^{-5}, dist between el in lattice is 10^{-8}.

$\therefore$ a light wave will displace very many el. in same manner.

frequency given by mp^2

$= 384 \times 8e^2 \Delta/m \; c/d$

c = dist where force changes from attrⁿ to repulsion

d = dist betw el.

i.e. there is a limit to the freq. of a wave wh. will pass thro a body.

Photo-el. effect

(1) emission gtr the shorter the λ of incident ray.

(2) Selective effect reaching a maximum at diff. values of λ for diff subs.

Max effect when resonance betw incident beam & electrons in solid.

mass = (at. wt) $\times 1.64 \times 10^{-24}$.

Vibration is slow & $\therefore$ recovery slow & may be unstable if the displacement is alternately up & down -

This could only be done by very short waves - of λ = dist betw 2 el -

Consider compression altering size not shape.

As it is in equilib $\therefore$ P.E. is a min

$\therefore$ any change means increase of P.E.

To get P.E. V.

pts P, Q e_p e_q charges. pq = dist.

$$V = \frac{1}{2} \sum \frac{e_p e_q}{pq}$$

Attempt to calculate V for each & each el leads to 2 divergent series but consider units +1 and $8 \times (-\frac{1}{8})$

Calc. P.E. with regard to 1 el.

It forms part of 8 units.

(1 with regard only to those 8 with their el & atoms $V = \frac{1}{2} \; 3.53 \; \frac{e^2}{d}$

d = side of a cube.

For other cubes out beyond only 3 or 4 % increase. 3.53 becomes 3.65.

11/3/22.

Compressibility $= C\left(\frac{\Delta}{M}\right)^4$

Δ = density.
M = At. wt.
C = const. depending on valency &c.

Gtst value of $\frac{\Delta}{M}$ is for diamond.
for Si. it is less.

C. has very small at. vol.
Si " " large " "
& yet compressibility is very similar.

Atoms of C not surrounded by octets of electrons but Si is. ∴ more work reqd to push two Si atoms near together. ∴ Si ~~not~~ more rigid than its high at. vol. wd. lead one to think.

Similarly Au has about 100 el. about its centre & ∴ much work reqd to compress.

Consider a salt. NaCl.
Electrons not in same regular lattice as in case of an element.

Na_+ Cl Here the octet is about the Cl in rock salt.

Na $\boxed{Cl}$ from Bragg's work.

$\boxed{Cl}$ Na

On compression work reqd. to alter distances of electrons.

Compressibility of rock salt measured by Richards. & is greater than on theory of just Cl_- and Na_+ alternately.

Type of Conductor

1. In case of elements – metals say – the lattice system was $\begin{matrix} - & - & - & - & - \\ + & + & + & + & + \\ - & - & - & - & - \end{matrix}$

Condn. of electricity means a movement along the lines of electrons – no motion of the atoms. Metallic Condn.

2. In the case of the salt structure the el. are held to the Cl. atom – not free to move by themselves.
∴ condn. not easy. but in Soln.

where atoms can move freely – then the Na's go one way + the Cl with octet of electrons goes other way –
Thus the Conduction is electrolytic + carried by atoms.

3. Units do not carry charge – Consider liquified O gas – Molecule O_2 is 2 octets with common face –
In liquid these systems are pushed closer + closer together until they adjoin. Individual unit is O_2 wh. is electrically neutral.
This does not correspond to 1 or 2 + even when liquid is a non conductor.

1. Metallic type
2. Salt ~~atoms~~ type –
3. Insulator whether solid or liquid.

3. Embraces substances wh. can form neutral mols –
N. O. Fl

Type 3 are nearly all volatile as wd be expected.

Specific Heat.

In K. Thy of Gases.
av. K.E. of any system ∝ Temp + does not vary for diff. masses –

Sp. Heat seems only adequate for atoms ignoring all the electrons –
Consider systems with definite time of vibration
Take •—× for rise of temp 1°
twice the energy reqd as for an atom without an el. on K. Thy.
This is true for very high temps but for low temps they act as one system + Spc. heat is as though for atom only –
Relations to period of vibration of el. about atom is the temp at wh. they begin to behave as separate units.
i.e. $R\theta > h\nu$.

Ordinarily vibration is very high
∴ θ very high ∴ electrons do not take up energy as does the atom + Spc. heat depends on

latter only –

One way of displacing electrons so as to have a very slow period – i.e. all electrons in one lattice move together relative to rest of lattice as though the chain were a rigid body – then θ is low. & the chain absorbs energy & adds to Spec. heat – the fraction $\frac{1}{n}$ if n be no of el. in chain – 10 000 perhaps acting like one unit.

Phil Mag. 1922 Oct.

Condn of El. in metals at low temps

Astronomy (in 6 lectures by Prof. H.A.S. Gillam)
Feb. 1924.

Prehistoric history.

B.C. 500 - Anaxagoras - Sun a flaming ball 1/2 size of Greece.

Early ideas of stars - relative posns constant
5 wanderers - Merc. Venus, Mars. J. Sat
+2. moon & Sun.

∴ 7 days in week.

Ancient diagram
perfect circle ⊙
7 pts in order of
speed. Take a
day in wh. 1st hour is governed by
Sun & go up to 24 hours.

Use corresponding Norse names of gods.
Only 24 hour division will give this diagram.

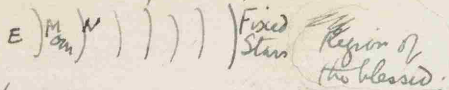
Week of 7 days.
28 " period for moon. Month.

Motion of Sun 1 year to regain its posn amongst stars.

All this is prehistoric

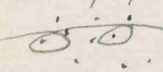
Observed path of Sun - Ecliptic
12 parts since 12 mos.
∴ Signs of Zodiac - 12 constellation.
measured angle very accurately.

Mechanism. ancient diagram of Greeks -
concentric crystal spheres



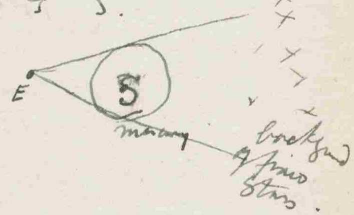
Spheres going about earth + grinding one on another
Pythagoras : music of spheres.

Plot of positions of planets in stars


∴ need for epicycle theory.

Ptolomy 200 AD. published Almagest containing all
this + it remained for 14 centuries.
∴ dead body of doctrine.
until Copernicus (Polish monk)
got church to stand sponsor for changed
ideas - ie. a Cardinal paid for + it was
dedicated to Pope.
Yet later ch. tried to overthrow C's ideas.
viz. No crystal spheres but rotation on axis.
Earth not centre but Sun.
S. M. V. E_M Mars J. S.

Explanation of Stationary pts
of planets



One of greatest observers.
Tycho Brahé danish nobleman.
Positions of stars by 2 sight instrument
+ made catalogue
his pupil J. Kepler genius of 1st quality
very poor 2 laws.
6 years calculation then 3rd law
based on observation of T. Brahé

Italian nobleman Galileo. (Pisa) died 1642.
2 bodies of diff wts fall together.
Thrown in prison - became blind + was visited ?
by Milton.

Dominic explained rainbow + died in prison.

Born Xmas Day at Grantham. Linc. - 1642 Isaac Newton - most
famous scientist who ever lived -
went to Camb. at 15 + by 21 had discovered
Binomial Thm + Fluxions (Dif + I. Calc)
+ at 26 was Lucasian Prof

Halley at Oxford propounded Question of
$\frac{m_1 m_2}{d^2}$ Inv. sq law of attrn.
gives orbit of ellipse was reply
of Newton

Principia - This, optics, spectrum
Tides etc. 3 laws - Gravitation etc.

This until 45 when he became master of Mint
+ lived in house on Berri St. off Jermyn St.
+ died at 85

All modern astron based on $F = G \frac{m_1 m_2}{d^2}$
not great. 100000 tons 1 mile. 100000 tons
force of attrn 1 pound.

Cavendish 1798. determination of G
+ weighing earth.

Gravitational effect of Tides area of moon = area of Europe / 2
frictional effect on moon gtr vel. + gtr distance²
∴ earth less rate of rotation ∴ longer day.

Sir Geo Darwin — $\frac{1}{100}$" per centur

Cumulative effect till day & month equal
& then only one side of earth will see
moon — dumbell motion.

Then month < day & moon will begin
to approach again until collision.

See this in more advanced stage in
Satellite of Mars — rapidly approaching
Mars — 4000 miles out now. weighs 200 tons
& when it hits it will generate enough heat
to be felt on Earth.

2nd lecture

Earths Attraction.

Newton — apple tree — cut down 1800, wood in R.S. London
he considered moon & attrn towards Earth —
& found it fell in 16 ft in 1 minute.
g on earth gives 16 ft in 1 sec.
But on theory it should fall 14 ft in 1 min.
For 16 years he put this aside
& then new measurement of radius of earth
was made & gave theory 16 ft.
& after 2 years he produced the Principia.
Then came Universal Law of Grav.

$\frac{m_1 m_2}{r^2}$ Force holding Moon to Earth
> force to sunder a rod of
steel (400 yds)^2 cross section
20 tons/in^2

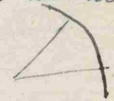
or ½ mil tons O — 1 mile — O ½ million tons
force 1 lb.

Compare 1 gm electrons — 1 yd — 1 gm electrons

Force 10^{25} tons —

Size of Earth —

Measure a line a few miles long. & by
triangulation get a distance on meridian.
& astronomer gets lat

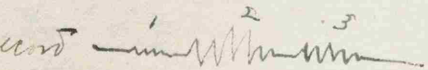


& fit in arc of it
length & so get radius of Earth. 4000 miles
& detect flattening towards pole —
Rate of rotation at equator 1000 miles/hr.
Proof of rotation — Foucault pendulum —
Gyroscope. compass
Time keeper — a pendulum — Galileo in Milan —
Airey's pendulum measurements for G. & M.

Interior of Earth.
Inc. of temp 58°F per mile.
Great pressure. 3 tons/in^2 at 1 mile down —
Mine pressures up to 20 000 tons/in^2.

Rigidity ① 27 miles flattening at poles.
If liquid it should be 100 miles
∴ rigidity is determined.

② Earthquake record



① comes str through 10 mi p. sec.
② " round ~~shortest~~ way 2 " "
③ " " longest " " "

∴ rigidity > steel.

then Solar tides tend to dec. Moons distance
& it will begin to spiral in. & will eventually
hit earth.

∴ limit to mans habitation on Earth
& feats of mind & works of art.

Laurentians mts were formed ~~Soon~~ after
moon was born.

3rd lecture

Eclipses & Phases of Moon

(S

Phases of moon & Spring & neap tides.

Moon in orbit progresses 12° per day
& ∴ earth takes 48' to catch up to similar
posn ∴ rising of moon & tidal phases
48' later each day.

Eclipse of Sun at new moon
" " moon " full moon.

Plane of moons axis

The Saros.
Chaldeans found that every 18 yrs $10\frac{1}{3}$ days
Sun moon & earth have exactly same rel posns
Observational rule.

Importance of Solar eclipse
Einstein shift 1.5" ?
Photos - May 28. 1919.

Eddingtons illustration
1 lb of light at 5c a unit
would be $1700000

& Sun gives 16 tons of light per day.

Use of eclipses in fixing historical dates
ie. 1924 AD is in error
Josephus records eclipse which
gives evidence that birth of Christ
was not 0 AD but 4 B.C.

Josephus - Eclipse was
(St Matt 2. 15) 15th Feb. 5 B.C.

Kepler's Laws - Empirical
Newtons Laws Explain the above.
from 2nd Law of K. N proved force towards
focus

Inv. Sq. Law.

from 3rd he proved above indept of
matter of wh. body is composed.

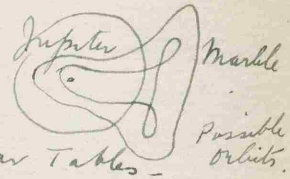
Laplace, Lagrange & all astronomers
based their work on Newtons Universal
Law of Gravitation.

Consider a marble & the Sun
If marble has no vel. it falls
str to Sun
If it has a small vel. it goes
in a ~~circular~~ elongated ellipse
v gtr. ~~elongated~~ ellipse
v " hyperbola parabola str line

These curves all studied by Gks as conic sections –

1. Value of pure math.
2. Problem of 3 bodies.

Sir Geo. Darwin . Sun



E. W. Brown (Yale) Lunar Tables –
3 body motion . Closest approximation
but not yet final –

$\frac{T^2}{a^3} = \frac{T'^2}{a'^3}$ = const. Get scale of Solar System

Mercury	.4	times dist of Earth
Venus	.72	
Earth	1	
Mars	1.52	
Jup	5.2	
Saturn	9.54	

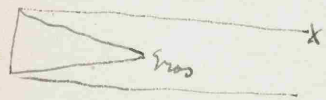
In 1777 Bode "a very ingenious person".

	0	3	6	12	24	48	96	192
+	4							
	.4	.7	1.0	1.6	2.8	5.2	10.0	19.6

Compare with above.
nothing corresponding to 2.8
∴ search for another planet
the Astronomical police
about 1800 Piazzi in Palermo . (not Palace)
seeking with transit ⊙
observed 50 stars each night for 159 nights –

+ saw that one was moving relative to others in 3 observations.
Problem was to find orbit from 3 posns on 3 nights i.e. small arc –
Gauss 22 yrs old one of the classical things in math – method of calculating orbits –
& he got thus first orbit of an Asteroid – & it fitted into 2.8
& was called Ceres
3 more discovered in 1807
1840 another
1891 Max Wolf (Heidelberg
idea of keeping plate by clockwork on stars & then asteroids leave a trail on plate.
Now 1000 are known all betw Mars & Jupiter

1898. De Witt of Berlin
discovered Eros as unique asteroid whose orbit comes near Earth. inside orbit of Mars –
This gives a method of getting dist to Sun.
Eros comes at closest 13000000 miles.



Survey method to get dist. to Eros . ∴
Scale of solar System determined with precision
(by AR Hincks at Camb)
1900

Planets beyond Saturn –

By Bodes law – what about 196

Wm. Herschel deserted German army at 19
+ came to England. + at 26
was musician of fashionable Ch. at Bath –
Read math + optics + began making telescopes
mirror grinding.

Tues 13th March 1781 he began to sweep as usual
+ found Uranus – not a spot but a disc.
Mass 4 times Earth dist 19.6 × Earth –
Geo. III very pleased.

What about next Bode figure 392.
Divergence during 60 years of Uranus posn
from that calculated – error of order
1 inch on a circle of circum. 100 yds –
1841 J. C. Adams heard from Sir Geo Airey.
Jan 1843 " grad. as Senior Wrangler.
+ in Oct 1845 he finished this problem.
+ sent it to Airy + Challis.
June 1846 – Le Verrier (31) Paris.
Airy wrote to Le V.
+ then to Challis to look for it
Challis looked but did not reduce his
results
Le Verrier wrote to German astron. Galle.
+ he saw it.

Neptune.

3000×10^6 miles fm Sun year = 165 years.

Earth 1 moon Uranus 2
Mars 2 Neptune 1
Jupiter 9
Saturn 9 + ring –

Sun to Nep 3000×10^6 miles.
Diam of System – 8 light hours 10 min.
Cf. nearest Sat α Centauri 4.4 years.
Scale 93 10^6 miles to inch.
Neptune is 30 inches from Sun
+ α Centauri is $4\frac{1}{2}$ miles –

Comets.

Kepler "there are more comets in the
heavens than fish in the sea".
before Newton – comets were regarded with awe
"hairy star" = comet
In 2nd edn of Principia Newton found
that comets were members of Solar System
some returning some 1 visit only
i.e. orbit ellipseal or)(a or ⟵la.

Halleys comet.		Inclin	Long. of ascending node
	1531	18°	
	1607	17°	49
	1682	18°	50
			50

∴ he predicted its return.
1st time of such a prediction.
+ it came only a month late
next time only 3 days late
" " " a few hours late. 1910

Path of H. Comet - an elongated ellipse
near Sun to just beyond Neptune.

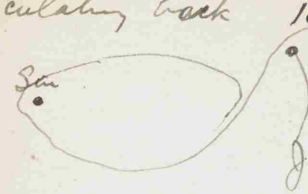
Encke's Comet. discovered by Caroline Herschel
3 year period - very small & goes thro
Suns atmos & is slowed up & will
eventually spiral into Sun.

? Biel Comet 6 yrs.
1840. to beyond orbit of Jupiter
1846 If J near its orbit wd be distorted
1852 was in two pieces.
pieces more apart.
1858
gone altogether
transformed into shower of
meteors - ribbon of shooting stars
to earth as it passed thro

Leonids. appears in November
Prof Newton in U.S. 1870. found that
every 33 years a very bright shower.
he traced this back to 500 AD.
like a ribbon 2000 miles across

& so long that
it takes 1 year to
pass any pt at 25 m p sec.

Calculating back 126 AD. there came
from afar a
comet on a ellipse
orbit & J was
at intersection.



& broke up head of comet
& swung it round

Slides of Planets:

Venus: has an atmosphere so dense that no
view is possible of the surface of the planet hence
impos. to tell its period of rotation. Not much
O or H_2O or CO_2 in its atmosphere ∴ not
likely to be life as we know it. Since it is
between us & Sun it has phases like moon.
Mars. thin ice cap at poles - definite land
configuration - no atmosphere - very cold.

Lecture 4. Foundations of Astrophysics.
1546 Danish nobleman T. Brahe.
Entered univ. at 13. Solar Eclipse 4 hours wrong
in time prediction ∴ he decided to make accurate
star catalogue.
Pivoted wooden sextant for direction.
Duels nose
King Fred II gave him £20 000 to build obsy at island Hven
20 years. Fred. II son turned him out.
exiled to Bohemia - another obsy. died at 55.

First use of 2 lens for magnification Lippasher apprentice to glass maker. 2 spec. lens showed wind vane on ch. top inverted.
Galileo heard of this + made telescope. + saw details of moon. measured mts 5 mi high
Saw double stars, many stars of M. way
& 4 moons of Jupiter + sun spots.
this was 1st refracting telescope — lenses.

Newton optics. no perfect image from a lens.
∴ something not understood about light
∴ prism experiment. + nature of light.
∴ Parabolic mirror & Reflecting Tel.
no chromatic effects.
Sextant invented by Newton.

Modern Transit O. 1st Transit invented by Roemer
1st Equatorial " " "

Slides of Mod. Tels.

Light. Red $\frac{1}{40000}$ inch violet $\frac{1}{70000}$ inch.

Law 1. Solid liq. or gas under pres. glowing gives continuous spectrum.
2. Glowing gas } gives lines Na 2 yellow.
low pressure } K. 1 red 1 orange
H. 1 orange + blues + violets
3. Absorption lines by a cooler gas.

Solar Spectrum. 16000 lines mapped

$\frac{8}{4}$ Ideal field only as yet.

Sun as nearest star + source for weather.
Sharp edge — photosphere — giving light.
every sq. cm of sun surface produces heat of a 9 h.p. engine.
as much energy from 1 sq metre as 90000 h.p.
Diam of Sun 866000 miles.
Total energy 10^{24} h.p.
In 48 yrs. a glowing ball of metal would cool to zero. ∴ not only heat of cooling

Meyer's Thy. meteorites.
Helmholtz. Thy. shrinkage 75 metres / year.
i.e. ½ sec. of arc less diam in $\overset{15}{\cancel{10}}000$ yrs.
i.e. 17 million yrs. till Sun's density wd. be that of earth + it wd. cool off.
i.e. 22 mil. yrs ago it was in nebulous state.
i.e. $39 \cdot 10^6$ yrs life wh. is inadequate.
as geologists want 200×10^6.

Radioactive Thy. shown inadequate by E.R.
who showed if it were all Ur
it would only add 5000 yrs.

Subatomic Energy Thy temp of edge 5000°C.
∴ inside temp very high.

Thin layer outside edge 500-1000 miles thick
of cooler gas - Reversing Layer -

Chromosphere - thin rarefied to 10000 miles -
+ Prominences from it out to 2-500 000 miles.

Sunspots. Hale of ~~Mt~~ Wilson

(Galileo estimated time of rotation of sun 1 month)
Gts rate of rotation of equator.

Equator 25 days rotation period
25° N S. 26 "
40° 28 "
60° 31 "
74° 32 "

Explanation on idea of concentric spheres
with inner going fastest ∴ biggest bulge
+ frictional effect on outer layer.

1843 Schwabe found 11 yr period.
Region 30° N + S of equator -
+ at max majority at 15° lat
getting nearer eq. until min - when
at 30° + gradual approach to equator.

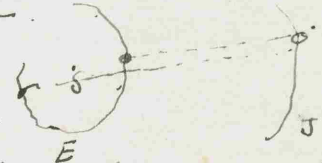
1908 Hale invented Spectroheliograph.
+ found vortex nature of sunspots
+ opp. dirns of N + S - spots -
+ " " of pairs -

Umbra Cooler centre may be 50.000 mi across -
Penumbra mi

1908-1912 Hale observed 26 groups of Sunspots.
1908 was time of maxm.
found magnetic effects - + of pairs one
was N. ~~+ other~~ S. pole
N S after 1912 when ~~[illegible]~~ was on
S N was order of pairs -
i.e. every 22 yrs they change their polarity.

5th Lecture

Keys Na absorption line experiment -
Vel. of light constant . C.
Time for signal from Montreal to London $\frac{1}{50}$ ".
1st determination by Dane Roemer. by eclipse
of Jupiters moon.
Noticed time different
when observed at
Spring & at autumn.



Duration of eclipse 42 h. 17 min
+ earth travels 3000000 miles.
15" extra for duration of eclipse from far side
∴ vel. of light 200000 mi

Atomic Structure

Consider drop of water - made up of mols + atoms.
Magnify to size of earth - mols wd. be as big as cricket
balls.
Atom - Rutherford - Bohr
Heavy nucleus + electrons -
H atom Speed of electron 10^{15} revolutions per sec.
radius of orbit $a = \cdot 54 \times 10^{-8}$ radius of ~~[illegible]~~ is $\frac{1}{25000}$ a

and radius of earth is about $\frac{1}{25000}$ radius of orbit

no of poss. orbits

$\overrightarrow{a}$: a 4a 16a

line due to falling in from outer orbit.

orange line 2^{nd} to 1^{st} orbit.

green 3 to 1
blue 4 to 1
violet 5 to 1^{st} orbit.

Star Catalogue -

1. Hipparchus
Ptolemy 1000 stars
40 constellation.
12 along ecliptic.

In last century obs^{rs} of S. hemisphere
raised no. of constellations to 88

Zodiac that part of sky where goats
rams bulls went to when dead.

Arcturus, Pleiades, Aldebaran (Arabic name)

Primitive idea of effect of Pleiades - year
dated from time it crossed meridian 17^{th} Nov.

Dipper,
Orion, Aldebaran, Sirius.

1887 Sir D. Gill till 1891 photographed S. hemisph.

mic measurement for cataloging done by
J C Kapteyn Dutch Groningen astronomer
finished in 1899 - 8 years - no pay for it.

Mizar 78 yrs.
alcor.

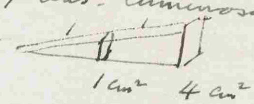
Pleiades (12 light years across) dust nebula.
Nebula of Orion . Chaos Gas.
H. He & nebulium

Hipparchus 20 brightest 1^{st} mag.
just visible 6^{th} "

Dipper 2^{nd} mag.
Alcor 5^{th} "

brightness of star depends on 2 things:
(1) distance (2) abs. luminosity.

Inv. Sq. Law.
of light intensity -



Modern mag. law.

2.512 x light from an n^{th} mag as from $\overline{n+1}^{th}$.
i.e. 6 mags. means 100 times less light.

1914. at Greenwich.

2nd mag.	41	
3	188	10000 to naked eye
4	454	
5	1480	i.e. 3 or 4 000
6	4710	visible at a time.
7	14960	

Visual & Photographic mag.

1914 Chapman & Melotte count of stars to
17^{th} mag.

4 million 17^m. stars reqd to give light
= a $1^m.0$ star

5 - 6 mag.	2 026	ratio
6 - 7	7 096 }	3.5
9 - 10	2,026	
10 - 11	42600 }	2.5
15 - 16	$14\ 10^6$	
16 - 17	$25\ 10^6$ }	1.4

Total . 55 000 000

Gives clue to finiteness of our galaxy -

By math if equal density throughout all space ratio should be 3.98

∴ not homogeneous but fewer & fewer.

$\times 10^3$ Result of these calc. n = no. of stars

2 8 5000 $770 \times 10^6 < n < 1800\ 10^6$

6400

6500

~~Consider~~ 6000 concentration.

Milky Way 7000 - or Galaxy -

9000

21 14000

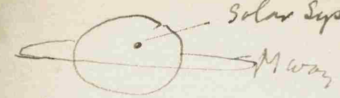
24 19000 10 Per unit area plane of M. Way. Take zones N & S

100 S

11 i.e. concentration towards M Way - & thinner in dir. ⊥r M. way -

9m 8m. i.e. Stellar universe is a lens

17m or bun with approx 1000 million with M way - mid edge



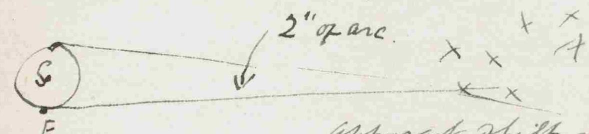
What are the other Suns - how large, history & destiny.

Dark lanes in M way probably due to dark matter either once luminescent or yet to be.

Distance of stars

Sun gives $10^9 \times$ light of a 1m star.

remove Sun to 100000 x its distance to make it 1m.



apparent shift of near stars against remote background.

cf. telegraph poles moving past trees in distance - Idea occurred to Ptolemy but could not measure it so said Earth was fixed.

Copernicus felt it a great mystery & Kepler " " " "

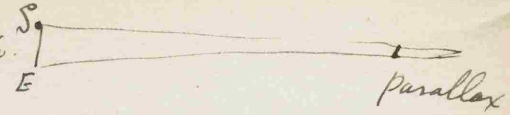
not realizing immense dist of stars & ∴ small angle only measurable with modern precise instruments

2" arc is l

206205 x l

Separation of eyes gives us an idea of distance i.e. dist. between eyes subtend 1" at 10½ miles -

Angle of $||^x$.



nearest star less than $||^x$ of 1".

Bradley attempted to measure $||^x$
but found aberration.

$||^x$ measured by 3 independently. 1838

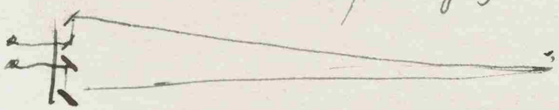
1 Englishman – Henderson – Cape T.
2 German Bessel
3 Russian. Struve

1. α Centauri	$||^x$	1"	actual	0.76
2. 61 Cygni	$||^x$	0.31	"	0.29
3. α Lyra	$||^x$	0.26	"	0.10

1. angle subtended by eyes at 15 miles
2. 30 "
3. 40 "

approx.

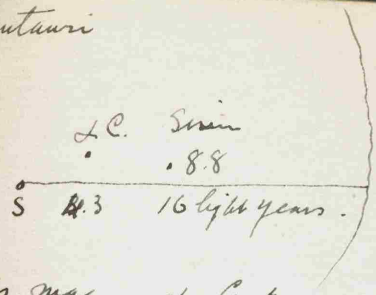
method 2. 3. was idea of range finder.



diam of orbit as base 186 000000 miles.

Modern photographic method
has given 2000 stars & has limitations

α Centauri



Inside are
19 stars only
i.e. emptiness
of space.

Abs. mag. α. Cent. 2 × brightness of Sun
Sirius 48 × " " "
19th in above $\frac{1}{200}$ × " " "

Only 9 visible to naked eye.
10 photographic only.

NB
Canopus very bright but so far off
that $||^x$ as yet unmeasured –

Motions of Stars.

Newtons Grav. law shows that eventually
even stationary stars wd gain a velocity

Radial vel. & angular vel of proper motion

p.m. easy by comparison of catalogues

Halley 1st to find out that fixed
stars are not fixed – by comparison
of positions in Ptolemys time.

Radial Vel. Sir Wm Huggins
& Doppler effect.
Spectral line shift.

Results accurate to $\frac{1}{10}$ mile sec.

Thus R. vel & P M gives Space Vel.
average 20 m.p.s.

Detection of Double Stars.

Visual binaries seen to move in orbits.
~~Spectroscopic~~ " " & spec. shifts —
$\frac{1}{5}$th stars are of this type.

Case of a planet (not 2 suns)
give a single line oscillating first towards red then blue —

1904 Kapteyn found Stellar motion as given in Boss ~~6188~~ Catalogue gave
2 Drifts — almost opp. in dirn & ||l to M. way.
Cf. clouds of bees.

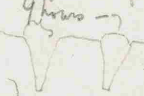
2 Drifts Theory done mathematically by A.S.E.

Solar Motion Sir Wm Herschel
7 stars
Cf. walking past trees in a wood. those in front opening up — those behind closing up.
Apex — Hercules — speed 13 m.p.sec.
average 25 m per sec.
One star a runaway 205 m.p.s.
but will take 3 million years to leave our telescope range —

6th lecture Variable Stars

1783 — J Goodricke — deaf & dumb — died aged 22 — 1st variable star. Algol
1784 2nd " " "

His insight gave an explanation verified in 1889
RS gave him Copley Medal (highest award)

Light variation


Algol (100 light yrs away) 2.8 days.
loses $\frac{5}{6}$ of its light in $4\frac{1}{2}$ hours.
stays dim 20 min
rises in next $4\frac{1}{2}$ hrs

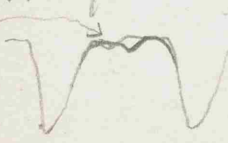
Arabian name means demon — Did they know its light variation?

Explan^n — Bright & dark stars. dark one smaller.

Vogel at Potsdam verified this by Spectroscope in 1889 —

diam of big one 1000000 miles
small 800000 miles
dist betw $3\ 10^6$ mi

Joel Stebbins (American) Selenium cell



found small bump due to reflected light from big one onto small one when latter is eclipsed when drop comes —

Big one is 240X bright as our sun

Plane of Algol orbit is 8°

1884 Goodrich discovered.

12 days →

2 great suns of glowing vapour (density of air)

9×10^6 miles (1)

somewhat smaller (2)

very close together & producing mutual tides.

All these variables lie in plane of M Way. or very close to it.

<u>Long Period Variable</u>   <u>Red</u> stars   Type

Mira Ceti.   period average 330 days.

erratic period & maximum brightness.

1868 max 5.2^m
1875 " 2.5

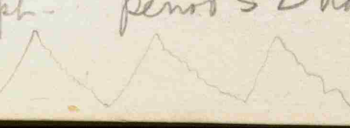
Not a binary. Somewhat a mystery
H gas is spouting up at a maximum.
"explosive disease"

<u>Cepheid Variables</u>   All giant stars low density.

Not a binary.

δ Ceph.   period $5\frac{1}{2}$ days.

Abs. regular.



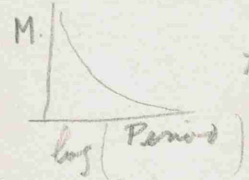
spectral lines oscillate with same period but out of phase ∴ not spec binary. & <u>no room</u> for 2 components i.e. dist betw cd have to be $\frac{1}{10}$ radius of one wh is impos.

Theory of pulsation.
difficult on account of necessary size of waves
i.e. 600 000 miles.
∴ still a mystery but important application.

<u>Abs. Mag.</u>   standard distance.

Miss Leavitt. correlation betw Abs Mag & Cepheid period.

Shapley



this given app. mag. & Period by obsⁿ
Curve gives M.

Ceph. Var. in glob. clusters gives their distance.
Nearest G. Cl. 21,000 light years.
farthest one (until recently) 220,000 l. years.
NGC 7606
New General Catalogue

Shapley 1924 NGC 6822 1×10^6 l. years.

Relation to M. way — 

10° N to 10° S of M.W.
is empty of gl. clusters
possibly grav. field within this zone breaks
up cluster form —

300 000 light years was diam —
of our univ.

roughly Shapley's idea 10^6 l. years
i.e. S. does not believe in "island universes" —

Evolution —

Newton got spectrum but did not see
black lines —
1st seen by Wollaston

1817. Fraunhofer — poor orphan boy
in optical factory in Munich —
became head of Telescope Dept —
died at 39. {Statue on Maximilian Strasse
named lines by letters 2 D lines of Na
2 K " " Ca

1849 Foucault explained
1859 Kirchoff reexplained
1860 Sir G. G. Stokes full explanation
as absorption by a gas or
vapour —

 gaseous star giving continuous spec-
cooler vapour absorbs —

Father Secchi Vatican Obsy
~~began~~ 4000 stars spectra sorted
into 4 classes —

(Beautiful Slide)
1 Orion full spec
2 less violet more lines. sun
3 TiO_2 flutings identified by A. Fowler
sharp toward violet

4 Hydrocarbons — flutes sharp toward red.

Huggins

Pickering made Draper Memorial Catalogue
1/4 million spectra

lettered thus O B A F G K M N
Secchi I II III IV

He H red darkred
Decimal Subdivision —

Temps.

Surface Temp	B 20000°C	F 7500	K 4200
(Potsdam)	A "	G 5000	M 3100
			N 2950

~~KMN~~ temp are too low
Our Sun about 6000° C

Early idea of Evolⁿ – began as very hot nebulous masses, cooling & shrinking.

1914. H. N. Russel Princeton Giants & Dwarfs

Start with B star – He where do the other elements come from.
SAHA – Electronic nature of matter
ionization & enhanced lines (brighter)
∴ degree of ionization ∝ ∴ temp.

F 9000 M 5000
G 7000 Hence other elements are
K 6000 pres. but only radiate when temp becomes right.
Compare Sunspot – it is up in chromosphere
∴ temp lower –

Novae. flames up – & dies away gradually
Example Aug. 20 1920 Nova Cygni.
Discovered by Denning – Bristol –
no trace of previous star there to 15ᵐ
Harvard Aug 9 – not there as a 9ᵐ star

Sweden	16th Aug.	7ᵐ star	
Denny	20 "	3ᵐ.5	
	25	2ᵐ.0	maximum.
	30	4ᵐ.0	
	30 Oct	9ᵐ.0	
		gone	

Brightest Nova –
Tycho Brahé brighter than Venus

Spectrum alters with light fluctuation
1st dark lines are shifted toward blue.
2nd then bright lines appear
& grow to gt intensity like a nebula spectrum
br. lines on dark ground
See Strattons paper – Camb 1924

possible explⁿ ~~dark~~ star enters a gaseous nebula (dark) approaching earth & by friction heats up –
This born out by
Nova Persei
bright nebula grew around star
was it thrown off from star
no vel. was c.
actually star enters black neb

+ light is propagated -

Novae usually in M-way.

Pleiades - Same type H + He Stars -

Slides of dark clouds -

If Sun ran into a dark neb.
heat wd go up to gigantic figure
blown us up -

Velocity

B	6.5	One of mysteries -
A	11	
F	14.4	Change in temp + colour brings change to vel. -
G	15	
K	16.8	
M	17.1	Km/sec -

H N Russel. Giants + dwarfs.
extremely bright or very faint -
M. Giant Betelgeus
Adams + Joy 1917 -

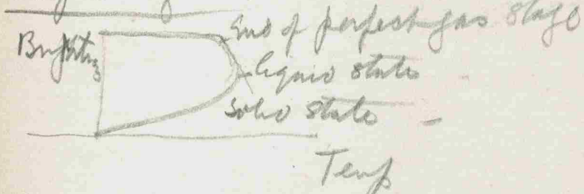
	M	K	G	F
Average mag	1.6	1.4	0.6	1.1
		7.1	5.3	4.1
	10.8	7.1		

∴ B star is not beginning of history but maturity -

begin large + diffuse + contract + heat up
+ then after A or B stage further
contraction brings cooling -

Russel got Gold Medal RAS for this -

Temp diagram



A S Eddington 1916-17 -

Inside of Giant Star -

Temp + vel - 10,000 °C 4.3 Km/sec -
10^6 °C vel. 43 Km/sec

2 kinds of Energy - Kinetic
Radiant.

200 yrs perhaps for rad. energy to work its way out -

In ordinary matter prop^n of Rad Energy negligible but increases faster than

K.E ($\because$ K.E $\propto T$ / R.E $\propto T^4$) as temp. increases

Clerk Maxwell :- El. mag. theory of light.

pressure of light — ray — glass reflector

Measured recently -

Sun = 1 Mass of Star	radiation pres. / Gravitation
$\frac{1}{2}$	0.04
1	0.16
3	·33
$4\frac{1}{2}$	·409
20	·8

Actual range of masses observed is chiefly $\frac{1}{2}$ to 20 × Sun -

Eddington :- Courage of a great pioneer. predicted diameter of typical stars as ang. diam

Ash. Antares .043"

1916-1917 Arcturus ·020"

(Distance between eyes subtend 1" at $10\frac{1}{2}$ miles)

Quarrel between Ash + JHJ

Measurement by Michelson

1920 Interferometer method suggested by Frenchman 1868 -

Idea of Interference on wave theory

When crest + trough overlap there is deadening.

Newtons rings - sup.

+ Wedge

gives lines

20 foot ~~beam~~ on Steel girder by Mt W. telescope

~~telescope~~

d

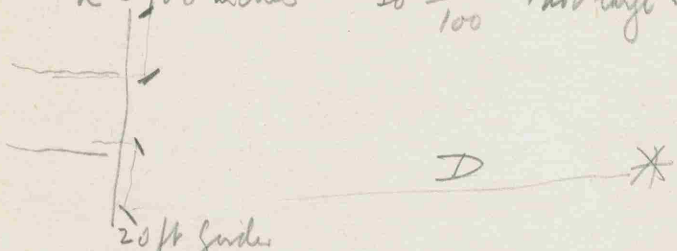
interference

D to star ——— *

D & d in inches —

$D = 5''/d$ when interference fringes just vanish —

R = 100 inches so $\frac{5''}{100}$ not large enough —



12th Feb. 1921 Mr S Peace Mt Wilson

got Arcturus $0''.024$

19 feet = d for dis appearance

Predicted 0.020 A.S.E.

∴ since dist* is known

its diam is $19\ 10^6$ miles

See Nicholson — Lines M.N.R.A.S. 1914?
— predicted before Bohr Thy —

Lodge — Nature — Mar. 1. /24 — Stationary Na & Ca in Space.

Evershed — Obsy. Feb. /24 " "

Plaskett. Dom. Obsy. — Ca & H&K lines stationary in double stars —

for Lecture 7 on Cosmogony & Evolution of Universe see 6 pages on —

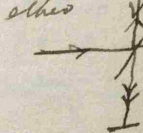
... Energy — The God of Love

Relativity. A. H. S. Gillson 22/4/24.

Century of revolution – painting – & nat. philos.

Cezanne died 1906 or 15 –
cubists – non Euclidean. simultaneity – time dimensions
Newtonian mechanics – inertia defining force. Cor. V. states
impos. to detect rel. motion if in a closed cage moving in ocean.
Observer outside sees a plot fall as a ⊂ a inside the cage
it is observed as a str. line. Newtonian Relativity.
now known not strictly true
abs. Space, abs. time – Newton – unaffected by observer –

Light is the controlling element.
Wave motion in ether. Attempt to measure vel. relative
to stationary ether



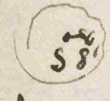
Michelson 1881 Morley 1887
accuracy to $\frac{1}{100}$ expected effect
but no diff.

∴ Vel. of light is same for all observers –
i.e. Let observer S' move at vel c rel. to S &
still vel. of light is c to each –

Fitzgerald & Lorentz. Shortening of arm to obviate
above difficulty – based on electron theory.

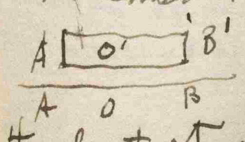
1905 Einstein (26 yrs old)

light signal sphere a sphere to both S & S'
∴ the Space & time must be diff
for each observer



Idea of Simultaneous. Synchronize clocks by light
signals if & only if the clocks are stationary rel. to
one another.

Light from A' to B' in time rel. to O'
from A to B rel. to O
but A' – B' > for O



thus lengths & time are relative.

l

$\rightarrow v \quad l\sqrt{1-\frac{v^2}{c^2}} \quad \therefore$ if $v=c$, $l=0$

Clock $\bigcirc \rightarrow v$ Stationary observer thinks clock goes $\frac{t}{\sqrt{1-\frac{v^2}{c^2}}}$

A.S.E. to get eternal youth cruise about the Univers with vel. c. You wd. also vanish –

Rel vel. $\frac{u+v}{1+\frac{uv}{c^2}}$

Shortening effect on train 1000 ft long. vel. 60 m.p.h. Observer on embankment sees it 999 999 999 999 999996 feet.

Earth round Sun at 18½ m.p.s. is flattened to observer on Sun by 2½ inches.

Alteration of Mass $\frac{m}{\sqrt{1-\frac{v^2}{c^2}}}$ Tested by β ptcls 99% c.

Energy $E = mc^2$ ∴ change mass ∴ change E. ∴ no energy i.e. heat it ∴ mass is [illegible]

1 gm cheese – $E = 1 \times (300\,000 \times 1000 \times 100)^2$ ergs would lift 2000 000 battleships 1000 miles.

Minkowski – Russian – chair at Gottingen.

World lines i.e. plot beetles moving on a plane in 3 dimens 2 of area + 1 of time.

∴ 3 of length + 1 of time gives our World lines – & the 4-dimensional Space-time is the World.

All this is the Restricted Theory of Relativity i.e. Uniform vel. rel. to one another.

General Thy – Newton's laws of grav. did not explain grav.

$\frac{m\,m}{r^2}$ – Uranus – Leverrier finds error 43" arc per century for Mercury – i.e. 1 in 10 000 000 error too gt. for cl. mechanics.

2 Kinds of mass inertial & gravitational are equal –

This was proved by Galileo in 14th Century – at Pisa. Newton did not get anything out of this known fact. Einstein made it the base of his conception of the unreality of grav. force.

People in a freely falling lift. no grav. force " . in accelerating upward vel. feel a grav. field.

Law of Equivalence impos. to distinguish betw a grav. field & an accel. force

gravitation = artificial accel. – directly due to Galileo's expt.

Effect of Grav. on beam of light –

Eclipse 29th May 1919. 1.98" ± 0.12
1.01 ± 0.30

Newton predicted .85"
Einstein " 1.70 ∴ Confirmed.

Suppose 2 discs 1 fixed 2 rotating rel. to fixed Stars.

× 2 Relativists — Same measuring rods & clocks.
× 1 Newtonians
×

fixed Stars.

1. $\frac{\text{circum}}{\text{diam}} = \pi$

2. measuring rod has contracted ∴ $\frac{\text{circum}}{\text{diam}} > \pi$

Non Euclidean Geom – $\frac{180 - \Sigma\text{ angles of }\Delta}{\text{area}} = R$.

Draw ⊙ on a sphere r great. circum gts until $r = \frac{1}{4}$ circum. when circ. is max. & then r gts. circum less.

Also max^m st. line is finite.

Distortion of ~~matter~~ space in vicinity of matter i.e. planets go about sun not due to grav attn of Sun but due to distortion of space

Distortion of space due to 1 ton gives
π = 3.141 592 653 589 793 338 261 613
instead of ^{2} as normally —

Put wt. in a stretched linen sheet
+ a marble rolls into the dint or warp.

W. K. Clifford - 25 yrs. old - d. at 34 foresaw 52 yrs ago i.e. 45 yrs before Einstein's paper.
the curvature of space in presence of matter & hence motion of matter —

Flat people on a sphere - finite area & unbounded.

Einstein 1905, 17.
de Sitter 1917
de S. calc. circum. of universe 100 10^6 light yrs.
3 space dimension spherical in a 4th.
& the max^m str line is this circum 100 10^6 l.y.
Total ~~matter~~ matter is. 100 trillion Suns.
" electrons 10^{78}
large but finite.

Haas says - 1 bacteria - doubles each hour.
2^{24} bacteria in 1 dy.
∴ in 6 days - if 1 bac = $\frac{1}{\text{billionth}}$ gm.
the total mass = mass of earth
in 10^{th} day mass of universe
∴ 11^{th} day no of electrons —

There may be other universes - we can never know - All energy & activity are bound up in our own —

Time & the Quantum Theory

2nd law of thermodynamics & all entropy work is too abstract. Importance of Irreversible Processes
In practice we use rev. proc. to evaluate inc. in entropy
Argument. of Inc. of entropy ∝ elapsed time.
3 classes of irrev. processes. (1) simple sweep in a closed system
gunpowder explosion.
Thermodynamic degeneration = inc. of entropy of system
(2) trailing sweep - heat water.
external changes
(3) steady sweep no external alteration of condns.

Tall energy for 2nd law pt. queer is (1) mechanical (2) heat.
i.e. el. or chem = heat energy.

Thus thermodyn degenn ∝ time.
∴ ∝ to work - & ∴ proportionality factor
is a function of T.
$\phi = f(T)\,W = \frac{W}{T}$ since simple argument
shows ϕ must go up as T goes down.
In Universe Statistical effect gives $\phi \propto \frac{W}{T}$
or inc of entropy ∝ time.

Consider purely mechan. system returns to exact same state except that time has elapsed.
(1) What is time? (2) Where is time?
The observer is not a purely mech. system & ∴
Time exists only for the observer - i.e. time has no reality for a purely mech. system
example of latter is Bohr atom.
Purely mech. world is timeless.
Observer is not a steady sweep ∴ jerky apprec of time. A Thinker introduces contemplation of time.
Clocks give us coincidence of instants.

What in nature corresponds to idea of forward flow of time
It is the irreversibility of natural processes i.e. the increase of entropy.
Consider oscillator $q = A \sin \omega t$
momentum $p = m\dot{q} = \omega\, m \cos \omega t$.

... get rid of all idea of time ...
... oscillator or ...
... state.
... ability - change
... during jump -
... in the
... time hence

Cosmogony

Solar System

Earth & Moon

Eclipses of Moon

" " Sun

Sun's atmosphere

Flash Spectrum.

Heat & Light from Sun

Life - Mind
Spirit

Nature very wonderful & in the beautiful ~~mighty~~ phenomena & mighty manifestation of force we are led to say with reverence Take off thy shoes from off thy feet for the ground on which thou standest is holy
But how much more do we stand with awe & reverence before the mighty power of the mind of man who can comprehend & explain & calculate & predict to the second the play of the forces of Nature - More still & with a deeper reverence do we bow our heads before The Creator - the Sustainer of all Energy - the God of Love

Time &

2nd law of thermod
abstract. Impo
In practice we w
Argument. of
3 classes of irre
gunpowder ex
thermodynami
(2) trailing sh
external ch
(3) steady sw

Tall energy f
i.e. el. or chem

Thus thermod
∴ α to
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Consider oscillator $q = A \sin \omega t$
momentum $p = m\dot{q} = \omega$ ~~[illegible]~~ $\cos \omega t$.

think of p & q & get rid of all idea of time –
thus observer must <u>not</u> watch the oscillator oscillate or
<u>he</u> introduces time into the argument.

Bohr <u>jumps</u>. nothing going on in steady state.
but in jumps – not complete reversibility – change
of <u>entropy</u> bound up with lapse of time during jump –
Jumps. to think of existence of atom in the
stationary state. non-existence in time hence
partly also in space –
Bohr's own feeling is along this line

Slater?

Lecture 7. Cosmogony — A. H. S. Gillson.

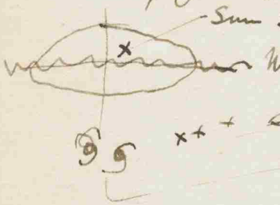
Nebulae 1. Irregular. in Milky Way — Orion Neb. its right side is receding, l. side approaching — ∴ ? rotation? The Pleiades. early type stars still connected by nebulosity.

2. Planetary Nebulae. Sir Wm. Herschel — Shell of gas rotating — no sharp edge — period of rot. 600 – 14000 yrs — density $\frac{1}{10^6}$ that of Sun ∴ $\frac{1}{10^6} \times 1.5$ density of water. Of this type are 3727, 3869, 4959, 5007.

3. Spiral neb. avoid the M. way. Andromeda is $\frac{1}{2}10^6$ yrs. away + an island universe system. Its spectrum is not that of a gas but many stars — i.e. br. lines & absn. lines — Other spiral neb. only 25000 l. years off are within our galaxy —

Van Maanen on Motion in Sp. neb. arms 85000 yrs to rotate.

Plan of galaxy — Webb (Camb)



— Sun 50,000 l.y. from centre.

Milky Way.

about 50 globular clusters averaging 200,000 l.y. from centre.

about $\frac{1}{2}10^6$ ~~sp. neb.~~ sp. neb. near poles — l.y. away.

Av. vel. of stars 20 km/sec.
clusters 300 km/sec.
Sp. neb. 600 km/sec.

Double Stars. In general if masses be m_1, m_2

$\frac{m_1}{m_2} = 1$ or 2 or 3 or 4 … 80

Cf. $\frac{\Sigma m \text{ of planets}}{M \text{ of Sun}} = \frac{1}{1000}$

[illegible] Neb. Hyp. on Saturn.

Early ideas of Evolution
N. Am. Indian — raven, Algonquins — Hare.
Kepler had no idea of gravitation ∴ each planet had a "soul".

Kant 1755. Vague neb. hyp. of universe
Laplace 1796 Rotating neb. Princ. of ang. momentum
Rings given off & condensed into planets
not now tenable.

Newton, 1680? Spin a liquid + it flattens
Maclaurin 1740 high rotation gives more flattening
+ Jacobi 1834 take a body with 3 unequal axes i.e. an ellipsoid will be in equilib. on rotation —
+ Poincaré 1885 showed that there were further stages ~~the~~ ellipsoid becoming furrowed & pear-shaped
Sir Geo. Darwin from 1901 — his death in 1910 tried to prove that fission would gradually result — (did prove it?)
Liapounoff proved it to be unstable & i. rapid fission.
J. H. Jeans confirmed & settled the instability & rapid fission — This is not applicable to gaseous nebulae.
Roche 187- considered gaseous body with solid nucleus + envelope of gas
(1) it flattens (2) becomes lens shaped (3) edges become sharper (4) gas is thrown off
a gradual trickle accompanied by inc. of vel. due to conservation of ang. momentum
The Condn. of a gas is specified by $\gamma = {c_p}/{c_v}$.
If [illegible] the matter is dense like water —

If $\gamma = 1.2$ the matter is gaseous.
thus $\gamma = \infty$ results in fission
$\gamma = 1.2$ " " Roché rings
$\gamma = 4.2$ is the critical value between these two
$\gamma < \frac{4}{3}$ the matter will not hold together at all but fly off into space.

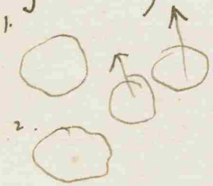
<u>Given some nebula</u> (origin unknown) then if $\gamma > \frac{4}{3}$ gravitation will hold it together & condensation will begin

Assume density non uniform – condensn will take place about many nucleii

J.H.J. says allow gas in an el. light bulb to fill St Pauls Cath. & the density is > that of this primal nebula.

The grav. effect of one nucleus on another brings about <u>velocities</u>

Tidal effects on near approach & the tidal lag tending to catch up in direction



tends to produce <u>rotation</u>

Thus mathematicians have a definite starting point

If after Roche – add a small tidal influence & get spiral neb. & each condensation along its arms is a giant Sun.

<u>Probability of Collision</u> 1 in 3000×10^6 years.

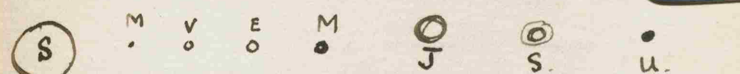
Roche separation cannot lead to planets but to planets about a Sun but to gas clouds too small & rare to condense – but only drift off as a cloud.

Red diffuse star heats as it contracts towards the B-stage & γ becomes larger & the Poincaré state begins. & may result in a double star

Thus far rigorous math. but a 3rd factor is not yet taken into account – <u>radn pressure</u> in the Poincaré work.

In Double star of close together tides are violent & ∴ recession & a possible further division of each. Multiple stars.

As yet no <u>Solar System</u>.



Bodes law of distance.

J.H.J. Approach of a stray star & tidal agitation & ejection of an arm large enough to prevent dissipation & ensure condensation into Planets. Similar idea for satellites due to tides from other planets.

Merc. & Venus born liquid or solid (?) ∴ have no satellites. Jup. etc born gaseous.

Earth-Moon System is an exception.

Frequency of a Solar System depends on the prob. of a close approach which is 1 in 3000 10^6 years. & then the relative vels. have to be small

Estimate of Life of Stellar System is
40.000, 10^6 yrs.
Rivers + Eliot Smith. new anthropology (since 1910)
indicate that culture is a freak - i.e. once in
Egypt. (1) ideal climate (2) ideal geographical
posn (3) fine race of people.
Everywhere the play of chance —

Miscellaneous questions + replies -
Star Streams: Turner (Oxford) suggests that
they are the remnant of sp. neb. arms of
our Galaxy -
Retrograde planets. F. J. M. Stratton has
showed that Tidal friction produces
retrograde planets.
Spiral Neb. degenerates into glob. cluster
+ latter disintegrates into open cluster
+ then — ? — .

Questions What is light? ENERGY
heat? "
elect.? "
matter? "

What is a star?
Is our Sun a star?

The Story of a Star.

Nebula clouds
spirals
clusters.

10^{12} years ago.
Tidal Theory. 8 planets

Sun	800 000 mi diam		
	9.000 mi. Ca		
	20 " " Hα prominences.		
	92 10^6 " from earth.		
		year.	
Mercury	2700 mi diam		
	36 10^6 " from Sun.	87 dy.	0 satellite ~~planets~~.
Venus	7500 mi. diam		
	67 10^6 " fr. Sun	224 dy.	0 "
Earth.	8000 " diam		
	92 10^6 " fr Sun.	365 dy.	1 Moon 24000 mi away $\frac{1}{81}$ mass of Earth. light comes $1\frac{1}{2}$"
Mars.	4000 " diam		
	141 10^6 " fr Sun.	687 dy.	2 satellites.
Jupiter	87000 " diam.		
	483 10^6 mi fr Sun	12 yrs.	8 satellites.
Saturn	~~87000~~ 76000 " diam		
	886 10^6 " fr Sun.	29 yrs.	10 "
Uranus	30 000 " diam		
	1780 10^6 " fr Sun	84 yrs.	4 "
Neptune	33000 " diam		1 "
	2790 10^6 " fr Sun	165 yrs.	

Moon $\frac{1}{4}$ size of Earth 240 000 mi

$\frac{1}{81}$ mass of earth

light comes $1\frac{1}{2}$ sec.

Neptune. 3000 10^6 miles from Sun.

light comes 8 hours 10^m

Stars α Cent 4.4 yrs.

Hipparchus 150 B.C.

Ptolemy 150 A.D.

Copernicus 1400 ?

Tycho Brahé 1546

Keppler. 1600

Galileo d. 1642.

Newton. b. "

Fraunhaufer orphan boy. 1817. Munich.

ount Wilson was conducted by Prof. alter Sydney Adams, using the 100 inch telescope.

Dr. Arthur Stanley Eddington had evolved the theory that the real brightness of a star depended only upon its weight, provided the star was in the early stage of evolution.

One property of perfect gas is that its volume diminishes as the pressure to which it is exposed increases. On earth this holds only for gases with densities less than that of water. At greater densities the size of the molecules becomes appreciable and overcrowding results, so that the volume no longer diminishes at the same rate as the pressure increases.

Robs Atom of Electrons.

Dr. Eddington showed that at the enormous temperatures of the stars the stellar gases would have their atoms stripped of their electrons. The size of the stellar atoms would be diminished to one-hundredth the size of terrestial atoms.

EINSTEIN THEORY IS BORNE OUT IN NEW STAR TESTS

Reveal Clews to Makeup of Stellar Bodies.

[Copyright: 1925: By the New York Times.]

CAMBRIDGE, England, July 25.—Announcements at the conference here of the International Astronomical union reveal that results obtained at the Mount Wilson observatory in California seem to confirm not only Einstein's relativity theory but also the Eddington theory of the internal constitution of the stars. The wo... Mount Wilson was co... Walter S... inch ...

have less than one-three-hundreth of its light. Its spectrum seemed to show that it was very small, but area for area, brighter than the sun.

This star, according to Eddington's theory, seemed to behave like a perfect gas. To test its apparent great density Dr. Eddington suggested that Einstein's test be applied.

Follow Einstein Test.

According to Einstein, a strong grazitational field slows down the atomic vibrations and produces a shift of the spectral lines. Dr. Eddington calculated that if his views were correct the shift in the spectral of the sattellite of Sirius should be thirty times greater than the Einstein shift, which had already been detected in the sun's spectrum.

Prof. Adams has photographed the spectrum of the satellite of Sirius and has found the exact shift predicted, thus giving a new confirmation of Einstein's theory and a confirmation of the Eddington theory. He showed the star to be only 12,000 miles in diameter, only slightly larger than the earth, although nearly as heavy as the sun, and that although it apparently behaves as a perfect gas, its density is 60,000 times the density of water.

Story of Science I Gks + Romans
Prof Stanley 20/4/26

Gk science not a short lived thing
Over 500 yrs -
Sir Thos. Heath says Archimedes was the
gtst math. genius of all time -

Archimedes had mechanical equipment -
surgical etc. math notation ample -

Anaximander taught evolution + variation
of species
Xenophanes like Huxley "every age of
men is incurably anthropomorphic" -
Archimedes dealt with shell fish etc.

Thales of Thamos - world is made of water.
" typifies "alchemy of thought -
influence of Babylon - astron
predicted eclipse 585 BC -
Infl. of Egypt - a practical mensuration
led Thales to pure math.
disc 5 of Euclid Bk I props -

Pythagoras b. 580 BC 60 yrs younger than
Thales. est. sch. a geometer. he
gave geom. a numerical content
not concerned with matter but
structure - octave, harmony - +
properties of numbers.
not what is matter but what is
matter like done yet again
by Leibnitz + Newton.
Pythag. says No. may not explain
matter but may be the key to manifestation
describing its phenomena. of matter

1st Gk Sci. of Gks is medicine + Surgery

Medical Guild — writings of Hippocrates fever – trepanning scull, dislocations, use of drugs, wine for washing wounds – duty of physician to assist Nature.

Physics & Astron.

Anaxagoras lectures in Athens when Soc. a youth knew moonlight is borrowed, universe in motion, & an atomic thy.

Democritus – math – travelled observer – a theory of atomic structure.

Pythagorean School.

Pure Math. 500 – 300 B.C. before Euclid

most of Euc I to IV & ~~III~~

Sound & harmony. Arith series, $\sqrt{\ }$, $\sqrt[3]{\ }$

& rational & irrational nos.

Pythag to Euclid = Newton to Einstein.

Few things in last 25 centuries have lasted like that math period. hard abstract thinking "a mental discipline than which nothing greater has been evolved"

Not a few isolated thinkers but many. i.e. Appolonius (conic sections) acknowledges help from many

written records before & after Euclid 900 yrs.

Pract. Mech. appliances: Syphon, Compound pully, engineering, optics & mirrors, Archimedes used statics & Ausglius used pneumatics in water appliances water clocks, torsion machines

Hipparchus 125 B.C. 29 d 12 h 44 m 2½ as Lunar Month. 1" error from modern mean.

Aristotle De Animalibus – accurate

classification basis. He had classed whales among mammals.
See D'Arcy Thompson on Aristotle as a Biologist
Following Anaximander – evolution + variation + descent of man.

Astron. Anaxagoras. Sun a molten mass
Pythag. said each star a sphere.
Democ. " ∞ in no + ∞ no
of atoms composing each.
obliquity of ecliptic +
Plato – impetus to astron.

Arist. spherical nature of earth is proved by shape of its shad. on moon.
~~Anaxim~~ Heraclides said venus rotates about sun.
Aristarchus of Samos heliocentric theory
310 – 230 B.C. + his theory (Copernican) accepted for 75 yrs.
Chorus of disapproval by religion.
He was followed by Eratosthenes
gave our pres. calendar + oblig of ecliptic – meas. circum. of earth. within ½° (~~Obsy at Alex~~
Hipparchus – 1" of length of lunar month
preces. of equinoxes.

Ptolemy (less sightine than any of these) based his writings on Hipp.
but Hip not a heliocentrist.

Copernicus ack. his debt to Aristarchus

Romans - applied science -
Engineering - sewers, tunnels thro
mts -
One of noblest pieces of lit is by Lucretius
Roman imbued with Gk. philos
powerful, unconventional mind -
poem preserved in Constantinople -
Mechanics of [illegible] [illegible]
magnetism / chem of digestion
Gk biology, evolution, etc.
Lyres of light heat sound &
atomic vibrations

Gk & Roman period long & laborious
& covering wide field -
mediaeval ch. by monstrous candle-snuffer
of thought - nothing so bad in Gk times
Sci. to be sci must be pure, abstract
Gk. sci - was a part of Gk imagination -
Curiously that through by the opposite of
anthropomorphic & anthropocentric
Aristotle says to study mind of man we
must study mind of God & dog -
Lucretius also free from anthropomorphic
(Sam Butler says a blade of grass reasons)

Storicism the enemy of Sci & it conquered -
Hebrew influence in modern mind is to
blame for return to anthropomorphism -

II The Dark & Middle Ages. Prof Waugh
27th Jany.

Roger Bacon:-
concealment of real ignor under display of learning
Dark ages & Middle 500 (Fall of Rome) to
1497 Discovery of America.
Pt. 1. Dark ages small pursuit & few addns
to learning. Pt. 2. a rebirth - Rise of
Universities & return to study.
not much sci. advance, - but many alert minds
Phys & Astrny less progressive in middle
ages than other branches.
Mediaeval man practical of mind - heaven
∴ popularity of philos & theology & wealth
or "building" churches - gothic growth
To become priest - eccles. dignitary or trade
& ∴ arith & law -
Inventions plumbing, chimney flues, specs.
& mech clocks & watches - to comfort
Medicine a remunerative pursuit. ∴ biology
Civic law & order
Foretell future - astrology.
Knowledge for its own sake not much
followed - evident even in Roger Bacon

What is Superstition (what one does not believe
oneself - sup to A, religion to B -

Superstitions inherited from Gks.
Aristotle was the philosopher to Mid Ages.
Geocentric - man a microcosm.
4 elements 4 humours hot cold, wet dry.
& his magic from Gks -
Alexander Necham. "Nature of things" book
1st mention of mariners compass & glass
mirrors - undeft mind & criticised
Gen 1. d 1217 abbot of Cirencester

Wm. of Auvergne protested ag. fettering of thought
+ experimental method — Bp of Paris
. Grosteste — magnifying lenses! force
subject to math law He became the
Bp of Lincoln d. 1253

Peter of Spain wrote "Treasury of Poor" sci.
experimental mind — → Pope John 21.
Albert the Gt. d. 1280 — advocate of sci
investigation, nature & experience at school
of Cologne — beatified by R.C.
Roger Bacon Franciscan friar instigated by Pope
John Peckham pupil of R.B. optics
became Archbp Cant.

∴ all sci investigation not opposed.
if central doctrines of Ch not denied —
Abelard (?) great sceptic mind of age but
never denied Ch. — Bernard of Clairvaux
finally got him quieted —
John Wycliffe den. transubstantiation
but only 20 yrs later began to burn Lollards

Persecution a last resort —

Magic + astrol. — denounced by Xtn fathers.
Ptolemaic authority too strong for "
∴ general belief in astrol — but still
free will not denied

Pure publn of Sci. truth never led to his
persecution — It was for magic, heresy etc.
Isolated case —
Execution of Czecho d'Ascoli burned
Florence 1327 but only after warnings —
pushed his astrology too far + to his magic
was in intercourse with Devil —

Failure of mid ages to add to Sci. not due to eccles. oppos.

Service to Sci of Middle ages —
(1) Transmitted Sci lore of ancient times —
(a) direct from Roman empire
Boethius 500 AD The consolation of Philosophy & trans. bits of Aristotle - all West Europe had of Aris for 600 yrs. He also trans. bits of Euclid, land surveying & arithmetic
Cassiodorus 600 AD
Isidore of S — 700 AD
Other writers handed on Zoology etc. astrol & magic writings no more credulous & more critical than those of later Gks —
(2) When Arabs (Moham) 7th cent hostile to learning until 9th cent - then got lore from India - decimal notation their gt scientists were of many nationalities including Jews
no gt. original works — but preserved learning of classical times.
10th & 11th cent Arab Sci entered Christian Europe via Spain in Arab hands 1000 AD —
This influence first in Astrology & later in 11th cent Constantinus Africanus travelled in East & then at Salerno & then a Benedictine monastery & died 1087 (wh the cong [illegible])
translated Arab works — medicine etc
1st of many translators of Arab & ∴ of Gk learning —

Adelard of Bath ^{1200} (Eng) brought this learning over Pyranees -
His version of Euclid from Arabic the best till 16th Cent.
Sci & religion - causes must be sought. Reason above authority -
Adelard in Sci like his contemporary Abelard in Philos. gt influence

Plato of Tivoli - trans Ptolemy's Astron -
Robt. of Chester " Algebra of Arabs
Gerard of Cremona trans Ptolemy's Almagest - Hippocrates & Galen's med & Aristotles sci.
Very many others
Metaphys. of Aris - translator unknown.

(3) Norman Kingdom of Sicily & Naples
3 languages current - & Kings were gt patrons of learning
Archdeacon Aristippus trans Plato
Others did Ptol & Aris -
all direct from Gk.

(4) North Italy - trading cities
chief translating in medicine
but Leonardo of Pisa - Arabic Arith & Algebra - (born in N Africa)
(the abacus - balls strung up)
This book introduced to Europe Arabic numerals (India-Arabic)
first used by Italian merchants
In England for many years Roman numerals used entirely

This recovery of Gk learning threw med ages back on authority

Thus grew up Algebra - (esp. Indian Algebra)
Arabic numerals (also fr. India)

§ Grostates theory of force
Optics - reflection & refraction
disc by Alhazin & Grostate -

Roger Bacon regarded Sci handmaid to theology
no original discov. of fact -
stressed value of languages & criticism
advocated "experimentum" = practical
applica of Sci. theory -
seldom a purposive expmt -
He was not original or solitary in this
but he did stress more the value
of math & physics as foundation
of all learning & need to test theory
by expmt.
No real persecution - & no contemp -
record of his imprisonment.
Famous in his lifetime - possibly
ordered not to write in his old age -

Emp Fred II ruled in Naples & Sc the nearest
to modern sci mind - free from relig -
Experiment - digestion - original language
Nat. hist - menagerie - giraffe - 1st in Europe
Treatise on Falconry - quotes Aristotle &
often contradicts him - real ornithology -
Barnacles - geese - etc -
12 & 13th Interest in Sci died away -
14th Cent no int
15th " revival esp in Math -

Renaissance was slow in accel. pace of sci. Astrol, magic &c to 17th cent
Renais. set authority more firmly in art & lit.
Sci, had to give up classical author.
Sci in Bertrand Russell

III	Copernicus Galileo & Newton.	Feb 3/26
	AHS Gillson	
IV.	Ampere, Oersted, Volta, Ohm	ESB.
V.	Helmholz, Kelvin	Ashwe.
VI.	Light - Subsequent advance	Foster
VII	Heat " "	Shaw
VIII	Sound " "	Reilly
IX.	Elect. & Mag. " "	Keys.
X	Astrophysics	AVD.

Apr. 19 11
Prof. A. N. Whitehead

The Ether of Events

1. The one genus of phys fact
2 Cartesian matt
3. Diff. " subject
4 Organic realism
5 ~~Enduring~~ Enduring relationships ~~organisms & extension~~
6 Space & Time
7 The Enduring organism
8 Cosmological order

Field where space has such photons -

19th Cent. ether pictured as a material entity mass & strain space motion
Activity within matter
does not lead to idea of continuous material ether
Ether a metaphys hypothesis
Showing of a mol in a dist. star clearly affects earth Ether no help for purpose of Sci.

19th Cent. concept of ether phys world a general type with modification -
1 genus of phys facts in phys world
ord mols modificati of cont ether

Substratum idea is artificial –
2. Cartesian materialism
Problem of ether is 1st constructive
task in laying foundation of
phy universe –
Cosmology of mod sci suggested by Descartes
He assents to 1 primary form of
phys facts –
Corporeal nature, Substance,
Extension, Endurance
These have haunted Sci ever since
Extens. is primary characteristic of body
phys body req nothing else but itself
"a community of self contained & exist
entities"

3. Difficulties of Cartesian Subjectivism
Community of nature an essential idea
but no explan^n indiv things
Imposes habits of language on facts
of nature –
Sci as mere description of partic fact
is no good –
Descript of community –
Influence of environment –

Perception – a

Bifurcation – real phys world
+ secondary } mental.
Perception becomes
Conceptivity
Perceptivity } no relation.

Kant accepted Humes sublimation of
Subjectivism of Desc. 1st statement
Desc. phys entities
req. nothing etc

4 Organic realism.

Kant Thoughts sans concepts empty
Intuition " " blind.
Latter fall in mentality –
There is the mental function knowing
blind perceptive function is physical
Complete concrete world is both
mental & physical
Nature is closed to mind in – ?

Mental occasion
Physical occasion – That complete
blind perception of phys world
Whereby phys world is what it is
relationships constitutes phys world

Phys world is a community of
phy. occasions
each occasion an ultimate buck
buk has a non actual factor
entering into mental occas
non actual factors – eternal objects
or universals

they express what
perceptivity

redness
what relation ?
what predicate ? } same answer

ultimate bucks are bound together
by eternal relationship of a
relator.
In Phys sci modern conception of
energy – active flow
vs. passive endurance

5 Primary relationship is extensive
instead of word attribute
Community of phys world is
" of extensive relationships
Notion of organism most fundamental
of all sci.

Common phys. world is
constituted
Perceptive occasion is the
Creativity by wh. it is itself
valuable –
A value something beyond itself –
Relationship of extens most
fundamental
derivative relationship – overlapping &
relationship of whole to part –
Relⁿ of extens fund
in notion of wh –
" " " each entity is
an organism –

6 Space & Time –
Spacialization of extension
Temporalization " "
Dimensions a
Nature never complete – creative advance
hence problem of Time –
Past & future take meaning from
standpoint of given effect
Past – causality.
A new occasion issues from its
environment – relevant gradations

7. Enduring organisms
1 genus < 2 species phys body
empty space
Desc. denied latter
i.e. a material ether –
Organic realism occasions with
& without phys atoms
Worldline of successive occasions
is host of electron
Individualism of creative activity

17
i.e. change of value along world line
– Environment draws its character
from the enduring organisms near
it
Stellar universe concentration
of analogous organisms favorable
to one another's existence —
Old doctrine space & time individual
new – electron

8. Electron behaves as it does bec
it is what it is
Ultimate order of world aesthetic
Electrons – outside realm of phys?
Phys laws are the creation of
the entities
electrons, mols, stars
they impose on nature the
laws of their own nature
Sci no promise of permanence
Sun 250 10^6 tons / min loss

Passage of time adventure
toward new realities -

Decay -

1 Univ physically wasting

2 " ascending to higher types

Slowness in conceivable to new
Conditions perhaps ultimately to develop
into ripple barely

distinguishable from monotony

Unimaginable Past -

" future

final phase of order
upon whose order all
phase of order depend

MacTaggart Spiritual time (not
antagonistic to Einstein's viewpoint
from physical aspect)

Structure an abstraction from full-blooded universe –
Extension fundamental.
Space & time a phase in creative progress
Pure physics worldlines of electrons not independent –
What electron is in itself depends on all other electrons –
i.e. mols in magnet oriented due to Earth mag field –
Hence an electron is perception –

I in knowing myself have a perspective of a common world –
Leibnitzian view of perspectives but each occasion replaces monads of Leibnitz

Visited Ottawa May 19 - 21 - 1926

Roy. Soc. Can.
McLennan, Gray, Beatty, Lash Williams -

Harper : 180 standard A stars 400 plates
correlation curves not yet complete.

A stars 29 % all stars. Gets linear
relation between Abs. mag.